

# **Risk Sharing and Redistribution in a Federal System with Population Mobility**

by

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## Abstract

This paper considers the implications of population mobility for risk sharing among individuals and among regions of a federation. There is an important interaction between risk sharing and interregional redistribution which precludes the regional authorities from fully exploiting gains from interregional risk sharing when population mobility is imperfect. However, the conditions characterising risk sharing arrangements among the individuals within each region in the Nash equilibrium correspond to those of the central authority. Finally, all gains from interregional risk sharing are fully exploited with perfect mobility.

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## I. Introduction

Studying the properties of risk sharing arrangements among countries has long been one of the concerns of macroeconomists. One strand in this literature (e.g., Lucas (1982) and Obstfeld (1995)) has, by and large, abstracted from government policies and has concentrated on the determination of asset prices. Another strand (e.g., Persson and Tabellini (1992a,b)), on the other hand, has abstracted from asset markets and has concentrated on the types of policies that governments should follow in order to maximize the welfare of their residents through risk sharing among individuals and among countries.

Persson and Tabellini's work is inspired by the recent discussions in the European Union (EU) regarding the optimal assignment of policy functions to different levels of government. They are, thus, concerned with identifying the reasons why decentralised risk sharing arrangements may be undesirable. To address these issues, they consider a federation consisting of two regions. Individuals in each region are subject to a region specific shock, and an individual specific shock. There is no population mobility. The regional authorities choose government expenditures and the level of risk sharing among the individuals in their region (intraregional risk sharing) through the appropriate tax/transfer policies. The federal government, on the other hand, decides on the level of interregional risk sharing and redistribution by the appropriate interregional transfers.

In this paper I examine the incentives that *population mobility* imparts on the regional authorities to choose the degree of risk sharing and redistribution in the federation. Unlike in Persson and Tabellini, here I allow the regional authorities to choose not only the degree of intraregional risk sharing, but also the degree of interregional risk sharing and redistribution.

This is an important extension of Persson and Tabellini's work because the possibility

of population mobility is one of the defining characteristics of a federation. The degree of population mobility will vary among federations. For example, it will be higher in Canada than in the EU, because the latter consists of more culturally diverse nationalities, with the members of each community exhibiting different degrees of attachment to home.

With population mobility, the regional authorities will take specific account of the effects of their policies on the distribution of population in the federation. The important implications of this has been recently emphasised by Myers (1990), Hercowitz and Pines (1991), and Mansoorian and Myers (1993). The present paper is most closely related to the latter (henceforth, referred to as MM). MM develop a model in which individuals derive varying degrees of non-pecuniary benefits from residing in the two regions that form the federation. Their attachment to home model adds imperfect mobility to the standard regional model in a tractable way.<sup>1</sup> Their important conclusion is that in the standard fiscal externality environment of Flatters, Henderson and Mieszkowski (1974) the regional authorities will make the interregional transfers that are necessary to obtain an efficient distribution of population in the federation. Thus, in that environment there is no efficiency role for a central authority. The central authority's role is to decide on the degree of interregional redistribution in the federation.<sup>2</sup>

I assume that, as in Persson and Tabellini, each individual faces a probability of being unlucky (or unemployed) and thus having no endowments. The endowments they receive if they are lucky (or employed) depends on the population size in their region. Moreover, the

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<sup>1</sup> See Wildasin (1986) for a discussion of the standard regional model.

<sup>2</sup> The MM model has been extended by Burbidge and Myers (1994a) and Wellisch (1994) in order to identify other sources of inefficiency with decentralisation.

proportion of the population that is lucky in the two regions are stochastic and independent of each other. Also, as in MM, individuals have varying degrees of attachment to the two regions. I show that the regional authorities will provide the efficient degree of intraregional risk sharing. I show, however, that with imperfect population mobility some mutually beneficial gains from interregional risk sharing are not fully exploited by the regional authorities.<sup>3</sup>

To understand the source of this inefficiency it is important to make a distinction between interregional risk sharing and interregional redistribution. Interregional risk sharing involves interregional transfers that are contingent on the state of nature. Interregional redistribution, on the other hand, involves interregional transfers that are not contingent on the state of nature; and are designed to increase the expected utility in one region at the expense of the other. Interregional risk sharing and redistribution are intertwined. With attachment to home, regional authorities disagree over the degree of interregional redistribution in the federation; each region desires a larger degree of redistribution in favour of its own residents. In a stochastic setting, each region can affect the degree of redistribution in favour of its residents only in the states in which it experiences a more favourable shock than the other region, and therefore decides on the size of the interregional transfers. Because they do not cooperate with each other in deciding on the size of these state contingent interregional

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<sup>3</sup> Persson and Tabellini justify abstracting from private financial markets in their models by arguing that, because it is costly for financial institutions to observe in this kind of setup whether an individual is lucky or unlucky, private insurances may not be feasible. The government, on the other hand, needs to observe output produced by firms (who employ lucky individuals); and can tax this output and redistribute it through lump sum subsidies to all individuals. Thus, the informational problems will not be so severe for governments.

transfers, this disagreement among them prevents them from exploiting all gains from interregional risk sharing.<sup>4</sup>

With no attachment to home this kind of inefficiency does not arise, because then free mobility implies that the same level of expected utility should prevail throughout the federation. Thus, in that case regional authorities do not disagree over the degree of interregional redistribution.

In another extreme case, if the regions are so asymmetric that one of the regions makes transfers to the other region in *all* states of nature then the degree of interregional risk sharing and redistribution is determined by that region *alone*. In that case again all gains from interregional risk sharing are fully exploited in the Nash equilibrium, because one of the regions is essentially passive in the determination of the degree of interregional risk sharing and redistribution.

The paper is organised as follows. The model is set out, and the Nash equilibrium is discussed in section II. The problem of the central authority is discussed in section III. Some concluding remarks are made in section IV.

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<sup>4</sup> In a recent paper, Burbidge and Myers (1994b) consider a model in which there are two types of individuals with different abilities. In their deterministic model there is perfect mobility by individuals of each type, and the regional authorities disagree over the degree of redistribution between the two types within each region (*intra-regional* redistribution). This disagreement leads to an inefficient outcome in their model. (The related papers by Wildasin (1991, 1995) are also concerned with intra-regional redistribution). The reason for inefficiency in the present model is fundamentally different. In the present model all individuals are identical before the state of nature is revealed. Moreover, both regional authorities are in complete agreement as to the degree of redistribution between the lucky and the unlucky within their region: both authorities treat the individuals equally, guaranteeing them equal levels of consumption in each state. The regional authorities in the present model disagree over the degree of *inter-regional* redistribution when population is imperfectly mobile, each wanting a higher income for all of its own residents in every state. (Persson and Tabellini (1992b) also concentrate on interregional redistribution.)

## II. The Nash Equilibrium

The federation consists of two regions, indexed by  $i$  ( $i=1, 2$ ). We consider an endowment economy which has much in common with that used by Persson and Tabellini.  $p_i$  is the fraction of the population in region  $i$  that are lucky and receive  $R(N_i)$  units of the good, where  $N_i$  is the population size in the region and  $R'(N_i) < 0$ .<sup>5</sup>  $(1 - p_i)$  is the fraction of the population in the region that are unlucky and receive nothing.

We assume that  $p_i$  can take on one of two values,  $\bar{p}$  or  $\underline{p}$ , with  $\bar{p} > \underline{p}$ . The state of nature is denoted by  $(p_1, p_2)$ . There are two possible states:  $(\bar{p}, \underline{p})$  with probability  $B(\bar{p}, \underline{p})$ , and  $(\underline{p}, \bar{p})$  with probability  $B(\underline{p}, \bar{p})$ . Of course,  $B(\bar{p}, \underline{p}) + B(\underline{p}, \bar{p}) = 1$ . Henceforth, for any variable  $z$  that depends on  $(p_1, p_2)$  we will denote  $\tilde{z} = z(p_1, p_2)$ .

The sequence of events are as follows. First policies contingent on the states of nature are announced. Then,  $(p_1, p_2)$  is chosen by nature. After this, individuals decide where to reside, a fraction  $p_i$  of the population that decide to reside in region  $i$  become lucky, a fraction  $(1 - p_i)$  become unlucky, and the policies that were promised are implemented.

Population size in the federation is normalised to unity. Thus, in each state  $(p_1, p_2)$  we have

$$N_1 + N_2 = 1. \quad (1)$$

Intraregional risk sharing in each region  $i$  is determined by the regional authority's choice of the consumption of the lucky ( $\tilde{c}_i$ ) and the unlucky ( $\tilde{b}_i$ ) in different states, through the

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<sup>5</sup> Persson and Tabellini, on the other hand, assume that  $R'(N_i) = 1 - \alpha N_i$ . The assumption  $R'(N_i) < 0$  captures the notion of congestion. Its role in the present model is discussed in footnote 7 below.

appropriate tax/transfer policies. Interregional risk sharing, on the other hand, is determined by their choice of the state contingent interregional transfers. The transfers from region  $i$  to region  $j$  in each state  $(p_1, p_2)$  are denoted by  $S_i$ .

In each state  $(p_1, p_2)$  region  $i$  has the resource constraint

$$N_i p_i \mathbf{R}(N_i) = N_i p_i \tilde{c}_i + (1 - p_i) N_i \bar{b}_i + S_i - S_j, \quad (2)$$

where the left hand side is total endowments in region  $i$  in that state, while the elements on the right hand side are, respectively, total consumption by the lucky, total consumption by the unlucky, transfers paid to the other region, and transfers received from the other region.

Next, consider the determination of the distribution of population in the federation. Suppose state  $(p_1, p_2)$  is given. Then for any individual residing in region  $i$  there is a probability  $p_i$  that he will be lucky and receive a consumption of  $\tilde{c}_i$ . There is also a probability  $(1 - p_i)$  that the same individual will be unlucky, and have a consumption of  $\bar{b}_i$ . The utility individuals derive from consuming  $x$  units of the good is given by  $U(x)$ , which is a von Neumann-Morgenstern utility function. Thus, the expected value of  $U(x)$  for any individual residing in region  $i$ , conditional on  $(p_1, p_2)$ , is

$$V_i(p_1, p_2) = p_i U(\tilde{c}_i) + (1 - p_i) U(\bar{b}_i). \quad (3)$$

To discuss the implications of the degree of population mobility, I assume that, as in MM, individuals derive non-pecuniary benefits from residing in the two regions. They are, moreover, heterogeneous only with respect to their degrees of attachment to the two regions. There is one individual of each type, denoted by  $n$ , and individuals are distributed uniformly over the interval  $[0, 1]$ . Individual  $n$  will derive a non-pecuniary benefit of  $k(1 - n)$  if he resides in region 1, and  $kn$  if he resides in region 2.  $k$  measures the degree of population mobility:

$k=0$  implies perfect mobility, and  $k=4$  implies no mobility.

After the state  $(p_1, p_2)$  is revealed the total expected utility for individual  $n$  will be  $V_1 + k(1-n)$  if he resides in region 1, and  $V_2 + kn$  if he resides in region 2. Individuals are free to choose their region of residence. Thus, in equilibrium, after  $(p_1, p_2)$  is revealed, there will be one individual, denoted by  $N$ , that will be indifferent between living in either region. For this marginal individual we will have

$$V_1 + k(1-N) = V_2 + kN. \quad (4)$$

All individuals with  $n < N$  will choose to reside in region 1, while all others will reside in region 2. Hence, from (1), we will have

$$N_1 = N, \text{ and } N_2 = 1 - N. \quad (5)$$

Now  $(p_1, p_2)$  is  $(C, \$)$  with probability  $B(C, \$)$  and  $(\$, C)$  with probability  $B(\$, C)$ .

Hence, using (3), the *unconditional* expected value of  $U(@)$  for any individual who resides in region  $i$  in both states of nature (referred to as a permanent resident of region  $i$ ) will be

$$W_i = B(C, \$) V_i(C, \$) + B(\$, C) V_i(\$, C), \quad (6)$$

I assume that the objective of the regional authority  $i$  is to maximize  $W_i$  by choosing the optimal degrees of intra- and inter-regional risk sharing. I choose this objective function for the regional authorities for the following reasons. First, this is the simplest objective function, and it allows me to compare my results with those of MM, where the regional authorities also maximize the part of the utility of their residents that is from consumption alone. Second, in the two extreme cases of no attachment ( $k=0$ ) and complete attachment ( $k=4$ ) this is the most sensible objective function for the regional authorities. In either of these cases, by maximizing

$W_i$  the regional authority will be maximizing the utility of a representative resident. Finally, if we assume that at the most only one third of the population in the whole federation migrates from one region to the other as we move from one state of nature to the other, then it would follow that by maximizing  $W_i$  the regional authority  $i$  would be maximizing the expected utility of its median voter in all states.

The problem of regional authority  $i$  is to choose  $\tilde{b}_i$ ,  $\tilde{c}_i$ , and  $S_i$  to maximize  $W_i$  subject its resource constraint (2), and the migration equilibrium conditions (4), and (5) for all  $(p_1, p_2)$ . In addition, it has the constraint  $S_i \geq 0$ , which prevents it from forcing the other region to make transfers to it. To perform this optimisation problem, first solve (2) for  $\tilde{b}_i$ , substitute the result into (3), and use (5) to eliminate  $N_i$ , to obtain  $V_i$  as a function of  $\tilde{c}_i$ ,  $S_i$ ,  $S_j$ , and  $N$ . Then substitute for  $V_i$  into (6) to obtain  $W_i$  as a function of the same variable. The optimality conditions for this problem can then be derived as

$$\frac{\partial W_i}{\partial \tilde{c}_i} \% \frac{\partial W_i}{\partial N} \frac{\partial N}{\partial \tilde{c}_i} = 0, \quad \forall (p_1, p_2), \quad (7)$$

$$\frac{\partial W_i}{\partial \tilde{S}_i} \% \frac{\partial W_i}{\partial N} \frac{\partial N}{\partial \tilde{S}_i} \neq 0, \quad \tilde{S}_i \geq 0, \quad \text{and} \quad (8)$$

$$\left[ \frac{\partial W_i}{\partial \tilde{S}_i} \% \frac{\partial W_i}{\partial N} \frac{\partial N}{\partial \tilde{S}_i} \right] \tilde{S}_i = 0, \quad \forall (p_1, p_2). \quad (9)$$

The migration responses that appear in these conditions are obtained from (4). As discussed above,  $V_i$  can be expressed as a function of  $\tilde{c}_i$ ,  $S_i$ ,  $S_j$ , and  $N$ . Hence, for each state  $(p_1, p_2)$ , (4) describes  $N$  as an implicit function of  $\tilde{c}_i$ , and  $S_i$  ( $i=1, 2$ ). Totally differentiating it, we obtain

$$\frac{\tilde{M}\tilde{N}}{\tilde{M}\tilde{c}_1} \cdot \left[ p_1 \frac{U'(\tilde{c}_1) \& U'(\tilde{b}_1)}{\tilde{J}^*} \right], \quad (10)$$

$$\frac{\tilde{M}\tilde{N}}{\tilde{M}\tilde{c}_2} \cdot \left[ p_2 \frac{U'(\tilde{c}_2) \& U'(\tilde{b}_2)}{\tilde{J}^*} \right], \quad (11)$$

and

$$\frac{\tilde{M}\tilde{N}}{\tilde{M}\tilde{S}_1} \cdot \left[ \frac{\tilde{M}\tilde{N}}{\tilde{M}\tilde{S}_2} \cdot \frac{U'(\tilde{b}_1)/\tilde{N} \& U'(\tilde{b}_2)/(1 \& \tilde{N})}{\tilde{J}^*} \right], \quad (12)$$

where  $\tilde{J}$  is the determinant of the Jacobian of the migration equilibrium in a particular state

$(p_1, p_2)$ , and should be negative for stability (as in Boadway (1982)). We have  $\tilde{J}$

$\tilde{J} = \tilde{J}_1 \& \tilde{J}_2 \& 2k$ , where

$$\tilde{J}_1 = \frac{\tilde{M}\tilde{W}_1}{\tilde{M}\tilde{N}} \cdot U'(\tilde{b}_1) \left[ p_1 \mathbf{R}'(\tilde{N}) \& \frac{\tilde{S}_1 \& \tilde{S}_2}{\tilde{N}^2} \right] \quad (13)$$

and

$$\tilde{J}_2 = \frac{\tilde{M}\tilde{W}_2}{\tilde{M}\tilde{N}} \cdot U'(\tilde{b}_2) \left[ \& p_2 \mathbf{R}'(1 \& \tilde{N}) \& \frac{\tilde{S}_1 \& \tilde{S}_2}{(1 \& \tilde{N})^2} \right]. \quad (14)$$

Substituting from (10) and (11) into (7), noting that  $\tilde{M}\tilde{W}_i/\tilde{M}\tilde{c}_i = \mathbf{B}p_i [U'(\tilde{c}_i) \& U'(\tilde{b}_i)]$  and  $\tilde{M}\tilde{W}_i/\tilde{M}\tilde{N} = \mathbf{B}\tilde{J}_i$ , we can show that

$$\tilde{c}_i = \tilde{b}_i \quad \text{for all } (p_1, p_2), \quad (i = 1, 2). \quad (15)$$

Hence, in the Nash equilibrium we will have complete risk sharing among the individuals within each region, regardless of the degree of population mobility (i.e., regardless of the value of  $k$ ).

To work out the degree of interregional risk sharing in the Nash equilibrium substitute from (12) into (8), noting that  $\tilde{M}\tilde{W}_i/\tilde{M}\tilde{S}_i = \frac{\mathbf{B}}{\tilde{N}_i} U'(\tilde{b}_i)$ , to obtain

$$\frac{\tilde{\mathbf{7}}_1}{1 \& \tilde{\mathbf{N}}} U'(\tilde{\mathbf{b}}_2) \% \frac{\tilde{\mathbf{7}}_2}{\tilde{\mathbf{N}}} U'(\tilde{\mathbf{b}}_1) \$ \frac{\&2kU'(\tilde{\mathbf{b}}_1)}{\tilde{\mathbf{N}}}, \quad (16)$$

and  $\mathbf{S}_1 \$ 0 \in (p_1, p_2)$  for region 1, and

$$\frac{\tilde{\mathbf{7}}_1}{1 \& \tilde{\mathbf{N}}} U'(\tilde{\mathbf{b}}_2) \% \frac{\tilde{\mathbf{7}}_2}{\tilde{\mathbf{N}}} U'(\tilde{\mathbf{b}}_1) \# \frac{2kU'(\tilde{\mathbf{b}}_2)}{1 \& \tilde{\mathbf{N}}}, \quad (17)$$

and  $\mathbf{S}_2 \$ 0 \in (p_1, p_2)$  for region 2.<sup>6, 7</sup>

Our next task is to determine whether the central authority can offer a degree of intra- and inter-regional risk sharing that will increase the value of the objective function of at least one regional authority without reducing the other.

### III The Problem of the Central Authority

The central authority will choose  $\tilde{c}_1, \tilde{c}_2, \tilde{b}_1, \tilde{b}_2$ , and the net transfers from region 1 to 2,  $\mathbf{S}$  ( $/ \mathbf{S}_1! \mathbf{S}_2$ ), in order to maximize a weighted sum of the objectives of the two regional authorities,  $\ast \mathbf{W}_1 + (1 - \ast) \mathbf{W}_2$  for  $\ast \in [0, 1]$ , subject to (2), (4) and (5) for all  $(p_1, p_2)$ . As discussed above, using (2), (3), (5) and (6),  $\mathbf{W}_i$  can be expressed as a function of  $\tilde{c}_i, \mathbf{S}_i, \mathbf{S}_j$ , and  $\tilde{\mathbf{N}}$ . Thus, the optimality conditions for this problem can be written as:

$$\ast \left[ \frac{\mathbf{M}\mathbf{W}_1}{\mathbf{M}\tilde{c}_i} \% \frac{\mathbf{M}\mathbf{W}_1}{\mathbf{M}\tilde{\mathbf{N}}} \frac{\mathbf{M}\tilde{\mathbf{N}}}{\mathbf{M}\tilde{c}_i} \right] \% (1 - \ast) \left[ \frac{\mathbf{M}\mathbf{W}_2}{\mathbf{M}\tilde{c}_i} \% \frac{\mathbf{M}\mathbf{W}_2}{\mathbf{M}\tilde{\mathbf{N}}} \frac{\mathbf{M}\tilde{\mathbf{N}}}{\mathbf{M}\tilde{c}_i} \right] = 0, \quad (i = 1, 2) \in (p_1, p_2), \quad (18)$$

and

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<sup>6</sup> The policies described by equations (14)-(16) are time consistent, for the following reason. Once  $(p_1, p_2)$  is revealed regional authority  $i$  will be maximizing  $\mathbf{V}_i$ , subject to (2), (4), (5), and  $\mathbf{S}_i \$ 0$ , by choosing  $\tilde{b}_i, \tilde{c}_i$ , and  $\mathbf{S}_i$  for that particular state. As  $\mathbf{M}\mathbf{W}_i/\mathbf{M}\tilde{\mathbf{X}} = \mathbf{B}\mathbf{M}\mathbf{V}_i/\mathbf{M}\tilde{\mathbf{X}}$  for any variable  $\tilde{\mathbf{x}}$  in state  $(p_1, p_2)$ , conditions (7)-(9) will hold for all  $i$  after  $(p_1, p_2)$  is revealed.

<sup>7</sup> To see the role played by the assumption  $\mathbf{R}\mathbf{N} < 0$  note that if  $\mathbf{R}\mathbf{N} = 0$  then from (13) and (14) at  $\mathbf{S}_1 = \mathbf{S}_2 = 0$  we will have  $\mathbf{7}_1 = \mathbf{7}_2 = 0$ , and so  $\mathbf{M}\mathbf{W}_i/\mathbf{M}\tilde{\mathbf{N}} = \mathbf{B}\mathbf{7}_i = 0$ . Now  $\mathbf{M}\mathbf{W}_i/\mathbf{M}\mathbf{S}_i = \mathbf{B}\mathbf{U}(\tilde{b}_i)/\mathbf{N}_i < 0$ . Thus, if  $\mathbf{R}\mathbf{N} = 0$  then, from (8) and (9), no transfers will be made by either region in any state.

$$* \left[ \frac{\mathbb{W}_1}{\mathbb{S}} \% \frac{\mathbb{W}_1}{\mathbb{N}} \frac{\mathbb{N}}{\mathbb{S}} \right] \% (1 \& *) \left[ \frac{\mathbb{W}_2}{\mathbb{S}} \% \frac{\mathbb{W}_2}{\mathbb{N}} \frac{\mathbb{N}}{\mathbb{S}} \right] = 0, \quad \forall (p_1, p_2). \quad (19)$$

Substituting from (10) and (11) into (18), noting that  $\mathbb{W}_i/\mathbb{N} = \mathbf{B}\mathbf{7}_i$ ,  $\mathbb{W}_i/\mathbb{S}_j = 0$  for  $j \neq i$ , and  $\mathbb{W}_i/\mathbb{S}_i = \mathbf{B}p_i [U^*(\tilde{c}_i) \& U^*(\tilde{b}_i)]$ , it can be shown that  $\tilde{c}_i = \tilde{b}_i$  ( $i=1,2$ ) for all  $(p_1, p_2)$ . Hence, with the central authority we will have complete risk sharing among individuals in both regions, as in the Nash equilibrium.

Now substitute from (12) into (19), and use the fact that  $\mathbb{W}_1/\mathbb{S} = \frac{\mathbf{B}}{\mathbb{N}} U^*(\tilde{b}_1)$  and  $\mathbb{W}_2/\mathbb{S} = \frac{\mathbf{B}}{1 \& \mathbb{N}} U^*(\tilde{b}_2)$  to obtain

$$\frac{\tilde{\mathbf{7}}_1}{1 \& \mathbb{N}} U^*(\tilde{b}_2) \% \frac{\tilde{\mathbf{7}}_2}{\mathbb{N}} U^*(\tilde{b}_1) = * \frac{\mathbf{B}2kU^*(\tilde{b}_1)}{\mathbb{N}} \% (1 \& *) \frac{2kU^*(\tilde{b}_2)}{1 \& \mathbb{N}} \quad \forall (p_1, p_2). \quad (20)$$

This is not consistent with (16) and (17) for all  $(p_1, p_2)$  when population is only imperfectly mobile (i.e., when  $k > 0$ ). To see this suppose, for simplicity, that in state  $(\mathbf{C}, \mathbf{\$})$  region 1 will make a transfer to region 2, because it has experienced a more favourable shock (recall that  $\mathbf{C} > \mathbf{\$}$ ). Thus, in that state (16) will hold with equality, and (17) with inequality. By the symmetry of the model, it follows that then in state  $(\mathbf{\$}, \mathbf{C})$  region 2 will be making a transfer to 1. Thus, in that state (17) will hold with equality and (16) with inequality. There is no  $*$  that will be consistent with these results. If  $* = 1$  then (20) will be consistent with (16) and (17) in state  $(\mathbf{C}, \mathbf{\$})$ , but not in state  $(\mathbf{\$}, \mathbf{C})$ . Similarly, if  $* = 0$  then (20) will be consistent with (16) and (17) in state  $(\mathbf{\$}, \mathbf{C})$ , but not in  $(\mathbf{C}, \mathbf{\$})$ . Finally, if  $0 < * < 1$  then (20) will be

inconsistent with (16) in state  $(C, S)$  and inconsistent with (17) in  $(S, C)$ .<sup>8</sup>

The extreme case of no population mobility ( $k=4$ ) will further illustrate the point. In that case the migration responses (12) will all be zero. Thus, as  $\partial W_i / \partial S_i < 0$ , from (8) and (9), we will have  $S_1 = S_2 = 0$ , for all  $(p_1, p_2)$ , and no interregional risk sharing. However, by the symmetry of the model we know that complete interregional risk sharing will be a Pareto superior allocation.

To see the reason for inefficiency in the Nash equilibrium first note that interregional risk sharing and interregional redistribution are inextricably linked. Interregional redistribution involves interregional transfers that are independent of the state  $(p_1, p_2)$ , and are designed to increase the expected utility in one region at the expense of the other. Interregional risk sharing, on the other hand, involves transfers that are contingent on the state, and are, moreover, *coordinated* across states.

With attachment, regional authorities disagree over the degree of interregional redistribution. To see this, abstract completely from interregional risk sharing by considering a world in which there is only one possible state  $(p_1, p_2)$ . Then the problem of region 1 will be to maximize  $V_1$  subject to (4), and the other constraints. Region 1 will thus, in effect, be maximizing  $V_2 + 2kN/k$ . On the other hand, region 2 will be maximizing  $V_2$ . Clearly, region 1 will desire a larger  $N$  than region 2. In such a world the degree of interregional redistribution

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<sup>8</sup> If in the Nash equilibrium no region makes a transfer in either state, then (16) and (17) will hold with strict inequalities in all states. It is clear that for each state we will be able to find a  $0 < \epsilon < 1$  such that (20) will hold for that particular state at the Nash equilibrium allocation. However, the  $\epsilon$ 's that we would require for different states will not be the same, again because there will be a coordination failure, as the regional authorities disagree over the degree of interregional redistribution.

in the Nash equilibrium will be determined by the rich region *alone*, through its choice of the transfer. In that environment, strategic interaction will not result in an inefficient outcome essentially because the poor region is passive in the game (it is constrained by  $S_i \geq 0$ ).<sup>9</sup> (See MM, p. 129, for a full discussion.)

On the other hand, in a world with more than one possible state of nature we need some *coordinated* state contingent transfers in order to exploit gains from interregional risk sharing. In the present, symmetric, model the state contingent transfers should flow from region 1 to 2 in state  $(C, \$)$  (when in the Nash equilibrium the expected marginal utility of consumption in region 1 is lower), and from 2 to 1 in state  $(\$, C)$  (when in the Nash equilibrium the expected marginal utility of consumption in region 2 is lower). As the regional authorities disagree over the degree of interregional redistribution in the federation, they disagree over the sizes of these transfers in different states. Each region determines the size of the transfer in the state in which it experiences a more favourable shock than the other region. Thus, the authorities' disagreement over interregional redistribution leads to a failure to coordinate the state contingent transfers, and, as a result, some gains from interregional risk sharing are not exploited.

The regional authorities cannot trade state contingent claims to overcome this problem, because they will not honour such claims once the state of nature is revealed (see footnote 6). For trading in such state contingent claims to work, we will need a central authority to enforce

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<sup>9</sup> With only one state of nature we can always choose a  $\alpha \in [0,1]$  such that (20) is satisfied, and is consistent with (16) and (17), at the Nash equilibrium allocation. If in the Nash equilibrium region 1 makes a transfer then we set  $\alpha = 1$ ; and if region 2 makes a transfer then  $\alpha = 0$ . If in the Nash equilibrium neither region makes a transfer then we can choose a  $0 < \alpha < 1$ .

them. But, then, once this is done the state contingent transfers will in effect be made by the central authority.

Some extreme cases will serve to further highlight the source of inefficiency in this model. First consider the case of an economy with asymmetric regions. Suppose one of the regions (region 1, say) is so rich that it makes transfers to region 2 in *all* the states. Then (16) will hold with equality, and (17) with inequality, in all the states. Then (20) will hold at the Nash equilibrium allocation with  $\alpha = 1$ . In this case the degree of interregional redistribution (and, hence, of interregional risk sharing) will be determined by region 1 (the rich region) alone. Region 2 will be essentially passive in the game, as in the deterministic model. Thus, in this case strategic interaction between the regions will not lead to inefficient degrees of risk sharing.

Finally, consider the case of perfect mobility (i.e., when  $k=0$ ). Then, (16) and (17) will imply (20): there will be only one efficient allocation, and the Nash equilibrium will coincide with it. In this case the regional authorities know that with free mobility the same level of expected utility will prevail throughout the federation. They, thus, do not disagree over the degree of interregional redistribution; and there is no coordination failure of the type described above.

#### **IV. Conclusions**

This paper has studied the risk sharing arrangements we may have in a federal system with mobile population. It was shown that in the Nash equilibrium there will be complete risk sharing among the individuals within each region. There is an important interaction between

interregional risk sharing and redistribution. With imperfect population mobility regional authorities disagree over the desired level of interregional redistribution in the federation. In general, each regional authority can affect the degree of interregional redistribution only in the states in which it experiences a more favourable shock than the other region. This disagreement results in an inefficient degree of interregional risk sharing in the Nash equilibrium.

However, if population mobility is perfect then the regional authorities will not disagree over the degree of interregional redistribution. Thus, in that case the Nash equilibrium will involve an efficient degree of interregional risk sharing.

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