Capital Structure with Opportunistic Stakeholders’ Coalitions

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Abstract

This paper shows that stakeholders’ multilateral opportunistic behaviour during financial distress may lead to premature liquidation of the firm. Consequently, the firm will use its capital structure to mitigate the costs of such opportunism. Specifically, the firm will reduce its debt so that the probability of multilateral opportunism is zero; namely, it will use only safe debt. The paper predicts that the debt-equity ratio will decrease with risk, the number of contracts, the difficulty in writing them and in achieving franchising arrangements, the supplier’s importance in opportunistic coalitions and a decrease in the firm’s size, or the supplier’s adjustment costs.

Key Words: Incomplete Contracts, Opportunistic Behaviour, Bankruptcy, Capital Structure.

JEL Classification: (D0, C7, G3, L2)

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1 Introduction

A vast literature on capital structure examines the interactions between real and financial markets. A common theme in this literature is that a firm may use its capital structure as an instrument to deal with various imperfections arising from agency problems, informational asymmetries, regulation, non-competitive markets, unions, etc.\(^1\)

More recently, the stakeholder theory of capital structure has been discussed in several theoretical and empirical studies.\(^2\) The basic idea behind this theory is that a firm is, essentially, the collection of its stakeholders (workers, suppliers, creditors, customers, equity holders, etc.) and the relationships among them. The stakeholder theory asserts that the financial decision affects the relationships among these stakeholders, thus affecting the value of the firm. For example, Titman (1984) argued that, due to “switching costs” (e.g., the costs of locating new business partners), the liquidation of a bankrupt firm hurts long term prospects of its suppliers and customers.\(^3\) Naturally, these costs will be taken into account by the suppliers and customers. Consequently, the volume of business with highly leveraged firms which are exposed to bankruptcy risk will be reduced in the presence of such switching costs. The fear of losing long term relationships due to post-bankruptcy switching costs, in turn, will prompt some firms to reduce levels of debt.

Following Titman (1984), the role of financing decisions in shaping relationships and bargaining power among stakeholders has been extensively discussed in the literature (see references in footnote 2). Most of the theoretical papers in this area focus on the impact of debt financing on stakeholders’ bargaining power in the case of bankruptcy and liquidation. A common feature in this literature is that each stakeholder acts alone to formulate strategic plans. The possible formation of coalitions of stakeholders is, therefore, not recognized in the analysis of a firm’s choice of its capital structure.

In reality, however, alliances are often attractive. By its mere size and leverage, a coalition of stakeholders may be able to attain more power and hence extract greater benefits than individual stakeholders can. The emergence of stakeholders’ coalitions and their consequences for the firm becomes most important in the face of bankruptcy, or even the mere possibility of bankruptcy\(^4\). Furthermore, as is well known, when agents cannot write complete contracts, switching costs and specific investments may create hold-up problems which may lead to opportunistic behaviour in “long term relationships” (see, for example, Hart (1995)).\(^5\) Such hold-up problems become much

\(^1\)The earlier literature on this subject began with Jensen and Meckling (1976) and was developed by a series of papers in the 80s and 90s. See Harris and Raviv (1991) and Allen and Winton (1995) for an early summary of this literature. For a more recent survey see Frank and Goyal (2008).

\(^2\)The following represents a small part of the growing literature in this area. Bronars and Deree (1991), Dasgupta and Sengupta (1993), Perotti and Spier (1993), Hennessy and Livdan (2009) and Matsa (2009) provide theoretical analysis, whereas Titman and Wessels (1988), Kale and Shahru (2007) and Banerjee, Dasgupta and Kim (2008), among others, provide empirical studies of the prevalence of low levels of debt. For a recent survey of this literature, see Parsons and Titman (2008).

\(^3\)The implications of switching costs and specific investments were first introduced in the Industrial Organization literature. See, for example, Williamson (1975).

\(^4\)In fact, as will be shown below, a coalition may even be able to force an opportunistic bankruptcy.

\(^5\)See also Hart and Moore (1998) who discuss a model of incomplete contracts with default and renegotiations,
more important and difficult to deal with in multilateral relationships. First of all, with multiple contracts, it is necessary to take into account the interactions of all contractual arrangements between the parties. Second, and more interesting, with multiple contracting parties we may face multilateral opportunistic behaviour.\textsuperscript{6} For example, under certain circumstances, opportunistic behaviour may lead some of the parties to form a coalition whose aim is to exploit the excluded parties.\textsuperscript{7} This implies that two new difficulties need to be addressed: (i) opportunistic coalitions may not be easy to prevent and (ii) initial contract cannot include, or specify, the terms of a potential future contract between the members of a defectors’ coalition.\textsuperscript{8} In this sense, the inability to control (defectors’) potential future contracts, makes initial contracts among the parties “more incomplete”. Consequently, efficiency is harder to achieve.

The primary objective of this paper is to introduce the possibility of the formation of opportunistic coalitions and to examine the impact on the choice of capital structure. The paper shows that multilateral opportunistic behaviour and the consequent ex-post conflicts among groups of stakeholders, especially in times of financial distress, introduce a previously unrecognized cost of debt financing, thus affecting the choice of capital structure. This is a novel source of endogenous bankruptcy costs associated with debt financing which has not been identified in the literature; it is the direct consequence of the possibility of multilateral opportunistic behavior.

Our model is partially motivated by numerous examples in which such coalitions were actually formed. One of the most commonly cited examples is the case of the Olympia and York Inc. Olympia and York filed for Chapter 11 bankruptcy, but a breakaway coalition of creditors, suppliers and other initial stakeholders took over the Canary Wharf venture in London, UK, and eventually drove out its original equity holders.

Other instances of opportunistic behaviour, bargaining and conflicts among sub-groups of stakeholders (which, at times, resulted in strategic bankruptcy) include the following: (i) the tussles involving LTV and its senior and junior creditors (NY Times, February, 15, 1992), (ii) the opportunistic multiple party bankruptcy bargaining, involving Wheeling-Pittsburgh Steel Corporation, its United Steelworkers of America workers and its creditors, (iii) the case of Campeau where suppliers pulled the plug and triggered bankruptcy (NY Times January 9, 1990), (iv) the recent case of GM, where a debt-equity swap deal was initially rejected by a tough bargaining position by creditors and unions (The Wall Street Journal, May, 28, 2009), can also be viewed

where the lender has foreclosure rights. They show that it is possible that the entrepreneur may not be able to prevent foreclosure in some situations. Other related papers are: Chiu (1998), who shows that with an incomplete contract, the possibility of loss of ownership may actually increase the asset loser’s investment incentive and Che and Hausch (1999) who look at “cooperative” investment and show that with unavoidable renegotiations, contracting may be abandoned altogether.

\textsuperscript{6}And of course, informational asymmetries make the problem much more complex. Since this paper assumes symmetric information, we do not pursue this here.

\textsuperscript{7}The problem of multilateral opportunistic behaviour is closely related to the general question of equilibrium coalition formation. Specifically, in a multilateral contracting environment, a coalition’s payoff often depends on its composition, as well as the composition of its complement. This is referred to as the problem of “externalities”. At this point, there is no general model in the literature on coalition formation that adequately deals with such externalities. For a discussion see: Bloch, F. (1996), Shenoy (1979), Yi (1997), Ray and Vohra (1999), Ray (2007). For a discussion of externalities in the context of multiple contracts, see Segal (1999).

\textsuperscript{8}In the same way that a (first) marriage contract cannot specify the terms of a potential future (second) marriage contract (that will be signed if the first marriage breaks down and the parties re-marry).
as implicit coalitional bargaining between parties in times of distress, (v) the case of Skeena Cellulose Inc., that after severe financial woes, partially attributed to a creditors-labour coalition, was taken over by its creditors, who later formed a joint venture with the BC government (as the majority stakeholder, thus in a sense, it was bailed out by the taxpayers)\(^9\) and (vi) the 1982 Johns-Manville (a major asbestos manufacturer), declaration of insolvency in order to avoid paying claims resulting from exposure to its products.\(^10\)

To examine the impact of possible opportunistic coalitions on the firm’s capital structure, we examine the firm’s decisions within the framework of a multi-stage contracting game between equity holders, debt holders and a “supplier” of a non-contractible, firm-specific input.\(^11\) While equity holders sign an complete contract with creditors, the contractual arrangement with suppliers is incomplete. The incompleteness of the contract with the supplier is due to three related problems. Namely, the firm’s inability to: (i) specify what/how much will be supplied, (ii) exclude the possibility of an opportunistic coalition with debt holders, (iii) control the outcome (contract) in a (potential future) supplier/debt holders opportunistic coalition.

We examine the nature of the subgame perfect Nash equilibrium of the contracting game and show that the firm will, indeed, use its capital structure to reduce the cost of opportunistic behaviour by its multiple contracting parties. We show that a lower debt reduces the probability of an opportunistic supplier/debt holders coalition, thus reducing the cost of the incompleteness. This, in turn, provides the supplier with a stronger incentives to supply the correct level of the firm-specific input.

Our main conclusion is, therefore, that when a firm depends on its stakeholders for procurement of crucial, non-substitutable inputs, which are not completely contractible and hence can lead to opportunistic coalitions, the use of debt becomes less attractive. Consequently, equity holders may either avoid issuing debt as a source of external financing, or at best they will maintain a lower debt-equity ratio. Our conclusion is in accord with the empirical literature.\(^12\)

The mechanism that creates a bias against the use of debt is intuitively very simple. A firm’s suppliers of scarce, firm-specific, non-substitutable and non-contractible inputs are, effectively,

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9 Upon insolvency, Skeena Cellulose Inc's ownership was transferred to Royal Bank and TD Bank. The Royal Bank did not want to be involved in operating Skeena, and was willing to sell its shares in the company. Eventually, after what seemed to be a controversial decision, the BC Government became a major stakeholder in a joint venture with the TD Bank, known as SCI Ventures. The BC government first became involved with Skeena Cellulose Inc in the 1997/98 fiscal year.

10 Continental Airlines' use of strategic bankruptcy threats to extract union concessions and Texaco's, attempts to use strategic insolvency to avoid damages payments to its archrival Pennzoil may be two other examples.

11 We use the term “supplier” to represent a third party to the contract, who may be an actual supplier, a manager, investor, partner, or other “upstream/downstream” participant in the firm’s activities, who provides an input/service/investment that can be used by the firm. The contribution of this party is referred to as an “input”.

12 A fairly large number of empirical papers (including Titman and Wessels (1988), Qian (2003), Strebulaev and Yang (2006), Strebulaev (2007), Kale and Shahru (2007) and Banerjee, Dasgupta and Kim (2008), among others) have examined the relationship between a firm and its primary stakeholders, such as suppliers and customers, and studied the effects of such relationships on the choice of capital structure. A major finding of these papers is that, controlling for other factors, firms that produce durable goods, use specialized and non-substitutable inputs and depend heavily on suppliers or supply chains for procurement of such inputs, tend to use relatively smaller amount of debt as a source of external financing.
junior creditors with considerable clout because they give rise to a hold-up problem. Namely, they can behave opportunistically, by withholding inputs needed for uninterrupted production and supply of finished goods and services. Such opportunistic behaviour can push the firm towards bankruptcy when the payoff to a coalition between the supplier and senior creditors, under bankruptcy, exceeds the payoff to a coalition with equity holders with no bankruptcy. In other words, the presence of a senior claimant (debt holder) allows a junior creditor (supplier) to engage in multilateral opportunistic behaviour, at a time when a firm is on the verge of financial distress. This may force equity holders to lose ownership, control and security benefits associated with the firm. Anticipating such an outcome, equity holders of firms that use such specialized, non-substitutable and non-contractible inputs may avoid issuing debt altogether. But, more generally (even if there is no corner solution), the marginal benefits of issuing debt (such as tax shields) will now be counterbalanced by the expected marginal cost of opportunistic behaviour. Consequently, such firms will issue less debt.\textsuperscript{13}

In contrast to our model, the existing literature in the context of the buyer-supplier nexus\textsuperscript{14} shows that debt enhances bargaining power of equity holders, because it disciplines suppliers, workers and other stakeholders via threat of bankruptcy. The conclusions reached in this literature hold when a firm issues public debt, so that its debt holders are numerous and disperse and thus cannot communicate with suppliers/junior creditors. On the other hand, if debt holders and suppliers are few in numbers and private (banks, for example), then they can communicate and form an opportunistic coalition, thus representing a credible threat (of pulling the plug) to equity holders. This paper shows that when stakeholder can form coalitions, the bargaining advantage of debt may disappear.

In addition to the general conclusion that equity financing becomes more attractive when facing multilateral opportunism, we also show that a firm’s capital structure decision is intrinsically linked to contractual arrangement with its stakeholders. Specifically, our model gives rise to the testable predictions that the debt-equity ratio will decrease with (i) the number of interdependent contracts, (ii) the difficulty of writing contracts with the suppliers, (iii) the importance of the supplier of the non-contractible input in an opportunistic coalition, (iv) the difficulties in achieving optimal franchising arrangements, (v) an increase in risk (in the first-order-stochastic-dominance, or mean-preserving-spread sense), (vi) a reduction in the firm’s size (as measured by its assets) and (vii) the cost of supply adjustment facing the supplier.

Finally, it should be noted that, in general, the literature on the hold-up problem\textsuperscript{15} focuses primarily on unilateral opportunistic behaviour, whereas this paper deals with the multilateral

\textsuperscript{13}The hold-up problems, leading to multiparty opportunistic coalition against equity holders is relevant when a firm is close to (but not in) bankruptcy. Situations like this have less bite if a firm is either in very good condition so that it can fulfil its obligations to all clients, or if it is in very bad shape, which forces liquidation resulting in payments to creditors according to seniority rules.

\textsuperscript{14}See Brander and Spencer (1989), Brander and Lewis (1986), Appelbaum (1993), Bronars and Deree (1991), Dasgupta and Sengupta (1993), Perotti and Spier (1993). In a recent paper, for example, Hennessy and Livdan (2009) show that the optimal leverage is the outcome of the trade-off between equity holders’ increased bargaining power due to higher debt and the overhang effect of higher debt which exacerbates the incentive problem by shrinking the set of feasible contracts to workers.

relationships in an incomplete contract setting. Moreover, while there are some studies of multilateral contractual problems, the emphasis of this literature is either on strategic default\(^{16}\), or on the probability of liquidation with complete contracts\(^{17}\) and not on the choice of capital structure with incomplete contracts\(^{18}\).

The plan of the paper is as follows. Section 2 outlines the time line of the model. In sections 3 and 4, we discuss the post-bankruptcy and multilateral bargaining games. Sections 5 and 6 present the supplier’s problem and the optimal contract with the supplier. Section 7 discusses the contract with debt holder and the choice of capital structure. Section 8 provides a conclusion.

## 2 The Structure and Time-line of the Game

Consider a relationship between a firm and a supplier. The supplier provides an input service that can be used in the production process. We assume that the input service is non-verifiable and consequently, not contractible.

The firm’s gross profits, \(\overline{R}\), are given by the function

\[
\overline{R} = R(K, y) + \theta
\]

where \(K\) denotes capital services, \(y\) is the level of input service actually provided by the supplier and \(\theta\) is a random variable, capturing the uncertainty facing the firm. We assume that the random variable \(\theta\), is distributed over the interval \([m, n]\) according to the density function \(g(\theta)\), where \(m < 0\) (and \(n\) is “sufficiently” large) and a corresponding cumulative distribution \(G(\theta)\).

The revenue function is assumed to be increasing and concave in \(K, y\), with \(R(K, y) \geq 0\), for all \(K, y\). We also assume that \(\overline{R}(K, 0) > 0\), for all \(K > 0\) (in other words, it is possible to carry out production even with a “basic level” of the input, \(y = 0\)) and that \(\partial \overline{R}(K, y)/\partial y\) is “sufficiently” high when \(y = 0\), but very low when \(y \to \infty\) (the usual Inada conditions). From the monotonicity, it follows that \(\overline{R}(K, y) > \overline{R}(K, 0)\), for all \(K > 0, y > 0\). Finally, the firm’s terminal assets, are given by \(R = \overline{R}(K, y) + (1 - \rho)K + \theta \equiv R(K, y) + \theta\), where \(0 \leq \rho \leq 1\) is the depreciation rate (and where the price of \(K\) is normalized to one).\(^{19}\)

Since we are not interested in explaining \(K\), we simplify the notation by dropping it in the following presentation (unless it is relevant); e.g., we will write \(R(K, y)\) as \(R(y)\).

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\(^{16}\) As in Bolton and Scharfstein (1996).

\(^{17}\) See Detragiache, Garella and Guiso (2002) and Brunner and Krahnenz (2002).

\(^{18}\) A very general discussion of multiple contracts is provided in Segal (1999). The paper examines and compares alternative multiple contracts, offered by a principle in the presence of externalities, and studies their efficiency properties. Two other related papers are McAfee and Schwartz (1994), where multilateral opportunistic behaviour in the context of vertical contracting is discussed, and Mailath and Postlewaite (1990), in which a bargaining model with many agents is studied. However, the primary focus of these papers is on game-theoretic issues of cost-sharing bargaining in a general economic framework as opposed to ours which focuses on the choice of capital structure.

\(^{19}\) A more general model will allow for both the firm and the supplier to have ex-ante and ex-post choice variables, to be determined in the last stage of the bargaining game. Thus, in general, we can take, \(\overline{R} = \overline{R}(K, Y, x, y, \theta)\), where \(Y\) is fixed \(\text{ex-ante}\), but \(x\) and \(y\) are "negotiable" \(\text{ex-post}\).
We consider a multi-stage contracting game of symmetric information\textsuperscript{20} with uncertainty. The game involves a principal (equity holders) and two agents: debt holders and the supplier.\textsuperscript{21}

2.1 Time-line

The time line, also described in Figure 1 below, is as follows.

- **Stage 1:**

  In the first stage, the firm (equity holder) makes its production and financing decisions. The production decision is captured by the choice of $K$. We are not concerned with this part of the decision, so that for our purpose we can think of $K$ as given. The financing decision involves the choice of capital structure and is summarized by the contract with debt holders. This contract specifies the level of debt, $b$, and the corresponding payment of, $D$ (in other words, $(D - b)/b$ is the rate of interest). Equity is then given by $e = K - b$. The capital market is assumed to be efficient, in the sense that there are no borrowing constraints.

- **Stage 2:**

  In the second stage, equity holders sign a contract with the supplier. They sign an incomplete contract that specifies that an “input” will be provided and in return the supplier will receive a share, $0 \leq \gamma \leq 1$, of net profits, after the production process is completed and profits are realized.\textsuperscript{22} In addition to profit sharing, they also agree on a side payment, $w$, to be paid at the signing of the contract. The contract, however, cannot specify precisely the amount (“nature”) of the input that will be provided.

- **Stage 3:**

  In stage 3, given the contract signed with equity holders, the supplier chooses the level of his input, $Y$. We distinguish between the level of the input that was chosen by the supplier, $Y$, and the level of the input that will actually be provided; $y$. The supplier’s costs are incurred at this stage. In the next period, the cost of his firm specific input is, therefore, a sunk cost.

- **Stage 4:**

\textsuperscript{20}The effects of asymmetric information on the firm’s capital structure have been discussed in the literature extensively (see Jensen and Meckling (1976), Leland and Pyle (1977)).

\textsuperscript{21}In principle, it is possible to consider different relationships, as long as: (i) there is at least one incomplete contract, (ii) it is possible to form subcoalitions. The results would not be different.

\textsuperscript{22}It is, of course, possible to consider alternative contracts with the supplier. In general, the contract will specify that the investor will receive some payment $w(R + \theta, D)$. To simplify matters, we take this as a share. Alternatively, a sharing arrangement may not be possible, so the contract may provide for a fixed payment. The results with a fixed payment are the same as the ones presented in this paper.
In the “beginning” of stage four, uncertainty about $\theta$ is resolved (and is common knowledge). Then, after the realization of $\theta$, the actual level of the input to be supplied ($y$) and consequently the firm’s solvency are determined, in a three-party bargaining/coalitional game (between equity holders, debt holders and the supplier).\(^{23}\) The firm has to meet its legal obligations to claimants: a payment of $D$ to debt holders and a share of profits to the supplier. This bargaining game is constrained by the realized state of the world, the previously determined contracts and the by existing legal structure. Specifically, the threat points in this bargaining game and consequently its outcome, are affected by bankruptcy laws and in particular by the seniority of claims.\(^{24}\)

Depending on the equilibrium coalitional structure (which in turn depends on the state of the world and the legal structure), there are three possible outcomes to this bargaining game. First, if the firm can meet all of its obligations we have solvency. In this case, the overall game ends. On the other hand, if the firm’s terminal assets, $R + \theta \geq 0$, are insufficient to meet its obligations in full,\(^{25}\) the firm is in default and goes into bankruptcy.\(^{26}\) But, there are two possible “types” of bankruptcy; strategic and non-strategic. Strategic bankruptcy occurs. Non-strategic bankruptcy occurs if the firm cannot meet its obligation due to a bad state of the world rather than opportunistic behaviour. That is, if even the supply of the input was internalized, the firm would still go bankrupt. Strategic bankruptcy can occur even if the firm is viable, due to the formation of an opportunistic coalition between the supplier and debt holders. In either case, if the firm goes into bankruptcy, its assets are distributed in accordance with the seniority of claims. We assume that debt holders are secured creditors (thus first claimants), where their claim can be applied against $R + \theta$.\(^{27}\) The supplier is an unsecured creditor, whereas equity holders are residual claimants.

- Stage 5:

If bankruptcy occurs in stage 4, debt holders and the supplier enter into a post-bankruptcy bargaining in which the level of the input to be supplied is, again, renegotiated and the parties’ shares of the gains from trade are determined. We denote the post-bankruptcy renegotiated input as $y'$ and assume that the supplier’s share of the gains from trade is $0 < \beta < 1$.

We now examine the subgame perfect Nash Equilibrium of this game.

\(^{23}\)In general, the supplier’s share of profits in this stage can (but does not have to) be different from the previously contracted share, $\gamma$.

\(^{24}\)The bankruptcy law provides the “guidelines” that govern this bargaining process. The constraints imposed by the bankruptcy rules and the parties’ relative strength in the in the bargaining process, determine actual payoffs. For example, according to the bankruptcy law in the US and Canada (See Altman (1983), Willes and Willes (2003), White (1980), (1989)), secured creditors receive the saleable value of the assets which are subject to security. If the value of the security is insufficient to satisfy the claims, the secured creditor is entitled to claim the remainder as an unsecured creditor. Unsecured assets are distributed, according to the US and Canadian laws in the following order: (i) administrative costs, (ii) taxes, (iii) wages and rents, (iv) unsecured creditors, (v) equity holders. If several claimants have the same priority, they are paid on a pro-rata basis.

\(^{25}\)Note that we do not include the side payment as part of the firm’s assets.


\(^{27}\)Other types of securities, such as specific liens and mortgages yield similar results. The question of why debtholders should be secured creditors is discussed, for example, in Stulz and Johnston, (1985).
3 Last Stage: Post Bankruptcy Bargaining

If bankruptcy occurred in stage 4 (the condition under which this happens will be discussed below), debt holders and the supplier engage in post-bankruptcy bargaining in the last stage. In this bargaining game, they renegotiate the input to be supplied, \(y\) (subject to the constraint \(y \leq Y\)) and the division of the gain from trade\(^{28}\). We assume that the supplier’s share is exogenously given by \(0 < \beta < 1\).

Consider the solution to this bargaining game. The threat payoffs of the supplier and debt holders are zero and \(R(0) + \theta\), respectively and, for any supply \(y’\), the gain from trade is given by \(R(y’) - R(0)\). Since the joint surplus, \(R(y’) + \theta\), is maximized at \(Y\) (which is determined in stage 3): it clear that the bargaining solution must be such that \(y’ = Y\). The supplier’s and debt holders’ equilibrium payoffs, \(p(\theta)\) and \(D(\theta)\) are, therefore, given by:

\[
\begin{align*}
    p(\theta) &= \beta[R(Y) - R(0)] \\
    D(\theta) &= R(0) + \theta + (1 - \beta)[R(Y) - R(0)]
\end{align*}
\]

Since bankruptcy has occurred, equity holders’ payoffs are

\[e(\theta) = 0\] (3)

If bankruptcy does not occur the game ends in stage 4.

4 Stage 4: Multilateral Bargaining

The state of the world is revealed in the beginning of stage 4. Subsequently, the actual level of input supplied, \(y \leq Y\) (as opposed to \(y’\), or \(y’\)) and consequently the parties’ payoffs and the firm’s solvency are determined, in this stage, in a three-party bargaining game. Bankruptcy may, or may not occurs in this stage. If the firm can meet its obligations, the game ends in stage 4. If, on the other hand, the firm cannot meets its obligations, it goes into bankruptcy. Clearly, bankruptcy may occur if the firm is not viable. But, as will now be shown, even a viable firm may sometime be forced into bankruptcy by a strategic supplier/debt holders coalition. In either case, if the firm goes into bankruptcy, the supplier and debt holders move to the post-bankruptcy game in stage 5, as described above.

Let us examine the three-party bargaining game in stage 4. Define the following regions:

\[
\begin{align*}
    A^3 &\equiv \{ (\theta, D) : R(K, 0) + \theta - D \geq 0, \ m \leq \theta \leq n \} \\
    A^1 &\equiv \{ (\theta, D, Y) : R(K, Y) + \theta - D \leq 0, \ m \leq \theta \leq n \} \\
    A^2 &\equiv \{ (\theta, D, Y) : (\theta, D, Y) \notin A^3 \cup A^1, \ m \leq \theta \leq n \}
\end{align*}
\]

For the problem to be meaningful and interesting, we assume that the three regions are not empty. For example, these regions are shown in Figure 2 below, in \((D, \theta)\) space (given \(Y\)).

\(^{28}\)Note that, at this point the supplier has already made his production decision, \(Y\) (in stage 3), and his actual supply decision, \(y\), in stage 4. But, we assume that he can adjust his supply subject to the constraint \(y' \leq Y\).
Alternatively, we can define these regions as follows. Define the cutoff states $\theta^1(Y,D)$, $\theta^2(Y,D)$ and $\theta^3(D)$, by the conditions:

\[
\begin{align*}
R(Y) + \theta^1 - D &= 0 \\
R(0) + \theta^3 - D &= 0 \\
\beta[R(Y) - R(0)] &= R(Y) + \theta^2 - D \\
\end{align*}
\]

It is easy to see that:

\[
\begin{align*}
\theta^1 &< \theta^2 < \theta^3 \text{ for all } Y > 0 \\
\theta^1 &= \theta^2 = \theta^3 \text{ if } Y = 0
\end{align*}
\]

The three regions above can now be written as:

\[
\begin{align*}
A^3 &\equiv \{ (\theta, D) : \theta \geq \theta^3(D), \ m \leq \theta \leq n \} \\
A^1 &\equiv \{ (\theta, D, Y) : \theta \leq \theta^1(D), \ m \leq \theta \leq n \} \\
A^2 &\equiv \{ (\theta, D, Y) : \theta^1(D) \leq \theta \leq \theta^3(D), \ m \leq \theta \leq n \}
\end{align*}
\]

It is easy that, within region $A^3$, bankruptcy cannot occur even if $y = 0$ is supplied, but within region $A^1$ bankruptcy occurs even if $y = Y$ is supplied. Within these two regions, strategic behaviour (forced bankruptcy) on the part of the supplier is, therefore, impossible. On the other hand, within region $A^2$, the supplier can force the firm into bankruptcy by not supplying all of his investment.

To examine the outcome of the multilateral bargaining game in stage 4, we have to consider the outcomes within the different regions.

- **Case I** : $(\theta, D) \in A^3$ - Solvency Region

In this case, the firm can always meet all its obligations to the claimants, regardless of how much of $Y$ is actually supplied. If the supplier provides a “basic” input, $y = 0$, he gets the contracted share $\gamma[R(0) + \theta - D]$, debt holders get $D$ and equity holders get a residual of $(1 - \gamma)[R(0) + \theta - D]$. But, by supplying $y = Y$, additional gains from trade of $R(Y) - R(0)$ can be divided between the supplier and equity holders. Since the cost of $Y$ is sunk, it is clear that $y = Y$ maximizes joint surplus. The outcome of the bargaining game will, therefore, involve the actual supply of $y = Y$ and a division of the gains from trade. We assume that the supplier’s ex-post share of the gains from trade in the bargaining game is $\gamma[R(y) - R(0)]$. In other words, his ex-post and ex-ante shares are the same. This assumption can be amended without affecting any of the results.

Let actual receipts of the supplier, debt holders and equity holders in the bargaining game be given by, $p(\theta)$, $D(\theta)$ and $e(\theta)$, respectively. Thus, in this region, we have $y = Y$ and the parties’ receipts are given by:
\[ p(\theta) = \gamma(R(0) + \theta - D) + \gamma[R(Y) - R(0)] = \gamma(R(Y) + \theta - D) \]
\[ D(\theta) = D \]
\[ e(\theta) = (1 - \gamma)(R(0) + \theta - D) + (1 - \gamma)[R(Y) - R(0)] = (1 - \gamma)(R(Y) + \theta - D) \]

**Case II: \((\theta, D, Y) \in A^2\)**

Within this region, the supplier can force the firm into bankruptcy by not supplying all of his input.\(^29\) To determine whether it is indeed in the interest of the supplier to exercise his option to force default, we have to compare his receipts if he forces bankruptcy with those when he does not. We know that if he actually forces bankruptcy, he will move to post-bankruptcy bargaining with debt holders in stage 5. His share of the gains from trade in post-bankruptcy bargaining is \(0 < \beta < 1\). Namely, he will receive \(\beta[R(Y) - R(0)]\).

What will be his receipts be if he can, but does not actually, force bankruptcy? Specifically, does the mere ability to force bankruptcy, in itself, endow him with extra benefits (in this region)? To see this, note that the supplier is now in a powerful position since (i) he can force bankruptcy and (ii) he is an indispensable party of any two-party coalition. We will now show that consequently, this does indeed endow him (under some circumstances) with extra power in this coalitional game and hence with extra benefits.

To demonstrate the solution within the \(A^2\) region, we need to partition it into two sub-regions, defined as follows:

\[ A^2_{nsb} \equiv \{ (\theta, D, Y) : \beta[R(Y) - R(0)] \leq R(Y) + \theta^2 - D, \ R(K, 0) + \theta - D \leq 0, \ m \leq \theta \leq n \} \]
\[ A^2_{sb} \equiv \{ (\theta, D, Y) : \beta[R(Y) - R(0)] \geq R(Y) + \theta^2 - D, \ R(K, Y) + \theta - D \geq 0, \ m \leq \theta \leq n \} \]

Region \(A^2_{nsb}\) represents a region where the supplier’s ability to force bankruptcy enables him to extract extra benefits, even without actually forcing bankruptcy. This is a no-strategic-bankruptcy (\(nsb\)) region. Region \(A^2_{sb}\) represents a region where the supplier can force bankruptcy but, in addition, he is better off actually forcing bankruptcy. Namely, this a is strategic-bankruptcy (\(sb\)) region. Again, for the problem to be interesting, we assume that the \(nsb\) and \(sb\) regions are not empty.

Let us now examine the outcomes within these two sub-regions.

**Case IIa: \((\theta, D, Y) \in A^2_{nsb}\) - No-Strategic-Bankruptcy Region**

The supplier can now force bankruptcy by withholding some of \(Y\) (see previous footnote). Assuming that multilateral negotiations and binding pre-bankruptcy agreements between the

\(^29\)Specifically, define the bankruptcy forcing level of investment, \(y(\theta)\), given the state \(\theta\), by the solution to the equation: \(R(y(\theta)) + \theta - D = 0\). For any state of the world \(\theta\), if the supplier provides \(y < y(\theta)\), the firm defaults on obligations to debtholders. On the other hand, if (within this range) \(y(\theta) < y \leq Y\) is supplied, an additional gain from trade can be obtained. Note that for \(\theta \geq \theta^3\) we cannot force bankruptcy, since \(Y(\theta) \leq 0\), for all \(\theta \geq \theta^3\).
supplier and debt holders about about post-bankruptcy payoffs are possible, and given the supplier’s pivotal role as a coalition partner, the supplier can now extract the full amount over and above the (legally constrained) receipts by debt holders in such a coalition: \( R(Y) + \theta - D \). This is the full value of the coalition between equity holders and the supplier. In other words, the supplier extracts all the value over and above his two rivals’ opportunity cost values of: \( D \) and zero, respectively. Since there are no bankruptcy costs here, the results are the same whether bankruptcy actually occurs or not. That is, the receipts by all three parties are the same whether the coalition is between the supplier and equity holders (without default), or between the supplier and debt holders.\(^{30}\)

Thus, we have:

**Theorem 1**  For all \((\theta, D, Y) \in A^2_{\text{sh}}\), the unique equilibrium of the game has \( y = Y \) and payoffs:

\[
\begin{align*}
    p(\theta) &= R(Y) + \theta - D \\
    D(\theta) &= D \\
    e(\theta) &= 0
\end{align*}
\]

**Proof.** See Appendix I. □

Theorem 1 suggests that the supplier’s pivotal role as a coalition partner and his ability to force bankruptcy, in this region, enable him to extract all the surplus in excess of the legally constrained receipts by debt holders \( (R(Y) + \theta - D) \)\(^{31}\) without actually having to force bankruptcy\(^{32}\).

- **Case IIb:** \((\theta, D, Y) \in A^2_{\text{sh}}\) - Strategic Bankruptcy Region

Again, the supplier can force bankruptcy by withholding some of \( Y \). But, since within this region, we have \( \beta[R(Y) - R(0)] \geq R(K, Y) + \theta - D \), he is better off actually forcing bankruptcy.

\(^{30}\)As an alternative scenario, we can consider the case where the supplier and debt holders cannot engage in pre-bankruptcy negotiations that lead to binding agreements about post-bankruptcy payoffs. Under this scenario, the supplier’s threat point will be different and hence the division of the surplus between him and equity holders will also be different. Specifically, the supplier’s threat payoff is now what he gets in the post-bankruptcy (rather than pre-bankruptcy) agreement, namely, \( \beta[R(Y) - R(0)] \). His payoff will, therefore, be \( \beta[R(Y) - R(0)] + \gamma\{R(Y) + \theta - D - \beta[R(Y) - R(0)]\} \), which consists of his threat point plus his share of the surplus (revenue, net of his threat point and the payment to debt holders). Since within this region we have \( \beta[R(Y) - R(0)] \leq R(Y) + \theta - D \), he is, indeed, better off not forcing bankruptcy. In the following theorem we show that when multilateral negotiations and binding pre-bankruptcy agreements between the supplier and debt holders about post-bankruptcy payoffs are possible, the supplier can extract extra surplus; in fact he can now extract the full amount of the surplus \( R(Y) + \theta - D \). Except for the division of the surplus, the results under these alternative scenario are the same.

\(^{31}\)This is the full value of the coalition between equity holders and the supplier. In other words, the supplier extracts all the surplus value over and above his two rivals’ opportunity costs, which are \( D \) and zero, respectively.

\(^{32}\)Since there are no bankruptcy costs here and the coalitional agreements are binding post-bankruptcy, the payoffs are the same whether bankruptcy actually occurs or not. That is, the receipts by all three parties are the same whether the coalition is between the supplier and equity holders (without default), or between the supplier and debtholders. It is, of course, possible to introduce costs of bankruptcy and “switching” management (from equity to debtholders-supplier management). This will make the potential gains to the supplier, within this range, smaller. His incentive to supply will, therefore, decrease. It is also well known (see Ravid (1988), Brander and Lewis (1986)) that bankruptcy costs provide an incentive for equity financing and we do not pursue this here.
When he forces bankruptcy, his receipts, in post-bankruptcy bargaining with debt holders, will be: \( \beta [R(Y) - R(0)] \).

Thus, for \((\theta, D, Y) \in A_{sb}^2\), we have \( y = Y \), and the parties’ receipts are given by:

\[
\begin{align*}
p(\theta) &= \beta [R(Y) - R(0)] \\
D(\theta) &= R(0) + \theta + (1 - \beta) [R(Y) - R(0)] \\
e(\theta) &= 0
\end{align*}
\]

This case implies that, under some circumstances, due to opportunistic behaviour, the firm will not be able to prevent bankruptcy, even if it is viable (in the sense that it can survive if \( Y \) is supplied). This result is reminiscent of Hart and Moore (1998), where the entrepreneur cannot prevent foreclosure.

- Case III: \((\theta, D, Y) \in A^1\) - Non-strategic bankruptcy Region

In this case, the firm will always default on its obligations to debt holders, so that equity holders are out of the picture. The supplier and debt holders, therefore, move to post-bankruptcy bargaining. Assuming that the sharing rule does not depend on whether the firm went bankrupt, or was forced into bankruptcy, the gains from trade and the outcome of bargaining will be the same as in the \( A_{sb}^2 \) region. In other words, we the same result as in Case IIb\(^{33}\).

These regions are shown in Figure 2.

## 5 Stage 3: The Supplier’s Problem

Given the outcome of the bargaining game, the supplier’s problem is\(^{34}\)

\[
\max_Y \delta E[p(\theta)] - Y + w \\
\equiv \max_Y J(K, D, Y, \gamma, \beta)
\]

where the supplier’s expected receipts are:

\[
E[p(\theta)] = \int_{\theta^2(Y,D)}^{\theta^2(Y,D)} \beta [R(Y) - R(0)] g(\theta) d\theta + \\
\int_{\theta^3(D)}^{\theta^3(D)} [R(Y) + \theta - D] g(\theta) d\theta + \int_{\theta^3(D)}^{\theta^3(D)} [R(Y) + \theta - D] g(\theta) d\theta
\]

\[
\delta \equiv \frac{1}{1+s} \quad \text{and} \quad s \text{ is the opportunity cost rate of return.}
\]

\(^{33}\)We are assuming that \( R(0) + \theta \geq 0 \). If we allow for the case where \( R(0) + \theta < 0 \), the result will be similar except that now the gains from trade are \( R(Y) + \theta - \max[0, R(0) + \theta] = R(Y) + \theta \), since here \( R(0) + \theta \leq 0 \).

\(^{34}\)The supplier’s cost function is taken to be linear, but the results remain the same for any increasing and convex cost function.
The Kuhn-Tucker conditions corresponding to the supplier’s problem are given by:

\[
\begin{align*}
\delta \frac{\partial R(Y)}{\partial Y} &- \pi(K, D, Y, \gamma, \beta) - 1 \leq 0 \\
[\delta \frac{\partial R(Y)}{\partial Y} - \pi(K, D, Y, \gamma, \beta)]Y & = 0 \\
Y & \geq 0
\end{align*}
\]

where

\[
\pi(K, D, Y, \gamma, \beta) \equiv \beta G[\theta^2(Y, D)] + \{G[\theta^3(D)] - G[\theta^2(Y, D)]\} + \gamma\{1 - G[\theta^2(D)]\}
\]

is a “discount factor” that is applied to the present value of the marginal benefits of \(Y\). This discount factor is simply the supplier’s expected share, which is the average of the shares in different states of the world. Being the supplier’s expected share, \(\pi\) must satisfy:

\[
0 < \pi(K, D, Y, \gamma, \beta) \leq 1, \text{ for all } 0 < \gamma \leq 1, 0 < \beta < 1
\]

Assuming that \(\partial R(0)/\partial Y\) is high enough (we need \(\partial R(0)/\partial Y > 1/\delta\pi(0)\)), it follows that when the input is zero we have: \(\delta \frac{\partial R(Y)}{\partial Y}\pi(0) > 1\). Assuming that \(E[p(\theta)]\) is concave in \(Y\) and given that \(\lim_{Y \to \infty} \partial R(Y)/\partial Y = 0\), there exists an input level \(Y^* > 0\), such that:

\[
\delta \frac{\partial R(Y^*)}{\partial Y} - \pi(K, D, Y^*, \gamma, \beta) = 1
\]

Since the supplier’s share in the post-bankruptcy bargaining will never be strictly equal to 1, \((\beta < 1)\), the only way to get \(\pi = 1\) is when \(\gamma = 1\) and the probability of a forced bankruptcy (with a supplier/debt holders coalition) is zero \((G(\theta^2) = 0)\). If, indeed, \(\gamma = 1\) and the probability of a coalition between the supplier and debt holders is zero, then we have \(\pi = 1\) for any value of \(\beta\). Alternatively, given that \(\beta < 1\), we have \(\pi < 1\) if either \(\gamma < 1\), or the probability of a coalition between the supplier and debt holders is strictly positive.\(^{38}\)

Since \(\pi \leq 1\), it follows that, at the optimal solution:

\[
\delta \frac{\partial R(Y^*)}{\partial Y} \geq 1
\]

\(^{35}\) Notice that when the firm does not use debt, we have \(E[p(\theta)] = \int_m^n \gamma[R(Y) + \theta]g(\theta)d\theta\), hence \(\pi = \gamma\) and the first order condition is \(\delta \frac{\partial R(Y)}{\partial Y} = 1\).

\(^{36}\) A zero probability of a forced bankruptcy can only happen if \(\theta^2(Y, D) \leq m\).

\(^{37}\) It can be easily shown that under the alternative scenario (discussed in footnote 30) when the supplier and debt holders cannot engage in pre-bankruptcy negotiations that lead to binding agreements about post-bankruptcy payoffs and consequently the supplier’s payoff is \(\beta[R(Y) - R(0)] + \gamma\{R(Y) + \theta - D - \beta[R(Y) - R(0)]\}\), the discount factor becomes \(\pi(K, D, Y, \gamma, \beta) \equiv \beta G[\theta^2(Y, D)] + (\beta + \gamma + \beta\gamma)[G[\theta^3(D)] - G[\theta^2(Y, D)]] + \gamma[1 - G[\theta^2(D)]]\). So, again, if \(\beta < 1\), the only way to get \(\pi = 1\) is when \(\gamma = 1\) and the probability of a forced bankruptcy is zero. Note also that if the supplier cannot form a coalition with debt holders, we would have \(G[\theta^2(Y, D)] = 0\), hence, \(\pi = \gamma + (1 - \gamma)[1 - G[\theta^2(D)]]\). This is the usual incomplete contract problem and it can be resolved by setting \(\gamma = 1\), which implies that \(\pi = 1\). Thus, in this case, the first best solution can be achieved.
Given the concavity of $R$, this implies that if either $\gamma < 1$, or the probability of a supplier/debt holders coalition is strictly positive (that is, when $\pi < 1$), the supplier will choose a level of $Y$ that is too low. To see this, note that the Pareto optimal (or integrated firm) level of input, say $Y^*$, equates marginal benefits and marginal costs:\footnote{See footnote 39 below.} $\frac{\partial R(Y^*)}{\partial Y} = 1$. Thus, for all $\pi < 1$ we have: $\delta \frac{\partial R(Y^*)}{\partial Y} = \delta \frac{\partial R(Y^*)}{\partial Y} \pi < \delta \frac{\partial R(Y^*)}{\partial Y}$. Given concavity, this implies that, for all $\pi < 1$, we have $Y^* < Y^i$.

The supplier’s first order condition, $\delta \frac{\partial R(Y^*)}{\partial Y} \pi(K, D, Y^*, \gamma, \beta) - 1 = 0$, defines his incentive compatibility constraint. Alternatively, it defines (implicitly) his input supply function, which can be written as:

$$Y^* = Y^*(K, D, \gamma, \beta)$$

From the first order condition we can easily obtain:

$$\frac{\partial Y^*}{\partial \gamma} = -\frac{[1 - G(\theta^3)]\delta \frac{\partial R(Y^*)}{\partial Y}}{\partial^2 J/\partial Y^2} > 0$$

(15)

Thus, an increase in the supplier’s share will increase $Y^*$.

6 Stage 2: The Contract with the Supplier

In stage 2, after the contract with debt holders is signed, equity holders sign a contract with the supplier. The contract with the supplier specifies that he will provide his services and in return, he will receive a share, $\gamma$, of profits\footnote{There are several ways to define the assets to be shared. For simplicity we assume that they share net terminal assets. Other arrangements yield similar results.} and a side payment of $w$ will be paid at the signing of the contract. The input service, however, is non-verifiable so that the parties cannot sign a contract based on its actual level.

Consider the firm’s problem. Given the contract with debt holders, the outcomes of the multilateral bargaining game and the supplier’s choice of $Y^*$, equity holders’ receipt in the last stage will be:

$$e(\theta) = \begin{cases} 
(1 - \gamma)[R(Y^*) + \theta - D] & \text{if } \theta \geq \theta^3(Y^*, D) \\
0 & \text{otherwise} \end{cases}$$

(16)

Equity holders choose a sharing arrangement, $\gamma$ (and the fixed payment) that maximizes the expected net present value of their receipts,

$$V = \delta E[e(\theta)] - e - w,$$

subject to the supplier’s participation and incentive compatibility constraints. If we define $E[p(\theta)]$, evaluated at the optimal solution, $Y^*$, as:

$$E[p^*(\theta)] = E[p(\theta)]|_{Y^*}$$
The equity holders’ problem can be written as:

$$\max_{\gamma, w} \{ \delta E[e(\theta)] - e - w : \delta E[p^*(\theta)] - Y^* + w \geq 0, \delta \frac{\partial R(Y^*)}{\partial Y} \pi(K, D, Y^*, \gamma, \beta) = 1, 1 - \gamma \geq 0 \}$$

(17)

Alternatively, instead of using the incentive compatibility constraint, we can simply use the solution for \(Y^*\) and write the problem as:

$$\max_{\gamma, w} \{ \delta E[e(\theta)] - e - w : \delta E[p^*(\theta)] - Y^* + w \geq 0, Y^* = Y^*(K, D, \gamma, \beta), 1 - \gamma \geq 0 \}$$

(18)

It is clear that the optimal solution does not leave any extra surplus to the supplier. In other words, his participation constraints will hold with strict equality. Thus, we can use it to solve for \(w\) as,

$$w = -\delta E[p^*(\theta)] + Y^*$$

Plugging this into the expected net present value of equity, \(V\), we get:

$$V = \delta \{ R(Y^*) + E(\theta) - E[D^*(\theta)] \} - e - Y^*$$

(19)

where \(E[D^*(\theta)]\) is the expected value of debt holders’ receipts, evaluated at \(Y^*\), given by:

$$E[D^*(\theta)] \equiv \int_{\theta_1^*}^{\theta_2^*} \{ [R(0) + \theta] + (1 - \beta)[R(Y^*) - R(0)] \} g(\theta) d\theta$$

(20)

where \(\theta_2^* \equiv \theta_2(K, Y^*, D)\).

Let the optimal contract be given by \((\gamma^*, w^*)\). Then, it is easy to show that:

**Proposition 2** The optimal contract involves a franchise arrangement; \(\gamma^* = 1\), and a franchise fee \(w^* = -\delta E[p^*(\theta)]|_{\gamma=1} + Y^{*1}\), where \(Y^{*1} \equiv Y^*(K, D, \gamma = 1, \beta)\).

**Proof.** See Appendix II.4

Finally, since we will need to use the value of the solution to above problem, in the determination of the optimal contract with debt holders, let us define maximum value function as:

$$V^{*1} \equiv V(\gamma^*) = V(1) = \delta \{ R(Y^{*1}) + E(\theta) - E[D^{*1}(\theta)] \} - e - Y^{*1}$$

(21)

where \(E[D^{*1}(\theta)] \equiv E[D^*(\theta)]|_{Y^* = Y^{*1}}\) with \(\theta_2^{*1} \equiv \theta_2(K, Y^{*1}, D)\).

41Thus, if the firm does not use debt, we have \(\pi = 1\), i.e., the first best supply of \(Y\).
7 The Contract with Debt holders:

In the first stage, equity holders sign a contract with debt holders. The contract specifies the level debt, $b$, and the corresponding payment, $D$.$^{42}$ Given the outcomes in stages 2, 3, 4 and 5, equity holders’ receipts are given by $V^{s*1}$ in equation (21) (i.e., it is given by $V$, evaluated at $Y^{s*1}$, $\delta E[D^{s*1}(\theta)]$). The contract with debt holders, therefore, must satisfy the debt holders’ participation constraint,

$$B(K, b, D, Y^{s*1}, \beta) \equiv \delta E[D^{s*1}(\theta)] - b \geq 0. \quad (22)$$

This participation constraint defines the market debt supply function.

Equity holders choose a contract with debt holders to maximize the expected net present value of their receipts, $V^{s*1}$, subject to debt holders’ participation constraint and the supplier’s incentive compatibility constraint, which is given by $\delta \partial R(Y^{s*1})/\partial Y \pi(K, D, Y^{s*1}, 1, \beta) = 1$. Note that since we used the supplier’s participation constraint in the derivation of $V^{s*1}$, we do not need to worry about it; its satisfaction has already been imposed. Furthermore, the optimal contract with the supplier ($\gamma^* = 1$, $w^* = -\delta E[p^*(\theta)]_{Y^{s*1}} + Y^{s*1}$ has also already been taken into account in the definition of $V^{s*1}$ in equation (21).

The equity holders problem can, therefore, be written as:

$$\max_{b, D} \{\delta \{R(Y^{s*1}) + E(\theta) - E[D^{s*1}(\theta)]\} - e - Y^{s*1} :$$

$$\delta \partial R(Y^{s*1})/\partial Y \pi(K, D, Y^{s*1}, 1, \beta) = 1, \ \delta E[D^{s*1}(\theta)] - b \geq 0\}$$

Again, it is clear that the optimal solution does not leave extra surplus to debt holders. In other words, their participation constraint will hold with strict equality:

$$\delta E[D^{s*1}(\theta)] = b \quad (23)$$

If we plug this strict participation constraint, (23), into the expression for $V^{s*1}$, we get the expected net present value of equity as:

$$\delta \{R(K, Y^{s*1}) + E(\theta) - (\rho + s)K\} - Y^{s*1} \equiv \delta [R(K, Y^{s*1}) + E(\theta) - (1 + s)K] - Y^{s*1}$$

which is simply the expected net present value of the firm: the expected value of profits, less the true costs of capital and supplier services.

The equity holders’ problem can, therefore, be written as:

$$\max_{b, D} \delta \{R(K, Y^{s*1}) + E(\theta) - (1 + s)K\} - Y^{s*1} :$$

$$\delta \partial R(Y^{s*1})/\partial Y \pi(K, D, Y^{s*1}, 1, \beta) = 1, \ \delta E[D^{s*1}(\theta)] = b, \ b \leq K \quad (24)$$

$^{42}$Note that, for any given $K$, the choice of $b$ also determines equity, $e$. 

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where there is no need to consider \( e \) explicitly, since it is given by: \( e = K - b \) (and \( K \) is taken as given).

It is convenient to proceed by first solving the incentive compatibility and participation constraints for \( D \) and \( Y^{*1} \) in terms of \( b \), thus eliminating these two constraints. The two constraints are:

\[
F \equiv \delta \frac{\partial R(Y^{*1})}{\partial Y} \pi(K, D, Y^{*1}, 1, \beta) - 1 = 0 \quad \text{and} \quad B(K, b, D, Y^{*1}, \beta) \equiv \delta E[D^{*1}(\theta)] - b = 0.
\]

Ignoring all variables other than \( D, Y^{*1} \) and \( b \), we can write these two constraints as:

\[
F(D, Y^{*1}) = 0 \\
B(b, D, Y^{*1}) = 0
\]

Let the solution to these two constraints be given by:

\[
D = e_D(b) \quad \text{and} \quad e_Y(b).
\]

The equity holders’ problem can, therefore, be re-written as:

\[
\max_b \{ \delta[R(\tilde{Y}(b)) + E(\theta) - (1 + s)K] - \tilde{Y}(b) : b \leq K \} \tag{25}
\]

Define the Lagrangian by:

\[
\mathcal{L} \equiv \delta[R(\tilde{Y}(b)) + E(\theta) - (1 + s)K] - \tilde{Y}(b) + \mu(K - b)
\]

where \( \mu \) is the Lagrangian corresponding to the inequality constraint, \( b \leq K \). The Kuhn-Tucker conditions are, therefore, given by:\(^{44}\)

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial b} & = (\delta \frac{\partial R(\tilde{Y}(b))}{\partial Y} - 1) \frac{d\tilde{Y}(b)}{db} - \mu \leq 0 \\
\frac{b \partial \mathcal{L}}{\partial b} & = b[(\delta \frac{\partial R(\tilde{Y}(b))}{\partial Y} - 1) \frac{d\tilde{Y}(b)}{db} - \mu] = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu} & = K - b \\
\mu \frac{\partial \mathcal{L}}{\partial \mu} & = \mu(K - b) = 0 \\
\mu & \geq 0
\end{align*}
\]

Let the solution the problem above be given by: \( \{ \tilde{Y}(K, \beta), \tilde{D}(K, \beta), \tilde{b}(K, \beta) \} \).

We can now show that:

**Proposition 3** With the optimal capital structure, the probability of an opportunistic forced bankruptcy is zero.

---

\(^{43}\) Appendix III confirms that the conditions for solving for \( \tilde{D}(b) \) and \( \tilde{Y}(b) \) (implicit function theorem) are satisfied.

\(^{44}\) We assume that the required curvature conditions are satisfied.
Proof. From the supplier’s first order condition (13) it follows that for any optimal solution (for the supplier), \( \bar{Y}(b) > 0 \), we have: 
\[
\delta \frac{\partial R(\bar{Y}(b))}{\partial Y} \bar{\pi}(b) = 1, \quad \text{where } \bar{\pi}(b) \equiv 1 - (1 - \beta)G[\bar{\theta}^2(b)] \text{ (since the optimal supplier's share is } \gamma^* = 1) \text{, and where } \bar{\theta}^2(b) \equiv \theta^2 \mathbb{E}[\bar{D}(b), \bar{Y}(b)]. \]
We prove the proposition by contradiction. First, note that (i) when debt is zero, the probability of forced bankruptcy is zero too, (ii) for the probability of forced bankruptcy to be strictly positive, debt itself must be strictly positive too. Now, suppose that we have a solution at \( b > 0 \), such that at \( b \) the probability of forced (i.e., opportunistic) bankruptcy is zero too, (ii) for the probability of forced bankruptcy to be strictly positive, debt itself must be strictly positive too. Note that since since \( \beta < 1 \), we must have \( \bar{\pi}(b) < 1 \), which implies that: 
\[
\frac{\partial \theta}{\partial Y} \bar{\pi}(b) < 1 \quad \Rightarrow \quad \frac{\partial \pi}{\partial b} > 0 \quad \text{for all } b \geq 0 \quad \text{such that at } b = 0, \text{ which is a contradiction. Hence, the optimal solution cannot be at a debt level, } b, \text{ for which } G[\bar{\theta}^2(b)] > 0. \]
We, therefore, conclude that with the optimal capital structure, say \( \bar{b}(K, \beta) \), the probability of an opportunistic forced bankruptcy is zero: 
\[
G[\bar{\theta}^2(\bar{b}(K, \beta))] = 0. \]

Proposition 1 shows that the firm will choose its optimal level of debt to be sufficiently low, so that the probability of strategic bankruptcy is zero. Note that since \( \theta^2 < \theta^1 \) for all \( Y > 0 \), this also implies that the optimal capital structure ensures that the probability of non-strategic bankruptcy (and its consequent post-bankruptcy debtholders/supplier coalition) is also zero.

The proposition above gives us an important property of the optimal capital structure. But, we want to go further and find out what the optimal capital structure will actually be. As is clear, the answer to this question will, generally, depend on the characteristics of the model. More specifically, it depends on whether the probability of an opportunistic strategic bankruptcy is possible at low levels of debt. To see this, define \( \bar{b} \) by the condition:
\[
G[\bar{\theta}^2(\bar{b})] = 0
\]
In other words, \( \bar{b} \) is the level of debt which reduces the probability of an opportunistic strategic bankruptcy to zero. Alternatively, since \( \theta \) is distributed over the interval \([m, n]\), we can simply define \( \bar{b} \) by the condition:
\[
\bar{\theta}^2(\bar{b}) = m \quad \text{(27)}
\]
By definition, the probability of strategic bankruptcy is zero for all levels of \( b \) below \( \bar{b} \):
\[
G[\bar{\theta}^2(b)] = 0, \quad \text{for all } b \leq \bar{b}
\]
Now, plugging the definition of \( \theta^2 \) (from equation (4)) into equation (27), we can re-write the definition of \( \bar{b} \) as:
\[
\bar{D}(\bar{b}) - (1 - \beta)R[K, \bar{Y}(\bar{b})] - \beta R(K) = m
\]
Thus, \( \bar{b} \) is a function of \( m, K \) and \( \beta \) and can be written as:
\[
\bar{b} = \bar{b}(m, K, \beta)
\]
Moreover, it is easy to verify\(^\text{45}\) that:

\[
\begin{align*}
\frac{db}{dm} &> 0, \quad \frac{db}{dK} > 0, \quad \frac{db}{d\beta} < 0
\end{align*}
\] (28)

In other words, as the distribution shifts up, or when \(K\) increases, the the highest value of debt such that the probability of a strategic bankruptcy is still zero increases. On the other hand, when the supplier’s share in post-bankruptcy bargaining increases, the highest value of debt such that the probability of a strategic bankruptcy is still zero decreases.

In general, however, we do not know whether \(b\) is positive or not. As the comparative statics results above indicate, this depends on the parameters of the problem. For example, for a sufficiently high value of \(m\) we will have a positive \(b\) (this case is shown in Figure 2), but for low values of \(m\), \(b\) may be negative. Let us, therefore, define the set \(Z\) as the set of parameters \((m,K,\beta)\) over which \(b > 0\):

\[Z \equiv \{(m,K,\beta) : b > 0\}\]

We can now address the question of what the optimal capital structure will actually be. This is given by the following proposition:

**Proposition 4** The firm’s optimal capital structure, \(\hat{b}(K,\beta)\), is given by:

\[
\hat{b}(K,\beta) = 0 \quad \text{if} \quad b \leq 0, \quad \text{i.e.,} \quad \text{if} \quad (m,K,\beta) \notin Z
\]

\[
\hat{b}(K,\beta) \in [0,\overline{b}] \quad \text{if} \quad b > 0, \quad \text{i.e.,} \quad \text{if} \quad (m,K,\beta) \in Z
\]

**Proof.** The previous proposition implies that the optimal solution cannot be at a debt level, \(b\), for which \(G[\theta^2(b)] > 0\). Thus, if \(\overline{b} \leq 0\), we must have a solution with zero debt (any strictly positive debt will yield \(G[\theta^2(b)] > 0\)).\(^{46}\) But if \(\overline{b} > 0\), we cannot have a solution for values of debt which are strictly higher than \(\overline{b}\). On the other hand, in this case, all values of debt which are (weakly) below \(\overline{b}\) will also yield \(G[\theta^2(b)] = 0\). Thus, in this case, the firm is indifferent between all debt levels between zero and \(\overline{b}\). Within this range the value of the firm is independent of its capital structure: this is a Modigliani-Miller-range. 

The results in the two propositions above provide another interpretation of the firm’s use of its capital structure. By limiting, or even some time avoiding the use of debt altogether, the firm eliminates the possibility of an opportunistic coalition. In doing so, it neutralizes the fact that it cannot control the outcome of an opportunistic supplier-debt holders coalition. From the definition of \(\pi\) (in equation (11)), it is clear that a firm whose level of debt, \(b\), is such that \(b > \max\{0,\overline{b}\}\), cannot achieve the first best outcome even if \(\gamma = 1\), because (i) there is a positive probability of a supplier/debt holders coalition and (ii) it cannot control the outcome (shares) in such a coalition. Control over the outcome of a potential post-bankruptcy supplier/debt holders

\(^{45}\)This follows immediately from the definition of \(\overline{b}\) and the fact that \(dY/db < 0\) and \(dD/db > 0\), as is shown in Appendix III.

\(^{46}\)Note that at \(b = 0\), we have \(K - b > 0\), so that \(\mu = 0\). Moreover, when no debt is used and with the optimal contract with the supplier (\(\gamma = 1\)), we have \(\pi = 1\) and for all \(Y > 0\) the supplier’s first order condition (13) becomes \(\delta \frac{\partial R(Y)}{\partial Y} - 1 = 0\). Thus, the Kuhn-Tucker condition above becomes: \(\frac{\partial L}{\partial Y}|_{b=0} = 0\).
coalition requires an *ex-ante* contract between equity holders and the supplier, which specifies the *ex-post* sharing rule in a post-bankruptcy supplier/debt holders coalition. This does not seem reasonable.\(^{47}\) By choosing a level of debt which is not higher than \(\max\{0, \bar{b}\}\), the firm neutralizes the costs of such a coalition. This is accomplished by reducing the probability of such a coalition to zero. This implies that rather than using debt to discipline the supplier, the firm uses its capital structure to avoid opportunistic coalitions, thus providing incentives to supply the correct level of input. The first best level of input (reached when \(\delta \frac{\partial R}{\partial Y} = 1\)) can only be achieved if \(\pi = 1\).\(^{48}\) Since \(\frac{\partial \pi}{\partial D} < 0\), the discount factor can be increased to its highest possible value (of 1), by reducing debt to levels that do not exceed \(\max\{0, \bar{b}\}\). Notice that if \(\max\{0, \bar{b}\} = 0\), this implies that the firm will be fully equity financed. On the other hand, if \(\max\{0, \bar{b}\} = \bar{b}\), the level of debt may be strictly positive, but indeterminate within the Modigliani-Miller-range \([0, \bar{b}]\).

The conclusion that the first best solution can be reached follows from the assumptions that: (i) the participation of debt holders is not necessary and (ii) it is possible to set the share, \(\gamma\), to one. Consider the first assumption. In our model, equity and debt are perfect substitutes in the generation of profits (in the \(R\) function). In many cases, however, the services of third parties may not be perfect substitutes. In fact, the participation of third parties may even be required. Under such circumstances, it may not be possible, or desirable, to reduce the participation of third parties to low enough levels that eliminate the possibility of strategic bankruptcy. When this happens, the first best solution will not be achieved. Furthermore, often there may be other motives for using debt. For example, with corporate taxation, the tax benefits of debt financing may be sufficiently high to offset the strategic considerations identified above. In such a case, the optimal level of debt may be greater than \(\max\{0, \bar{b}\}\). Under these circumstances, the firm may not be able to elicit the first best level of input. As for the second assumption, it may not always be possible, or desirable, to set \(\gamma\) to one.\(^{49}\) If this cannot be done, the first best solution cannot be achieved. This is, of course, the standard result, given the non-contractibility.

Finally, it is worth summarizing the general predictions and empirical implications of the model.

- First, our model identifies a new cost of debt financing. This new cost is due to two factors:

  - (i) the possibility of opportunistic behaviour \((G(\theta^{21}) > 0)\),

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\(^{47}\)It also does not seem reasonable that equity holders will be able to include this as part of their contract with debtholders.

\(^{48}\)The integrated firm’s problem is given by: \(\max_{Y, b} V = \delta \{R(Y) + E(\theta) - (s + \rho)K\} - Y\). where \(Y\) is defined by the first order condition: \(\delta \frac{\partial R(Y)}{\partial Y} = 1\). Since \(V\) does not depend on \(b\), we have the usual Modigliani-Miller result that the value of the firm does not depend on its capital structure.

\(^{49}\)For example, (i) there may be additional sources of incompleteness/imperfection, (ii) there may be insurance considerations (with a risk averse supplier), (iii) the participation of equity holders is required (or desirable) because their “input” is not perfectly transferable, (iv) profits are not verifiable, so that profit sharing is problematic. Even when we have an *ex-ante* agreement with \(\gamma = 1\), the first best solution may not be achieved if the firm retains some residual authority (for example, if it can still block/refuse some \(Y\)), so that re-negotiations occur in the last stage. This issue is related to the question of what is a firm. It will not be pursued here. For a discussion of this issue, see for example, Hart (1995) Grossman and Hart (1986), Hart and Moore (1998), Tirole, (1990).
- (ii) the firm cannot control the contract of a potential post-bankruptcy supplier/debt holders coalition (it cannot control the value of $\beta$).

Due to this new cost, the firm faces a new consideration that tends to limits its use of debt. Since, in general, there are other motives for using debt, our conclusion simply implies that under these conditions, the debt-equity ratio will tend to be lower. At the same time, it is important to remember that is the incompleteness of the contract with the supplier that facilitates opportunistic behaviour in the first place.

In addition to the prediction above, our model also has the following implications:

- As the firm’s contacts become more complex (in the sense that they involve a greater number of interdependent incomplete contracts), the probability of opportunistic behaviour tends to increase and as a result, the cost of debt financing will increase, leading to a decrease in the debt-equity ratio. To test this prediction, one would need to examine the effects of changes in an appropriately defined and calculated measure of complexity on the debt-equity ratio. It is interesting to note that this result is related to the literature on bilateral versus multilateral cooperation. The consensus in this literature is that, in general, it is easier to achieve bilateral than multilateral cooperation (because the former is less “costly” to achieve and sustain).\(^5\) This is the equivalent to the result here, that multilateral opportunistic behaviour is more costly.

- As the party whose contract is incomplete (in our case the supplier) becomes stronger (more valuable) in an opportunistic coalition, his share in such a coalition will tend to increase ($\beta$ tends to be larger). As a result, again, the cost of debt financing will increase, leading to a decrease in the debt-equity ratio. To test this prediction, one would need to examine the effects of changes in an appropriately defined and calculated measure of bargaining power\(^5\) on the debt-equity ratio.\(^6\)

- Since it is the incompleteness of the contract with the supplier that facilitates opportunistic behaviour, it is clear that the more difficult it is to write contracts with the supplier (in our example), the greater the chances of opportunistic behaviour, hence the cost of debt financing increases and consequently, the debt-equity ratio will be lower.

- A similar implication will result from greater difficulties in achieving optimal franchising arrangements.

- If the adjustment costs (of changing $Y$) facing the supplier decrease, opportunistic behaviour becomes easier and hence more costly to the firm. As a result, the debt-equity ratio will decrease.


\(^6\)As is done in the unions power bargaining literature, for example.

\(^5\)Both of the above predictions (the effects of greater complexity, or power) may be consistent with the results in Bolton and Scharfstein (1996), Detragiache, Garella and Guiso, (2002) and Brunner and Krahnenz (2002), since the reduction in the debt equity ratio (holding everything else fixed), tends to reduce the probability of default.
• From the comparative statics results in equation (28), it follows that the probability of opportunistic behaviour and hence its cost tend to decrease with the firm’s assets (with \( K \)). The implication is, therefore, that the debt-equity ratio will increase with its size (as captured by \( K \)).

• From the comparative statics results in equation (28), it also follows that the probability of opportunistic behaviour and hence its cost tend to increase when the distribution of \( \theta \) becomes less (first order) stochastically dominant, or when it become riskier (in the mean preserving spread sense). The implication is, therefore that the debt-equity ratio will decrease with these two types of increase in risk.

• Finally, there may be implications regarding the likelihood of governmental bailouts (as in the Skeena case that was mentioned in the introduction) and hence strategic capital structure and default decisions. For example, if we add labour as one of the affected parties, loses from a breakup due to a defecting coalition may, depending on the political environment, make it more likely for the government to provide a bailout (this possibility will, of course, be taken into account by the parties involved).

8 Conclusion:

This paper considers the effects of multilateral opportunistic behaviour on the firm’s capital structure. We consider a multi-party contract between equity holders, debt holders and a provider of a non-contractible input. We show that with multiple parties, the contract with the supplier becomes more incomplete, because the firm cannot, generally, avoid opportunistic coalitions and it cannot control the terms of the contract in such potential supplier/debt holders coalitions. This identifies a new cost of debt financing. A higher debt-equity ratio increases the probability of an opportunistic coalition, thus increasing the costs of opportunistic behaviour. In fact, by limiting its use of debt financing, the firm can avoid the costs of opportunistic coalitions altogether. Thus, in the absence of all other motives for using debt, the firm will reduce its debt to a level that ensures that the probability of opportunistic coalitions is zero. Since, in general, there are other motives for holding debt, our conclusion simply implies that under these conditions, the debt-equity ratio will tend to be lower.

We also find that by choosing a level of debt that reduces the probability of opportunistic coalitions to zero, the firm provides the supplier with incentives to supply the correct level of the firm-specific input. Specifically, if the firm can be franchised (without any residual authority to equity holders), such level of debt yields a first best outcome. On the other hand, if the participation of debt holders is (i) required, (ii) not a perfect substitute for equity, or (iii) desirable for other reasons, the first best solution may not be possible to achieve. Similarly, if the firm cannot be franchised, the first best solution cannot be achieved.

The results in this paper suggest that, in general, the firm’s debt-equity ratio will decrease with the number of interdependent incomplete contracts, the importance of the provider of the non-contractible input in an opportunistic coalition, the difficulty of writing contracts with
the supplier, the difficulties in achieving optimal franchising arrangements and certain types of increased risk. On the other hand, the firm’s debt-equity ratio will increase with its size and the cost of adjustment facing the supplier.

9 Appendix

I. Proof of theorem 1:

First, remember that we assume that the supplier and debt holders can engage in pre-bankruptcy negotiations that lead to binding agreements about post-bankruptcy payoffs. Under this scenario, the supplier’s threat point is the post-bankruptcy payoff that can be agreed on in pre-bankruptcy negotiations with debt holders.

Now, since \( R(Y) + \theta > R(0) + \theta \), it is clear that the only possible equilibrium outcome will have \( y = Y > 0 \). But, in this region, for all outcomes with \( y = Y > 0 \), the firm is not in default, so that debt holders must be paid their full claim of \( D \). Now, consider any outcome with \( y = Y > 0 \), in which the firm was not actually forced into bankruptcy. Since the firm is not in default it must pay debt holders their full claim of \( D \), but it has no reason to pay more than \( D \). The amount left to divide between supplier and equity holders is, therefore, \( R(Y) + \theta - D \).

A feasible division requires that it adds up to \( R(Y) + \theta - D \). Now, consider a feasible division in which the supplier does not receive the full amount of \( R(Y) + \theta - D \). For example, take \( p(\theta) = \lambda[R(Y) + \theta - D] \) and \( e(\theta) = (1-\lambda)[R(Y) + \theta - D] \), where \( 0 \leq \lambda < 1 \). This cannot be an equilibrium division since the supplier can break away and form a coalition with debt holders in which the division is, for example, \( p(\theta) = \lambda[R(Y) + \theta - D] + .5(1-\lambda)[R(Y) + \theta - D] \) and \( D(\theta) = D + .5(1-\lambda)[R(Y) + \theta - D] \). This outcome is better for both and adds up to \( R(Y) + \theta \), so it is feasible for a debt holders/supplier coalition.

Similarly, take any division of \( R(Y) + \theta \) between the supplier and debt holders in a debt holders/supplier coalition, in which the supplier does not receive the amount of \( R(Y) + \theta - D \). For example, take the division \( p(\theta) = R(Y) + \theta - D - q \) and \( D(\theta) = D + q \). This division is feasible since it adds up to \( R(Y) + \theta \). But, it cannot survive since the supplier can do better in a coalition with equity holders in which the division is, for example, \( p(\theta) = R(Y) + \theta - D - .5q \) and \( e(\theta) = .5q \). This outcome is better for both and adds up to \( R(Y) + \theta - D \), so it is feasible for an equity holder/supplier coalition. Thus, in this region, the only possible outcome is with \( y = Y > 0 \) and receipts of: \( p(\theta) = R(Y) + \theta - D, \ e(\theta) = 0, \ D(\theta) = D. \)

II. Proof of Proposition 1:

Plugging in the supplier’s incentive compatibility constraint, we obtain the corresponding Lagrangian as: \( L(K,D,Y^*(K,D,\gamma,\beta),\gamma,\beta) \equiv V + \phi(1-\gamma) = \delta\{R(Y^*) + E(\theta) - E[D^*(\theta)]\} - e - \gamma^* + \phi(1-\gamma) \) where \( \phi \) is the Lagrangians of the share constraint. The Kuhn-Tucker conditions are, therefore, given by:\(^{53}\frac{\partial L}{\partial \phi} = \frac{\partial V}{\partial \phi} - \phi \leq 0 \leq \gamma \) and \( \frac{\partial L}{\partial \gamma} = 1 - \gamma \geq 0 \leq \phi \), where the notation \( \frac{\partial L}{\partial \phi} \leq 0 \leq \gamma \) is used to denote: \( \frac{\partial L}{\partial \phi} \leq 0, \frac{\partial L}{\partial \gamma} z = 0, \) and \( z \geq 0 \). But, from the definition of \( V \) we have: \( \frac{\partial V}{\partial \phi} = 1 \delta\{\frac{\partial R(Y^*)}{\partial \gamma} - \frac{\partial E[D^*(\theta)]}{\partial \gamma}\} |\gamma^* - 1\{\delta \frac{\partial R(Y^*)}{\partial \gamma}\} (1-\beta)G(\theta^2)\} - 1\} \frac{\partial V}{\partial \gamma} = 1 - (1-\beta)G(\phi^2) > \pi \). But, from the definition of \( \pi \) we have: \( 1 - (1-\beta)G(\phi^2) - \pi = (1-\alpha)(1-G(\theta^3) > 0, \) so that \( 1 - (1-\beta)G(\phi^2) > \pi \).

\(^{53}\)We assume that the required curvature conditions are satisfied.
We also showed above that $\frac{\partial V^*}{\partial \gamma} > 0$. This implies that: $\frac{\partial V}{\partial \gamma} = \delta \frac{\partial R(Y^*)}{\partial \gamma} (1 - (1 - \beta)G(\theta^2)) \frac{\partial V^*}{\partial \gamma} > \delta \frac{\partial R(Y^*)}{\partial Y} \pi \frac{\partial V^*}{\partial \gamma} > \frac{\partial V}{\partial \gamma}$ (since $\delta \frac{\partial R(Y^*)}{\partial Y} \pi = 1$ from the first order condition for the supplier’s problem in equation (13)). Now, from the Kuhn-Tucker conditions we know that if $\gamma < 1$ we have $\phi = 0$. But since $\frac{\partial V}{\partial \gamma} > 0$, we have $\frac{\partial Y}{\partial \gamma} > 0$. In other words, if $\gamma < 1$, $V$ can be increased by increasing $\gamma$ as much as possible. Hence we cannot have a solution at $\gamma < 1$. We must have a solution where $\gamma = 1$, so that $\phi > 0$ and then $\frac{\partial L}{\partial \gamma} = \frac{\partial V}{\partial \gamma} - \phi = 0$. The optimal solution, therefore, involves a franchise arrangement with $\gamma = 1$.

III. Solution and Comparative Statics for $Y^*$ and $D$:

To calculate the effects of a change in $b$ on $Y(b)$, we must take into account that the two constraints, $F(D,Y^*) = 0$ and $B(b,D,Y^*) = 0$, must hold simultaneously. Thus, $D$ and $Y^*(D)$ are interdependent. But, these two equations are simply the debt holders’ strict participation constraint, $\delta E[D^1(\theta)] - b = 0$, and the supplier’s incentive compatibility constraint $\delta \frac{\partial R(Y^*)}{\partial Y} \pi (K, D, Y^*, 1, \beta) - 1 = 0$, which is simply his first order condition, evaluated at $Y = Y^*$, $\gamma = 1$, namely, it is simply $\frac{\partial J(K, D, Y^*, 1, \beta)}{\partial Y} = 0$. From these two equations we get:

$$\begin{bmatrix} J_{YY} & J_{YD} \\ B_Y & B_D \end{bmatrix} \begin{bmatrix} \frac{\partial Y}{\partial b} \\ \frac{\partial D}{\partial b} \end{bmatrix} = \begin{bmatrix} 0 \\ -B_b \end{bmatrix}$$

(29)

where, $B_b \equiv \frac{\partial B}{\partial b} = -1$, $B_D \equiv \frac{\partial B}{\partial D} = \delta (1 - G(\theta^2)) > 0$, $J_{YD} \equiv \frac{\partial^2 J}{\partial Y \partial D} = -(1 - \beta) \delta \frac{\partial R}{\partial Y} g(\theta^2) < 0$, $B_Y \equiv \frac{\partial B}{\partial Y} = \delta (1 - \beta) \int_m^\theta R(Y^*) g(\theta) d\theta > 0$ and $J_{YY} \equiv \frac{\partial^2 J}{\partial Y^2} < 0$, by the concavity of $E[p(\theta)]$.

The effects of debt are, therefore, given by:54

$$\frac{dY}{db} = -\frac{J_{YD} B_b}{|X|} < 0$$

(30)

$$\frac{dD}{db} = -\frac{J_{YY} B_b}{|X|} > 0$$

(31)

where $|X|$ is the determinant of the matrix in (29). The overall effect of an increase in debt is to increases $D$, which in turn decreases $Y^*$.55

10 References:


54Where we assume that the “stability” condition: $|X| < 0$ is satisfied. This condition ensures that the system is stable, in the sense that when $b$ changes, the effect of investment is not “explosive”. A unit change in $Y^*$ affects $D$, which in turn affects $Y^*$.

55Note that $J_{YD} < 0$ and of course $J_{YY} < 0$. 

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Kale, J.R, and H. Shahrur, (2007) “Corporate capital structure and the characteristics of


![Figure 1: Time Line](image)

Figure 1: Time Line
Figure 2: Multilateral Bargaining Regions