On Cash in Advance Constraints for Open Economies

by

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Abstract

The implications of different cash in advance (CIA) constraints for open economies are worked out. If CIA constraints are only for consumption expenditures, changes in the rate of growth of money will have no steady state effects. If all transactions, even those involving bonds, are subject to CIA constraints, an increase in the rate of growth of money will reduce savings and steady state consumption, and have no steady state effects on capital. If investment is not subject to CIA constraints, an increase in the rate of growth of money will increase steady state capital.

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I. Introduction

In the Macroeconomics literature concerned with policy issues for open economies, money is usually introduced into the model by assuming that instantaneous utility is a function of consumption and real money holdings, as in Sidrauski (1967). The seminal papers in this literature are by Obstfeld (1981a,b). In this literature relatively small attention has been paid to the implications of introducing money through cash in advance (CIA) constraints, as in Stockman (1981).[i, ii]

An important result of Stockman's is that the implications of his model, which is for a closed economy, are sensitive to whether there is a CIA constraint on consumption alone or on consumption and investment. He shows that when there is a CIA constraint on investment as well as on consumption then an increase in the rate of growth of money reduces the steady state capital stock, as the CIA acts as a tax on investment. On the other hand when there is a CIA constraint on consumption alone then there is no effect on the steady state capital stock.

In this paper, I consider the implications of different CIA constraints for a small open economy. I start with a utility maximizing representative household with time separable preferences. In that setting, I show that if there is a CIA constraint on consumption alone then, again, changes in the rate of growth of money will have no steady state effects. Then I assume that all transactions, including transactions involving internationally traded bonds, are subject to CIA constraints. With this assumption, which is very much in line with the traditional IS-LM-BP approach to open economy macroeconomics, I show that time separable utility functions, with a fixed rate of time preference, are no longer suitable.[iii] The reason is that with CIA constraints on bonds, the real rate of return on bonds are not constant. Thus, the fixed rate of time preference can no longer be set equal to a fixed rate of interest in order to ensure the existence of a steady state.

To circumvent the above problem, I proceed by employing Uzawa (1968) preferences, which assume that the rate of time preference is an increasing function of instantaneous utility. These are the same preferences which are employed by Obstfeld (1981a,b). It, therefore, allows me to contrast the policy implication of models with CIA constraints with models with money in the utility function.

Obstfeld (1981), abstracting completely from investment, shows that with Uzawa preferences and money in the utility function, an increase in the rate of growth of money will result in a fall in steady state real money holdings and a rise in steady state consumption. The reason is that in his model the rate of time preference must be equal to the real rate of interest in the steady state.[iv] The increase in the rate of growth of money increases the inflation rate. This increases the cost of holding real balances. Hence, there must be a substitution away from real money holdings, and towards consumption, along the same indifference curve in the steady state. In order to increase steady state consumption, the representative agent must be accumulating assets along the adjustment path. Thus, in the short run after the increase in the rate of growth of money there will be a sharp fall in consumption and real money holdings, leading to a current account surplus. After that, both consumption and real money holdings increase along the adjustment path to the new steady state equilibrium.

I show that if we maintain the Uzawa preferences, but introduce money into the model through a CIA constraint on all transactions (even transactions involving bonds) then the adjustment of the economy will be in sharp contrast to Obstfeld's. With CIA constraints on bond purchases, an increase in the rate of growth of money will reduce the rate of return on bonds. This will in the short run reduce savings, which will reduce consumption and real money holdings in the new steady state. Consumption and real money holdings will rise in the short run, and will then be falling along the adjustment path to the new steady state.

I proceed by introducing investment without adjustment costs into the model. I show that if there are CIA constraints on all transactions (including transactions involving assets and investment goods) then an increase in the rate of growth of money will not have any effects on the steady state capital stock.[v] The reason is that then the higher rate of growth of money taxes both bond purchases and investment in the same way.

Next, I consider the case in which investment expenditures are not subject to CIA constraints, while all other transactions are. I show that, then, an increase in the rate of growth of money will increase the steady state capital stock. The reason is that, in this case, the increase in the inflation rate acts as a tax on bonds while it does not act as a tax on capital.[vi] Finally, I conclude that if investment is subject to adjustment costs, some of which are not subject to CIA constraints, because they are intangible (e.g., costs resulting from congestions in the firm), then an increase in the rate of growth of money should increase the steady state capital.

The paper is organized as follows. The problem of the representative agent with time separable preferences is set out in Section II. The effects of changes in the rate of growth of money with Uzawa preferences are considered in Section III. Investment is introduced into the model in Section IV. Some concluding remarks are made in Section V.

II. The Model with Time Separable Preferences

Some of the assumptions of the model correspond closely to Obstfeld's (1981a). The foreign currency price of the single good in the model is fixed at P^*. The economy is small and takes P^* as given. P^* is set equal to 1. The domestic currency price of this good is $P=EP^*$, where E is the exchange rate (the price of foreign currency in terms of domestic currency). The rate of inflation is equal to the rate of depreciation of the domestic currency ($\dot{E}_t/E_t$), which is denoted by ϵ_t.

The preferences of the representative agent are given by a time separable utility function:

$$\int_0^\infty e^{-\theta t} U(c_t)\, dt, \qquad (1)$$

where θ is his (fixed) rate of time preference, and c_t is his consumption at time t.

The agent is endowed with y units of the good at any time t. He also receives monetary transfers with real values of τ_t from the government. There are two kinds of assets in the model, money balances and internationally traded bonds. These bonds have a fixed price of unity in terms of the foreign currency (or in terms of goods, as $P^*=1$), and each bond pays r units of the foreign currency at any time. The real assets of the representative agent are

$$a_t = m_t + b_t, \qquad (2)$$

where b_t is his bond holdings, and m_t his real domestic money holdings.

His flow budget constraint is

$$\dot{a}_t = y + rb_t + \tau_t - c_t - \epsilon_t m_t; \qquad (3)$$

and he should satisfy the intertemporal solvency condition

$$\lim_{t \to \infty} e^{-rt} a_t \geq 0. \qquad (4)$$

I will be considering different types of CIA constraints. I will start with the simplest case, in which the CIA constraint is on consumption only:

$$m_t = c_t. \qquad (5)$$

the representative agent's problem is to maximize (1), subject to (2)–(5), and the initial condition a_0, taking the paths of $\{\tau_t\}$, and $\{\varepsilon_t\}$ as given. Along a perfect foresight path, the agent's expectations about $\{\tau_t\}$ and $\{\varepsilon_t\}$ coincide with the actual paths of these variables.

In this paper we abstract completely from government expenditures on goods and services, and concentrate exclusively on monetary policies.[vii] The government chooses the real monetary transfers τ_t in order to satisfy its flow constraint, which says that it should finance its expenditures (τ_t) from the interest on its bond holdings and the inflation tax:

$$\tau_t = rR_t + \varepsilon_t m_t , \qquad (6)$$

where R_t is the central bank's holdings of foreign reserves, which are in the form of internationally traded bonds. I will assume that the exchange rate system is flexible, in which case, with no central bank intervention on the foreign exchange market, R_t is constant.

To solve the representative agent's problem maximize (1) subject to (2)–(5), and the initial condition a_0. Using (2) and (5), the current value Hamiltonian for his problem can be written as

$$H = U(c_t) + \lambda_t \left[r(a_t - c_t) + y + \tau_t - (1 + \varepsilon_t) c_t \right], \qquad (7)$$

where λ_t is the shadow price of a_t.

The optimality conditions are:

$H_c = 0$:

$$U'(c_t) - \lambda_t (r + \varepsilon_t + 1) = 0 , \qquad (8)$$

$$-H_a + \theta_t \lambda_t = \dot{\lambda}_t : \quad (\theta - r) \lambda_t = \dot{\lambda}_t , \qquad (9)$$

and the transversality condition.

From (9) it is clear that a steady state can be reached only if

$$r = \theta . \qquad (10)$$

This is a standard assumption that is made in the literature for models involving a small open economy with a fixed rate of time preference. (See, for example, Sen and Turnovsky (1989) and Mansoorian (1996).)

One can easily show that with the CIA constraint on consumption alone, changes in the rate of growth of money will leave the steady state unaffected. This will be proven in the next section, because at this stage I would like to show that assumption (10) cannot be made when the CIA constraint is placed on all transactions, even those involving bonds.

As explained in footnote 3 above, in the traditional IS-LM-BP models it was assumed that all transactions, even those involving bonds, are subject to CIA constraints. With such CIA constraints, (5) should be replaced by

$$m_t = c_t + {}_t, \qquad (11)$$

where ${}_t$ is the representative agent's bond purchases at time t. This constraint can be re-written as

$${}_t = m_t \,!\, c_t. \qquad (12)$$

The representative agent's problem will then be to maximize (1) subject to (2)!(4), (12), and the initial conditions a_0, b_0. Using (2) to eliminate m_t, the current value Hamiltonian for his problem can be written as

$$H = U(c_t) + \lambda_t [r b_t + y + \tau_t - c_t - \epsilon_t (a_t - b_t)] + \mu_t [(a_t - b_t) - c_t], \qquad (13)$$

where 8_t and μ_t are the shadow prices of a_t and b_t, respectively.

The optimality conditions are:

$H_c = 0$:

$$U'(c_t) - \lambda_t - \mu_t = 0, \qquad (14)$$

$$-H_a + \theta_t \lambda_t = \dot{\lambda}_t : \quad (\theta + \epsilon_t) \lambda_t - \mu_t = \dot{\lambda}_t, \qquad (15)$$

$$-H_b + \theta_t \mu_t = \dot{\mu}_t : \quad (\theta + 1) \mu_t - (r + \epsilon_t) \lambda_t = \dot{\mu}_t, \qquad (16)$$

and the standard transversality conditions.

Equations (15) and (16) are inconsistent with a steady state. Setting ==0, one can derive the following relationship which must hold in a steady state:

$$(1 + \theta) = \frac{(r + \epsilon)}{(\theta + \epsilon)}. \qquad (17)$$

In the steady state **,** (the inflation rate) will be equal to the rate of growth of money (which is determined by the government), while r and θ are fixed. Thus, (17) will in general not hold; and one can conclude that time separable preferences are not useful in studying the implications of this model.

The reason for the above conclusion is that with CIA constraints on bond purchases, changes in the rate of growth of money affect the real rate of return on bonds. There is, therefore, not a fixed real rate of return to which the rate of time preference could be set equal, in order to ensure a well defined steady state, as is usually possible in small open economy models with fixed rates of time preference. For this reason, in the next section Uzawa preferences will be used in order to work out the implications of different CIA constraints.

III. The Model with Uzawa Preferences

With Uzawa preferences, the rate of time preference is an increasing function of instantaneous utility:

$$\int_0^\infty e^{-\int_0^t \theta_v \, dv} \, U(c_t) \, dt, \tag{18}$$

where $\theta_v=\theta(U(c_v))$, with $\theta'(\cdot)>0$, $\theta''(\cdot)>0$, and $\theta(\cdot)-\theta'(\cdot)U(c_v)>0$.

This assumption, regarding the rate of time preference, circumvents the problem encountered in the previous section. It, therefore, allows me to work out the effects of different CIA constraints for the open economy.

With CIA constraints on consumption alone, the problem of the representative agent will now be to maximize (18), subject to (2)–(5) and the initial condition a_0. In order to solve this problem, it is easiest to follow Uzawa and Obstfeld and employ a change of variable from actual time t to psychological time as follows. First note that the preferences described by (18) can be expressed as

$$\int_0^\infty e^{-\Delta_t} \; U(c_t) \; dt. \tag{19}$$

where

$$\frac{d\Delta_t}{dt} \; = \; \theta(U(c_t)) \tag{20}$$

From (20), $dt \; = \; d\Delta / \theta(U(c_t))$. Use this result in (19), and also in (3), noting that $\dot{a}_t \; = \; \frac{da}{d\Delta} \; \theta(U(c_t))$, in order to change the variables from actual time t to psychological time). The Hamiltonian for this problem will then be:

$$\hat{H} \; = \; \frac{H}{\theta(U(c))}, \tag{21}$$

where H is given by (7).

One can readily show that condition (9) should still hold (except that now **2** is not fixed, but is a function of instantaneous utility), while condition (8) is replaced by

c=0:

func { U' (c sub t) `` - `` lambda sub t `(r `` + `` epsilon sub t `` + `` 1)`` -``{ theta ' (U(c sub t)) ` U'(c sub t) } `` H hat ``= ``0 . }(22)

From (9) it is clear that the steady state level of consumption will not be affected by changes in the rate of growth of money (or the steady state inflation rate). Using the government budget constraint (6) in the representative agent's flow constraint (3), it is clear that the steady state bond holdings of the representative agent is also unaffected. (Recall that R_t is constant.) It follows that the country's steady state net foreign asset position (R+b) will also be unaffected.

This is analogous to Stockman's result for a closed economy. With the CIA constraint on consumption, an increase in the inflation rate increases the costs corresponding to consumption in the present period. The CIA constraint on consumption also increases the costs corresponding to consumption resulting from future interest payments on bonds. Hence, the increase in the inflation rate reduces the benefits from current consumption and the benefits from consumption of future returns on current bond holdings by the same amount, leaving the steady state unaffected.

Next, consider the case in which there are CIA constraints on all transactions, even those involving bonds. The problem of the representative agent will now be to maximize (18), subject to (2)!(4), (12) and the initial conditions a_0 and b_0. Switching from real to psychological time, and writing the current value Hamiltonian =H/(**2**(U(c)), where H is given by (13), one can readily show that conditions (15) and (16) should still hold (except that now **2** is not fixed, but is a function of instantaneous utility), while condition (14) is replaced by

c=0:

func {
U' (c sub t) `` - `` lambda sub t `` - `` mu sub t `` -
``{ theta ' (U(c sub t )) ` U'(c sub t )} `` H hat `` = `` 0 . }(23)

As before, with ==0, (17) must hold in the steady state. Thus, with **2** a function of instantaneous utility, we will have

func{ {d c bar } over {d epsilon } ~ = ~ { - theta} over { theta ' `` left \[ ` 2 theta `` + `` epsilon `` + `` 1 `right \] } ~ < ~ 0 . } (24)

Steady state consumption will fall after an increase in the rate of growth of money (an increase in steady state inflation ,). From (11), the steady state real money holdings of the representative agent also falls.

Using the government budget constraint (6) in (3), it is clear that the steady state bond holdings of the representative agent will fall with the rise in the rate of growth of money. With smaller, and R unaffected, the country's steady state net foreign asset position deteriorates.

The intuition for these results is as follows. An increase in the rate of growth of money acts as a tax on bond purchases. This reduces the return on savings, increasing consumption (and likely also the real money holdings) in the short run.[viii] The country, therefore, runs a current account deficit, and both c and m fall along the adjustment path to the steady state equilibrium.

It will be instructive to contrast these results with Obstfeld's (1981a,b), who also uses Uzawa preferences, but introduces money into the model as an argument in the instantaneous utility function. In Obstfeld's model there is a unique level of utility which must be maintained in the steady state. This is determined by the equality of the rate of time preference with the real rate of interest (which is unaffected by monetary policy). Thus, an increase in the rate of growth of money, by increasing steady state inflation, will result in a fall in steady state real money holdings and a rise in steady state consumption. In order to increase steady state consumption, the representative agent must be accumulating assets along the adjustment path. Thus, in the short run after the increase in the rate of growth of money there will be a sharp fall in consumption and real money holdings, leading to a current account surplus. After that, both consumption and real money holdings increase along the adjustment path to the new steady state equilibrium.

Hence, the results obtained with CIA constraints are in sharp contrast to Obstfeld's. As pointed out in footnote 3, the assumption that all transaction, including those involving bonds, are subject to CIA constraint was implicit in the traditional models involving the IS-LM-BP framework.

IV. The Model with Investment

Until now the discussion has been confined to an endowment economy. In this section, I assume that output is produced with a neo-classical production function, $y_t=f(k_t)$, where k_t is the capital stock at time t, fN(")>0, and fNN(")<0. Substituting for y into (3), we obtain the representative agent's flow budget constraint

$$\dot a_t = f(k_t) + rb_t + \tau_t - c_t - \epsilon_t m_t, \quad (25)$$

Assuming for simplicity that all domestic capital is held by the representative agent, his total asset holdings are now

$$a_t = m_t + b_t + k_t. \quad (26)$$

When all expenditures are subject to CIA constraints, we have

$$m_t = c_t + {}_t + I_t, \quad (27)$$

where I_t denotes Investment expenditures at time t. One can re-write (27) as

$${}_t = m_t \,!\, c_t \,!\, I_t. \quad (28)$$

Without depreciation and perfect reversibility of capital, we will have

$${}_t = I_t. \quad (29)$$

The problem of the representative agent is now to maximize (18), subject to (25), (26), (28), (29), (4), and the initial conditions, a_0, b_0 and k_0. Using (26) to eliminate m_t, and switching from real time t to psychological time **)**, the current value Hamiltonian for his problem can be written as

$$\hat H = \frac{1}{\theta(U(c))}\left[U(c_t) + \lambda_t\left[r b_t + f(k_t) + \tau_t - c_t - \epsilon_t(a_t - b_t - k_t)\right] + \mu_t\left[(a_t - b_t - k_t) - c_t - I_t\right] + \omega_t I_t\right], \quad (30)$$

where **8**$_t$, μ_t, and **T**$_t$ are the shadow prices of a_t, b_t, and k_t respectively.

The optimality conditions are:

$_c = 0$:

$$U'(c_t) - \lambda_t - \mu_t - \theta'(U(c_t))\, U'(c_t)\hat H = 0, \quad (31)$$

$_I = 0$: **T**$_t$! $\mu_t = 0$ (32)

$$-d[H e^{-\Delta}]/da = d[\lambda e^{-\Delta}]/d\Delta: \quad (\theta + \epsilon_t)\lambda_t - \mu_t = \dot\lambda_t, \quad (33)$$

$$-d[H e^{-\Delta}]/db = d[\mu e^{-\Delta}]/d\Delta: \quad (\theta + 1)\mu_t - (r + \epsilon_t)\lambda_t = \dot\mu_t, \quad (34)$$

$$-d[H e^{-\Delta}]/dk = d[\omega e^{-\Delta}]/d\Delta: \quad \theta_t\omega_t + \mu_t - [f'(k_t) + \epsilon_t]\lambda_t = \dot\omega_t, \quad (35)$$

and the standard transversality conditions.

From (32), **T**$_t$ = μ$_t$ for all t. Using this in (35), and then noting that in a steady state ==0, it follows from (34) and (35) that the steady state capital stock, , is determined by

$$fN() = r. \qquad (36)$$

With r fixed, the steady state capital stock is unaffected by an increase in the rate of growth of money. The reason is that, with CIA constraints on both investment and bond purchases, changes in the rate of growth of money acts as a tax on both capital and bond holdings by exactly the same amount.[ix]

Moreover, (33) and (34) imply that (17) must hold in the steady state; and, therefore, an increase in the rate of growth of money must reduce the steady state consumption (result (24)). As in Section III, the increase in the rate of growth of money reduces the real rate of return on bonds and investment, reducing savings and steady state consumption.

Now consider the case in which investment is not subject to CIA constraint, while all other transactions are. Then, the representative agent will still face constraints (25), (26), (29) and (4), as before, while constraint (28) will be replaced by

$$_t = m_t \,!\, c_t. \qquad (34)$$

Writing the current value Hamiltonian with psychological time, it can be easily shown that conditions (31), and (33)!(35) will be the same as before, while condition (32) becomes

$$\mu_t = 0. \quad (38)$$

Substituting for μ$_t$ from (38) into (33) and (35), it can be easily shown that in the steady state we should have

$$fN() = \mathbf{2}(U()). \quad (39)$$

Moreover, (33) and (34) also imply that (17) must hold in the steady state; and, therefore, an increase in the steady state inflation rate **,** (or an increase in the rate of growth of money) will lead a fall in steady state consumption (result (24)). The fall in , from (39), implies that the steady state capital stock must increase (recall that **2**N(")>0 and fNN(")<0).

The reason for these results is as follows. An increase in the rate of growth of money, by increasing the inflation rate, acts as a tax on bond holdings, while without CIA constraints on investment, it leaves the return on capital unaffected. As a result, investment in capital becomes more attractive. At the same time, as in Section III, the lower return on bonds reduces savings, and steady state consumption.

One can conclude that as long as there are some adjustment cost for investment which are not subject to CIA constraints, then an increase in the rate of growth of money will lead to an increase in investment, and the steady state capital stock.[x]

V. Conclusions

In the literature concerned with the policy issues for open economies, money is usually introduced into the model by assuming that instantaneous utility depends on real money holdings. In this literature, relatively little attention has been paid to the alternative method of introducing money into the model ! through CIA constraints.

This paper has explored the implications of different types of CIA constraints for open economies. It was shown that if only consumption expenditures are subject to CIA constraints then changes in the rate of growth of money would not have any steady state effects.

It was argued (footnote 3) that the implicit assumption in the traditional literature, employing the IS-LM-BP model, was that all transactions, including those involving bonds, were subject to CIA constraints. It was shown that with CIA constraints on all transactions, utility functions involving fixed rates of time preference are no longer useful. The reason was that, then, changes in the rate of growth of money affect the rate of return on assets, and, therefore, there is no fixed real rate of interest to which the rate of time preference could be set equal.

To circumvent the above problem, Uzawa preferences were used to show that if all transactions are subject to CIA constraints, then an increase in the rate of growth of money will reduce savings, bringing about a current account deficit, and reducing steady state consumption and real money holdings. The results were compared to Obstfeld's, where Uzawa preferences were also used, but money was introduced into the model by assuming that instantaneous utility depends on real balances. It was shown that the results with CIA constraint were in sharp contrast to those with money in the utility function.

Finally, investment was introduced into the model. It was shown that if all transactions were subject to CIA constraints then an increase in the rate of growth of money would leave the steady state capital unaffected. The reason was that then the higher inflation rate would reduce the return on bonds and capital in the same way. On the other hand, if investment is not subject to CIA constraints then an increase in the rate of growth of money will increase steady state capital, because, then, the higher inflation would reduce the return on bonds, and not the return on capital. It was argued that if there are some investment adjustment costs which are not subject to CIA constraints then an increase in the rate of growth of money would increase steady state capital.

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Appendix

Proof of the Results on Page 9

With Uzawa preferences we have

func { {INT _0^ INF } ~ e sup {- DELTA sub t } `` U(c sub t ) `` dt. }   (19)

where

func{ {d DELTA sub t} over {dt} `` =`` theta (U(c sub t))

.

}(20)

From (20),

func{ {d t} `` =`` {d DELTA} / { theta (U (c sub t)) } .}Using this result in (19), we can change time from

actual time t to psychological time **)**. Preferences can then be written as

func { {INT _0^ INF } ~ e sup {- DELTA } `` U(c ) `` d DELTA. }

(A.1)

Moreover, from (20) it follows that

func{ a dot sub t `` =`` {d a sub t} over {dt} `` =`` {theta (U(c sub t))}``{ d a } over { d DELTA}

.

}(A.2)

Using (A.2) in (3), we can express the law of motion for a in psychological time as

func{ {d a } over { d DELTA} `` = `` 1 over {theta (U(c))} `` left \[ ` y `` + `` r` b `` + `` tau `` - `` c `` - `` epsilon ` m ` right \]

.

}(A.3)

Hence, with psychological time the Hamiltonian for this problem will be:

func{ H hat `` = `` H over {theta (U(c))}

},

}(21)

where H is given by (7). The optimality conditions will be

$_c$=0:

func { U' (c sub t) `` - `` lambda sub t `(r `` + `` epsilon sub t `` + `` 1)`` -``{ theta ' (U(c sub t)) ` U'(c sub t) }

`` H hat ``= ``0, }(22)

and

func{

{d` left \[ e sup {- DELTA} ` lambda right \]} over {d` DELTA} `` = `` - `{d` left \[e sup {- DELTA} ` H hat right \]} over {d`

a}

} Y

$$-\lambda + \frac{d\lambda}{d\Delta} = -\hat{H}_a . \qquad \text{(A.4)}$$

Using (20) in (A.4), we obtain

$$-\lambda + \frac{1}{\theta}\frac{d\lambda}{dt} = -\hat{H}_a ,$$

which is equation (9) in the text.

The results in the rest of the paper can be similarly derived. Roughly, with Uzawa preferences, the differential equations for co-state variables along the optimal path (such as (9)), and the optimality conditions with respect to any control variable other than c will be as with time separable preferences. The optimality condition with respect to c will have an extra term !2NUN on the left hand side.

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[i] On exception to this is Calvo (1987), who considers a balance of payment crisis in an open economy with a cash in advance constraint on consumption.

[ii] According to Turnovsky (1997, p. 20), the reason for such scant attention paid to the CIA approach may be that "One difficulty with this approach is that the introduction of the various constraints, embodying the role played by money in transactions, can very quickly become intractable. Accordingly, a shorthand alternative to this . . . is to introduce money into the utility function." See also Turnovsky (1995, p. 265). Similarly, Blanchard and Fischer (1989, p. 155) state that "Models based explicitly on (CIA) constraints . . . can quickly become analytically cumbersome. Much of the research on the effects of money has taken a different shortcut, that of . . . putting real money services directly in the utility function."

[iii] In the IS-LM-BP framework with perfect capital mobility, when domestic interest rates were above (below) foreign rates, there was a huge capital inflow (outflow), and balance of payments surplus (deficit). This implied an excess demand for (supply of) the domestic currency on the foreign exchange market, which raised (lowered) the value of the domestic currency. The implicit assumption here is that all transactions (even the ones involving bonds) are financed with money.

[iv] With money in the utility function the real rate of return on bonds is not affected by changes in the rate of growth of money.

[v] This is contrary to Stockman's conclusion for a closed economy. In his model, if there is a CIA constraint on investment then the steady state capital will fall when there is an increase in the rate of growth of money.

[vi] Again, this is contrary to Stockman's findings for a closed economy. He shows that if there is no CIA constraint for investment then the steady state capital is unaffected by an increase in the rate of growth of money.

[vii] Obstfeld (1981a) allows for government purchases of goods and services. He, however, assumes that such government expenditures are unproductive, or, alternatively, that the utility of

the representative agent is strongly separable between private consumption and public goods. We could make this assumption without changing any of our results.

[viii] From (11), if in the short run c_t increases while $_t$ falls, m_t may fall or rise.

[ix] This results for the open economy is in contrast to Stockman's result for a closed economy. In Stockman's model, when there was a CIA constraint on all transactions, an increase in the rate of growth of money reduced the steady state capital stock, because in his model the increase in the rate of steady state inflation rate taxed holdings of capital, reducing savings and thus investment.

[x] Investment adjustment costs which would not be subject to CIA constraints would include intangible costs resulting from congestion in the firm.