Fiscal Externalities with Imperfect Population Mobility: The Three Region Case

by

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Abstract

We consider a federation with three regions and an imperfectly mobile population. If in the Nash equilibrium one region makes transfers to the other two then the outcome is efficient. If in the Nash equilibrium two regions make transfers to a third region the outcome may be inefficient, because each of the transfer making regions ignores the effect of its transfers on migration out of the other rich region, and the resulting benefit to that region. Nevertheless, the Nash equilibrium is efficient when the two transfer making regions are strongly tied together by migration.

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I. Introduction

The traditional model of a federal system with population mobility in the fiscal federalism literature includes two regions with a homogeneous population that is perfectly mobile.[1] Flatters Henderson and Miezkowski (1974) (FHM), using such a model, argued that there is need for interregional transfers in order to obtain an efficient distribution of population in the federation. Myers (1990), after explicitly modelling the behaviour of the regional authorities in the FHM framework, showed that in their model the regional authorities have the incentive to make the efficient transfers voluntarily even if one allows for more than two regions.

The result obtained by Myers hinges on the assumption of a perfectly mobile population, and the associated implication that in equilibrium the same level of utility should prevail throughout the federation. This implies that the regional authorities will not disagree over the distribution of resources in the federation, and thus make the transfers which are necessary for efficiency.

Mansoorian and Myers (1993) (henceforth referred to as MM) extended the fiscal externality model with two regions to incorporate imperfect population mobility. In their model individuals derive different degrees of non-pecuniary benefits from residing in the two regions (attachment to home).[2] This imperfect mobility precludes the same level of utility from prevailing throughout the federation. Hence, because the regional authorities are concerned

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[1] An alternative formulation has a continuum of regions. See, for example, Wilson (1987). Recent surveys of the literature are provided by Wildasin (1986), and Mieszkowski and Zodrow (1989).

[2] Hercowitz and Pines (1991) have an alternative (pecuniary) formulation of migration costs.

with the welfare of different subsets of the population, they disagree over the distribution of resources in the federation. MM show that in their model the regional authorities will still make the necessary transfers to obtain an efficient distribution of population in the federation. The reason for their result is that with two regions the transfers should flow from one region to the other. Thus, the rich region alone decides on the distribution of resources in the economy, while the poor region is passive in the game. (See MM, p. 129.)[3]

The next important step in extending the traditional model of a federal economy is to work out the implications of imperfect population mobility in a model with more than two regions. This is the purpose of this paper. We extend the MM model to incorporate a third region.

The three region framework with imperfect population mobility gives rise to the potential for a richer game between the regional authorities. In this framework we obtain results that correspond closely to that derived in MM, and also contrasting results. With respect to the former, we show that if one of the regions in the federation is so rich that in equilibrium it makes voluntary inter-regional transfers to the other two regions then the resulting Nash equilibrium will be efficient. The reason for this is that in this case, as in MM, the rich region alone decides on the distribution of resources in the economy. The other two regions are passive in the game, as they are constrained by the condition that they cannot make negative transfers.

We also show, however, that if two of the regions are relatively rich then it is possible

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[3] Of course, it is possible that neither region decides to make a transfer. MM show that even such a Nash equilibrium will be efficient.

that the Nash equilibrium will be inefficient. This possibility arises from the strategic interaction (or externalities) between the two relatively rich regions. When two regions have to make transfers to the third region, each of these regions ignores the fact that if it withholds part of its transfers it will induce some migration into the other rich region, reducing utility of the residents in that region. Thus, it is possible that, starting from an efficient set of transfers for such a federation, each of the two relatively rich regions will have an incentive to withhold part of its transfers, making the efficient allocation untenable as a Nash equilibrium.[4]

We prove the inefficiency result for the case in which the two rich regions are more closely tied to the poor region through migration, than they are to each other. When we consider the alternative case in which migration ties the two rich regions together we find efficient Nash equilibria. The logic underlying this result is that strong migration ties lead to strong incentive ties among the rich regions. An extreme illustration would be when migration leads to equalized utility (or incentive equivalence) among the rich regions.

The paper is organised as follows. The model is presented in Section II, the properties of the Nash equilibrium are discussed in Section III. Some concluding remarks are made in Section IV.

II. The Model

The model is the same as that used in MM, except for the inclusion of a third region.

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[4] This is reminiscent of the intuition that was provided by Stiglitz (1977, Section II.2) for an inefficient outcome in the traditional fiscal externality model with a perfectly mobile population. Myers (1990), however, showed that Stiglitz's intuition will not hold in that model. Below we show that even with imperfect mobility this logic may not hold.

The three regions that form the federation are indexed by i ($i=1,2,3$). There are two factors of production, labour and land. As in MM, the regional authorities own the land in their regions (T_i), and decide on the head taxes and the interregional transfers. Population size in the federation is normalised to unity. Each individual is endowed with one unit of labour, and can reside in any region that he desires.

Individuals derive utility from consumption of a single private good. They also derive non-pecuniary benefits from residing in a region. Individuals are heterogeneous only with respect to their degrees of attachment to the regions. There is one individual of each type, denoted by n; and individuals are distributed uniformly over the interval [0,1]. Specifically, the preferences of individual n are given by

$$V^n = \begin{cases} U(x_1) + (1 - n) & \text{if he lives in region 1} \\ U(x_2) & \text{if he lives in region 2} \\ U(x_3) + n & \text{if he lives in region 3,} \end{cases} \tag{1}$$

where x_i is his consumption if he resides in region i, while $(1-n)$ and n are the non-pecuniary benefits he derives from residing in regions 1 and 3, respectively. Region 2 does not yield non-pecuniary benefits to anyone.

Output in region i is produced by competitive firms with a constant returns to scale production function $f_i(N_i,T_i)$, where N_i is the population size in the region. An individual residing in region i receives a wage equal to the marginal product of labour, denoted by $F_i(N_i, T_i)$, and pays a head tax of τ_i. Thus, his budget constraint is $x_i=F_i - \tau_i$.

As in MM, we allow the regional authorities to make voluntary interregional transfers to control the population size in their regions. The transfers from region i to j are denoted by

S_{ij}. The regional authority i faces the budget constraint

$$S_{ij} + S_{ik} = N_i\tau_i + R_i + S_{ji} + S_{ki} \qquad j \neq i \text{ and } k \neq i \tag{2}$$

where R_i denotes total land rents ($R_i = f_i - N_iF_i$).

Using the individuals' budget constraints, and the budget constraints of the authorities, one can readily derive the regional feasibility conditions

$$x_i = \frac{f_i - (S_{ij} - S_{ji}) - (S_{ik} - S_{ki})}{N_i} \quad \forall\ i, \text{ and } i \neq j,\ i \neq k, \text{ and } j \neq k. \tag{3}$$

With free location choice only one individual (denoted by n_1) will be indifferent between living in regions 1 and 2; and only one individual (denoted by n_2) will be indifferent between living in regions 2 and 3. Thus, we will have

$$U(x_1) + (1 - n_1) = U(x_2) \tag{4}$$

for individual n_1, and

$$U(x_2) = U(x_3) + n_2 \tag{5}$$

for individual n_2.

Individuals with $n \in [0, n_1]$ will reside in region 1, those with $n \in (n_1, n_2]$ will reside in region 2, and all others will reside in region 3.[5] Hence,

$$N_1 = n_1,\ N_2 = n_2 - n_1, \text{ and } N_3 = 1 - n_2. \tag{6}$$

To derive the migration responses, substitute from (3) and (6) into (4) and (5) and totally differentiate the resulting system to obtain

[5] If there is migration out of regions 1 or 3 it flows into region 2. Thus, regions 1 and 3 are not as closely tied to each other as are regions 1 and 2, or regions 2 and 3.

$$\begin{bmatrix} \mathbf{7}_1+\mathbf{7}_2-1 & -\mathbf{7}_2 \\ -\mathbf{7}_2 & \mathbf{7}_2+\mathbf{7}_3-1 \end{bmatrix} \begin{bmatrix} dn_1 \\ dn_2 \end{bmatrix} = \begin{bmatrix} \dfrac{U_1'}{n_1}+\dfrac{U_2'}{(n_2-n_1)} \\ \dfrac{-U_2'}{(n_2-n_1)} \end{bmatrix} dS_{12} + \begin{bmatrix} \dfrac{U_1'}{n_1} \\ \dfrac{U_3'}{(1-n_2)} \end{bmatrix} dS_{13} + \begin{bmatrix} \dfrac{-U_2'}{(n_2-n_1)} \\ \dfrac{U_3'}{(1-n_2)}+\dfrac{U_2'}{(n_2-n_1)} \end{bmatrix} dS_{23}, \quad (7)$$

where, U_i' denotes the derivative of $U(x_i)$, and $\mathbf{7}_i = U_i' \dfrac{F_i - x_i}{N_i}$. The determinant of the Jacobian of the system is $|A| = (\mathbf{7}_1 - 1)(\mathbf{7}_2 + \mathbf{7}_3 - 1) + \mathbf{7}_2(\mathbf{7}_3 - 1)$. For stability of the migration equilibrium $|A|$ should be positive.[6] The migration responses to policy change are obtained by applying Cramer's rule to this system. These migration responses will be used by the regional authorities when they decide on their desired policies.

The problem of regional authority i is to maximize $U(x_i)$ subject to (3)-(6) and $S_{ij} \geq 0$, for all $j \neq i$. Note that by maximizing $U(x_i)$ regional authority i maximizes the utility of everyone who ends up living in its region, as the non-pecuniary benefits that individuals derive from residing in a region is just a parameter in its utility function.[7] The optimality conditions for this problem are

$$\frac{\partial U_i}{\partial S_{ij}} \equiv \frac{-U_i'}{N_i} + \mathbf{7}_i \frac{\partial N_i}{\partial S_{ij}} \leq 0, \qquad S_{ij} \geq 0, \qquad \text{and} \qquad (8)$$

$$\frac{\partial U_i}{\partial S_{ij}} S_{ij} = 0 \quad \forall j \neq i.$$

The next natural step would be to substitute the migration responses into these

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[6] A sufficient condition for this is that the regions head subsidize (i.e., $\mathbf{J}_i = F_i - x_i < 0$). MM (footnote 15) show that $\mathbf{J}_i < 0$ always holds in their model.

[7] See Mansoorian and Myers (1995) for a discussion of the importance of the choice of governmental objective functions in models with mobile population.

conditions in order to provide a more specific characterization of the Nash equilibrium. However, we will take a more direct route to proving our results. Hence, at this point we turn to the problem of the central authority.

II. The Central Authority

The central authority chooses the net interregional transfers $\bar{S}_{12}$ ($\equiv S_{12} - S_{21}$), $\bar{S}_{13}$ ($\equiv S_{13} - S_{31}$), and $\bar{S}_{23}$ ($\equiv S_{23} - S_{32}$) to maximize a weighted sum of the utilities of everyone in the economy subject to (3)-(6).[8] Its objective function is

$$L = \int_0^{n_1} \omega_n \left[U(x_1) + (1 - n)\right] dn + \int_{n_1}^{n_2} \omega_n U(x_2)\, dn + \int_{n_2}^{1} \omega_n \left[U(x_3) + n\right] dn,$$

where ω_n is the weight attached to the welfare of agent n.

The optimality conditions for this problem are:

$$\frac{\partial L}{\partial \bar{S}} = \delta_1 \frac{\partial U_1}{\partial \bar{S}} + \delta_2 \frac{\partial U_2}{\partial \bar{S}} + \delta_3 \frac{\partial U_3}{\partial \bar{S}} = 0 \qquad \left(\bar{S} = \bar{S}_{12}, \bar{S}_{13}, \bar{S}_{23}\right), \tag{9}$$

where

$$\delta_1 = \int_0^{n_1} \omega_n dn, \qquad \delta_2 = \int_{n_1}^{n_2} \omega_n\, dn, \qquad \text{and} \qquad \delta_3 = \int_{n_2}^{1} \omega_n\, dn.$$

Clearly, $\delta_i \geq 0$ (i=1, 2, 3), and $\delta_1 + \delta_2 + \delta_3 = 1$. The conditions in (9) give us a set of transfers from which it is impossible to make a change without reducing U_i for some i.

The following lemma will be very useful in proving our results:

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8 From the definition of $\bar{S}_{ij}$ it is clear that the derivative of any variable with respect to $\bar{S}_{ij}$ will be the same as its derivative with respect to S_{ij} (ij=12, 13, and 23).

<u>Lemma 1</u>: *The system of equations in (9) are linearly dependant: satisfaction of two equations implies satisfaction of the third.*

<u>Proof</u>: We first prove that

$$\frac{\partial U_i}{\partial S_{12}} + \frac{\partial U_i}{\partial S_{23}} - \frac{\partial U_i}{\partial S_{13}} = 0 \qquad (i = 1, 2, 3). \tag{10}$$

Consider the case for i=1. Note that $\partial U_1/\partial S_{12}$ and $\partial U_1/\partial S_{23}$ are given in (8), while $\partial U_1/\partial S_{23} = \lambda_1 \partial N_1/\partial S_{13}$. Substituting these results, using (6) and applying Cramer's rule to (7) to obtain the appropriate migration responses, one finds that (10) will hold for i=1. Similarly, one can prove (10) for i=2 and 3. Next, substitute for $\partial U_i/\partial S_{13}$ (i=1,2,3) from (10) into $\partial L/\partial \bar{S}_{13}$ to derive the result that if $\partial L/\partial \bar{S}_{12} = \partial L/\partial \bar{S}_{23} = 0$ then $\partial L/\partial \bar{S}_{13} = 0$. $\blacksquare$

The intuition for Lemma 1 is that with balanced budgets any net transfers between the three regions can be achieved by any two of $\{\bar{S}_{12}, \bar{S}_{23}, \bar{S}_{13}\}$. Equipped with this lemma, we now proceed to analyze the main implications of the model.

<u>Proposition 1</u>: *If in the Nash equilibrium one region alone makes transfers to the other two then the resulting equilibrium will be efficient.*

<u>Proof</u>: Without loss of generality suppose in the Nash equilibrium region 1 makes transfers to regions 2 and 3. Then we will have $\partial U_1/\partial S_{12} = \partial U_1/\partial S_{13} = 0$. The question is whether we can find a set of δ's that will make the optimality conditions described in (9) consistent with this Nash equilibrium. If we set $\delta_1 = 1$, $\delta_2 = \delta_3 = 0$, and use Lemma 1, it will follow that $\partial L/\partial \bar{S}_{12} = \partial L/\partial \bar{S}_{13} = \partial L/\partial \bar{S}_{23} = 0$ at the Nash equilibrium allocation. $\blacksquare$

The logic for the result described in this proposition is analogous to the logic for the result obtained in MM. Consider the efficient allocation where $\delta_1 = 1$, and S_{12} and S_{13} are positive. This allocation can be supported as a Nash equilibrium because no region would have any incentive to deviate from it. Region 1 will not deviate because the allocation maximizes U_1. Regions 2 and 3 would like to deviate, but are constrained by the condition that they cannot make negative transfers. Thus, in this case the "rich" region alone decides on the distribution of resources.[9]

Proposition 2: *If in the Nash equilibrium two regions are making transfers to a third region then it is possible that the outcome will not be efficient.*

Proof: In Appendix 1 we prove this proposition assuming that in the Nash equilibrium regions 1 and 3 make transfers to region 2. ~

If in the Nash equilibrium regions 1 and 3 make transfers to region 2 then $MU_1/MS_{12}=MU_3/MS_{32}=0$ while, as shown in Appendix 1, $MU_i/MS_{12}>0$ for i=2 and 3, and $MU_j/MS_{32}>0$ for j=1 and 2. Thus, starting from the Nash equilibrium, one can increase U_i for i=1, 2, and 3 by appropriately increasing S_{12} and S_{32}. The externality at work here can be explained as follows. Rich regions make transfers because they are overpopulated. In the present model when regions 1 and 3 are rich each of them ignores the fact that when it makes a transfer to region 2 it necessarily reduces the population size in the other rich region, and thus imparts external benefits to that region.

It is important, however, to emphasize that two regions making transfers to a third in the Nash equilibrium will not necessarily imply inefficiency. Notice that the economy used in the proof of Proposition 2 was special in that the rich regions were not connected to each other

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[9] Throughout this paper we use "rich" region(s) to denote the region(s) which is (are) inefficiently overpopulated in the absence of transfers.

by a marginal individual. We have chosen this setup for its simplicity, and its ability to highlight the externality involved in the most direct way.

To fully work out the limitations of Proposition 2 we would need a much more complete model, in which *all* regions may be tied together by direct migration. The driving force for an efficient outcome in such a complete model would be a high degree of mobility between the two rich regions (1 and 3), as compared to the rich regions and the poor region. Then if, for example, region 1 makes a transfer to 2 it will lead migrants out of region 3 into region 2, as before. But now, in addition, there will be migration from 1 into 3, as the transfer makes region 1 relatively poorer. If this second effect on the population size of region 3 is not dominated then the larger transfer from 1 to 2 will not benefit region 3. In that case Pareto improvements may not be possible, and the Nash equilibrium would be efficient. Thus, strong migration ties between the rich regions will lead to strong incentive ties between them, and they will make the efficient transfers voluntarily.

Although the present model cannot fully capture this type of outcome, it can give rise to a Nash equilibrium with similar characteristics. Suppose regions 1 and 2 were the rich regions. Then, the rich regions will be more closely tied to each other by migration than the rich regions and the poor region (region 3). The reason is that now there is no direct migration between regions 1 and 3 (also see footnote 5). This will result in an efficient Nash equilibrium. The following Proposition is somewhat stronger than required to show this.

Proposition 3: *The outcome of a Nash equilibrium in which region 2 is making a transfer to one of the other regions will be efficient, regardless of whether there is another region that is also making a transfer.*

<u>Proof</u>: Suppose region 2 makes a transfer to 3, and thus $\partial U_2/\partial S_{23}=0$. Then, totally differentiating (4) and using (6), we obtain

$$\frac{\partial U_1}{\partial S_{23}} = \frac{\partial N_1}{\partial S_{23}}. \tag{11}$$

But $\partial U_1/\partial S_{23}=\lambda_1 \partial N_1/\partial S_{23}$, while in Appendix 2 we prove that in such an equilibrium $\lambda_1 \neq 1$. Hence, for (11) to hold we must have $\partial N_1/\partial S_{23}=0$, which implies that $\partial U_1/\partial S_{23}=0$. With $\partial U_1/\partial S_{23}=\partial U_2/\partial S_{23}=0$, we will have $\partial L/\partial \bar{S}_{23}=0$ if we set $\delta_3=0$.

Our next task is to show that we can choose $0\leq\delta_1\leq 1$, and $\delta_2=1-\delta_1$ such that $\partial L/\partial \bar{S}_{13}=0$; that is, $\delta_1 \partial U_1/\partial S_{13}+(1-\delta_1)\partial U_2/\partial S_{13}=0$. It would suffice to show that $\partial U_1/\partial S_{13}$ and $\partial U_2/\partial S_{13}$ cannot both be positive or negative. From (8) $\partial U_1/\partial S_{13}\leq 0$. Now, as $\partial U_2/\partial S_{23}=0$, from (10) $\partial U_2/\partial S_{13}=\partial U_2/\partial S_{12}$. Also, from (8) $\partial U_2/\partial S_{12}=-\partial U_2/\partial S_{21}\geq 0$. Thus, $\partial U_2/\partial S_{13}\geq 0$, and so $\partial U_2/\partial S_{13}$ and $\partial U_1/\partial S_{13}$ cannot both be positive or negative.

Hence, if we set $\delta_3=0$ we can always choose $\delta_1\in[0, 1]$ and $\delta_2=1-\delta_1$ to make $\partial L/\partial \bar{S}_{13}=\partial L/\partial \bar{S}_{23}=0$ at the allocation resulting from a Nash equilibrium in which region 2 makes a transfer to region 3, regardless of whether there is another region that is also making a transfer. $\square$

An implication of Proposition 3 is that if in the Nash equilibrium regions 1 and 2 are both making transfers to 3 ($\partial U_2/\partial S_{23}=\partial U_1/\partial S_{13}=0$) then the outcome will be efficient, with $\delta_1=1$. The reason for this result is that regions 1 and 2 have strong incentive ties. Migration into region 1 is from region 2; and all migration from 3 should flow through 2. This is, therefore, a special case of the more general model discussed above, because in it we have severed the direct migration ties between one of the rich regions and the poor region. The fundamental reason for the efficient outcome is the more direct migration ties between the two rich regions than between the rich regions and the poor region.[10]

IV. Conclusion

The purpose of this paper has been to extend the MM model of a federation with imperfect population mobility to incorporate a third region. With such an extension we obtained results that were analogous to the one obtained in MM, and also contrasting results. The number of regions which have to make transfers in order to obtain an efficient allocation, and the way in which migration ties regions together, play important roles in determining whether the Nash equilibrium is efficient.

If there is more than one region that has to make a transfer for efficiency then the outcome with decentralization may be inefficient, essentially because each rich region ignores the effect of its transfer on the migration out of the other rich region. This inefficiency result was proven for the case in which the rich regions had relatively stronger migration ties with the poor region than with each other. When the migration ties between the rich regions were stronger the externality in question was internalised by the rich regions, and the Nash equilibrium was efficient. The MM framework, by abstracting from this externality, allows one to evaluate the importance of other externalities that may arise in a federation.

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[10] It is this strong migration, and incentive, ties between regions 1 and 2 that explains the result that $MU_1/MS_{23}=0$ when $MU_2/MS_{23}=0$, which was fundamental to the proof of Proposition 3. This result implies that regions 1 and 2 agree on the size of S_{23}.

Appendix 1

In this Appendix we provide a proof of Proposition 2. Suppose in the Nash equilibrium regions 1 and 3 make transfers to region 2. Then in the Nash equilibrium we will have $\partial U_1/\partial S_{12}=0$ and $\partial U_3/\partial S_{32}=0$. Hence, from (9) we can see that for the Nash equilibrium to be efficient we should be able to find a set of $\delta_i \geq 0$ $(i=1, 2, 3)$ and $\delta_1+\delta_2+\delta_3=1$ such that at the Nash equilibrium allocation

$$\delta_2 \frac{\partial U_2}{\partial S_{12}} + \delta_3 \frac{\partial U_3}{\partial S_{12}} = 0, \tag{A.1}$$

and

$$\delta_1 \frac{\partial U_1}{\partial S_{23}} + \delta_2 \frac{\partial U_2}{\partial S_{23}} = 0. \tag{A.2}$$

To show that we cannot find such a set of δ's we will show that at the Nash equilibrium allocation $\partial U_2/\partial S_{12}$ and $\partial U_3/\partial S_{12}$ are strictly positive, while $\partial U_1/\partial S_{23}$ and $\partial U_2/\partial S_{23}$ are strictly negative.

To show that $\partial U_2/\partial S_{12}>0$ first note that from (4) and (6)

$$\frac{\partial U_2}{\partial S_{12}} = \frac{\partial U_1}{\partial S_{12}} - \frac{\partial N_1}{\partial S_{12}}. \tag{A.3}$$

We know that $\partial U_1/\partial S_{12}=0$. This, from (8), implies that $U_{N}^{1}/N_1=\lambda_1 \partial N_1/\partial S_{12}$. Hence, $\partial N_1/\partial S_{12}\neq 0$, and from (A.3) $\partial U_2/\partial S_{12}\neq 0$. Now from (8) $-\partial U_2/\partial S_{12}=\partial U_2/\partial S_{21}\leq 0$.[11] It must, therefore, be the case that $\partial U_2/\partial S_{12}>0$.

Similarly, to show that $\partial U_2/\partial S_{23}<0$ first note that (5) and (6) imply that

[11] As for all $i\neq j$ S_{ij} and S_{ji} enter the model in the net form $S_{ij}-S_{ji}$, $\partial U_i/\partial S_{ij}=-\partial U_i/\partial S_{ji}$. This result comes up several times in this appendix, and should be born in mind from the outset.

$$\frac{MU_2}{MS_{23}} \; ' \; \frac{MU_3}{MS_{23}} \; \& \; \frac{MN_3}{MS_{23}}. \qquad \text{(A.4)}$$

But we know that $MU_3/MS_{32}=0$. This, from (8), implies that $U_3N/N_3=\mathbf{7}_3MN_3/MS_{32}$. Hence, $MN_3/MS_{23}...0$, and from (A.4) $MU_2/MS_{23}...0$. Now from (8) $MU_2/MS_{23}\#0$. It must, therefore, be the case that $MU_2/MS_{23}<0$.

To show that $MU_1/MS_{23}<0$ first note that, as $MU_1/MS_{12}=0$, from (10) $MU_1/MS_{23}= MU_1/MS_{13}$. We know from (8) that $MU_1/MS_{13}\#0$. Thus, if we prove that $MU_1/MS_{23}...0$ then we will have shown that $MU_1/MS_{23}=MU_1/MS_{13}<0$. The proof will be by contradiction. Note that $MU_1/MS_{23}=\mathbf{7}_1MN_1/MS_{23}$. For this to be zero we will need $MN_1/MS_{23}=0$. The reason is that $MU_1/MS_{12}=0$ (as $S_{12}>0$), which implies, from (8), that $U_1N/N_1=\mathbf{7}_1MN_1/MS_{12}$ and so $\mathbf{7}_1...0$. Suppose $MN_1/MS_{23}=0$. From (4) and (6)

$$\frac{MU_2}{MS_{23}} \; ' \; \frac{MU_1}{MS_{23}} \; \& \; \frac{MN_1}{MS_{23}}. \qquad \text{(A.5)}$$

We have already shown that $MU_2/MS_{23}<0$. Hence, from (A.5), if $MN_1/MS_{23}=0$ then it must be that $MU_1/MS_{23}<0$. Thus, by contradiction MU_1/MS_{23} cannot be zero: it must be negative.

Finally, we need to show that $MU_3/MS_{12}>0$. As $MU_3/MS_{32}=0$, from (10) $MU_3/MS_{12}= MU_3/MS_{13}$. We know from (8) that $MU_3/MS_{13}=!MU_3/MS_{31}\$0$. Thus, if we prove that $MU_3/MS_{12}...0$ then we will have shown that $MU_3/MS_{12}=!MU_3/MS_{31}>0$. The proof will again be by contradiction. Note that $MU_3/MS_{12}=\mathbf{7}_3MN_3/MS_{12}$. For this to be zero we will need $MN_3/MS_{12}=0$. The reason is that $MU_3/MS_{32}=0$ (as $S_{32}>0$), which implies, from (8), that $U_3N/N_3=\mathbf{7}_3MN_3/MS_{32}$ and so $\mathbf{7}_3...0$. Suppose $MN_3/MS_{12}=0$. From (5) and (6)

$$\frac{MU_2}{MS_{12}} \; ' \; \frac{MU_3}{MS_{12}} \; \& \; \frac{MN_3}{MS_{12}}. \qquad \text{(A.6)}$$

We have already shown that $MU_2/MS_{12}>0$. Hence, from (A.6), if $MN_3/MS_{12}=0$ then it must be that $MU_3/MS_{12}>0$. Thus, by contradiction MU_3/MS_{12} cannot be zero: it must be positive.

We have shown that if in the Nash equilibrium regions 1 and 3 make transfers to region 2 then it must be the case that $MU_i/MS_{12}>0$ for $i=2$ and 3, while $MU_j/MS_{23}<0$ for $j=1$ and 2. It is thus impossible to find a set of *****'s to satisfy (A.1) and (A.2) at the Nash equilibrium allocation. ~

Appendix 2

In this Appendix we prove that if region 2 makes a transfer to 3 ($MU_2/MS_{23}=0$) then **7**$_1…1$. This result was used in the proof of Proposition 3.

The proof will be by contradiction. Let **7**$_1=1$. Then $MU_1/MS_{13}=\,!\,U_1^{)}/N_1+MN_1/MS_{13}$. This, together with (4) and (6), imply that

$$\&\frac{U_1^{)}}{N_1}\;'\;\frac{MU_2}{MS_{13}}. \tag{A.7}$$

Now, from (10), if $MU_2/MS_{23}=0$ then $MU_2/MS_{13}=MU_2/MS_{12}$. This and (A.7) imply that in such an equilibrium we should have $MU_2/MS_{12}=\,!\,U_1^{)}/N_1$; and, therefore, $MU_2/MS_{21}=\,!\,MU_2/MS_{12}>0$. But this is inconsistent with (8). Thus, by contradiction **7**$_1…1$. ~

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