A Note on Endogenous Time Preference and Monetary Non-Superneutrality

by

Eric Kam,
Arman Mansoorian,
and
John Smithin,

Department of Economics
York University
Toronto, Ontario
Canada, M3J 1P3

Abstract

We suggest a simple variant of Uzawa preferences which has the same predictions as his formulation, but is less prone to criticism. We assume that the rate of time preference is an increasing function of the total value of current financial assets. It is shown that an increase in the rate of money growth will initially reduce the real value of financial assets, reducing the rate of time preference, increasing savings and the steady state capital stock. This provides a restatement of the Mundell-Tobin effect in an optimizing framework.

JEL classifications: F31, F32, F41
Keywords: Monetary Non-Superneutrality, Time Preference, Financial Assets

1. Introduction

The Uzawa (1968) utility function, in which the rate of time preference is an increasing function of instantaneous utility, has received considerable attention as well as criticism. Some authors have argued that the assumptions underlying the Uzawa function are counterintuitive. In this note we suggest a simple reformulation of the Uzawa function which emphasizes the similarities between his formulation and the more traditional aggregative (IS-LM type) models.

The original intention of Uzawa was to break the superneutrality result of Sidrauski (1967) with regard to the effects of monetary policy in a model with money in the utility function. Sidrauski showed that in a model with a fixed rate of time preference, changes in the rate of growth of money will not have any effect on the steady state capital stock, which is determined by the equality of the rate of time preference and the marginal productivity of capital.

Uzawa showed that with his formulation, an increase in the rate of growth of money would increase the steady state capital stock. The reason is that an increase in the rate of growth of money increases the opportunity cost of holding real balances, rendering the initial steady state equilibrium too costly to maintain. This results in a fall in consumption and real money holdings, and a corresponding increase in savings and the capital stock.

The Uzawa function has also received a good deal of attention in the open economy literature concerned with the Harberger-Laursen-Metzler (H-L-M) effect. In a seminal paper, Obstfeld (1982) showed that with Uzawa preferences, a deterioration in the terms of trade would result in an increase in savings and a current account surplus for a small open economy facing a fixed world rate of interest. Obstfeld's findings were contrary to the intuition of H-L-M, and this gave rise to a large literature on examining the effect of a terms of trade deterioration in optimizing frameworks.[1]

Obstfeld's result depended critically on the assumption that for stability of the steady state equilibrium, the rate of time preference must be an increasing function of instantaneous utility. This assumption is viewed as being "arbitrary, and even counter-intuitive" (Persson and Svensson, 1985, p.45). Blanchard and Fisher (1989, pp. 74-75) go further and argue that "the Uzawa function…is not particularly attractive as a description of preferences and is not recommended for general use."

In this note, we suggest a simple variation of the Uzawa function which has the same predictions as the Uzawa formulation, but has the important advantage that it is consistent with the older literature which used aggregative (IS-LM type) models. In this literature, it was customary to view consumption as an increasing function (and savings a decreasing function) of financial wealth. Mundell (1963) and Tobin (1965) used such a formulation in order to discuss the effects of an increase in the rate of growth of money on the capital stock. Such specifications of the savings function were also the centerpiece of the monetary approach to the balance of payments, as exemplified by the important contributions of Dornbusch (1973a, b).

In this paper, we assume that the rate of time preference is not an increasing function of instantaneous utility, but an increasing function of the total value of real financial assets held by the representative agent. Such a formulation is consistent with regarding the savings function as being a decreasing function of financial wealth, as was done in aggregative models. To remain consistent with these models, in our formulation, reducing the real value of financial assets will increase the rate of growth of money, reduce the rate of time preference and increase both savings and the steady state

[1] Some of the prominent papers in this literature are by Svensson and Razin (1983), Persson and Svensson (1985), Matsuyama (1987, 1988), and Sen and Turnovsky (1989).

capital stock. These conclusions are consistent with those derived by Uzawa, but they are not prone to the same criticisms. Our formulation provides a restatement of the Mundell-Tobin effect in an optimizing framework. In terms of the open economy literature, it also implies that the results derived by Obstfeld were not very much out of line with the more traditional approach to the balance of payments.[2]

2. The Model

The representative agent maximizes:

$$U_0 = \int_0^\infty u(c_t, m_t) e^{-\int_0^t \theta_v dv} dt \tag{1}$$

where c and m represent, respectively, consumption and real money holdings; and the rate of time preference θ_v is assumed to be an increasing function of the total value of the financial assets (a_v) held by the agent at time v:

$$\theta_v = \theta(a_v) \text{ with } \theta' > 0 \tag{2}$$

By assumption, u_c, u_m and $u_{cm} > 0$, while u_{cc} and $u_{mm} < 0$. The agent's flow budget constraint is:

$$\dot{a}_t = f(k_t) + x - c_t - \pi_t m_t \tag{3}$$

where $f(\cdot)$ is a constant returns to scale production function, k is the capital stock, x is the real value of government's transfers of money and π is the inflation rate. The agent also faces the stock budget constraint:

$$\lim_{t\to\infty} a_t e^{-\int_0^t r_v dv} \geq 0 \tag{4}$$

where r represents the real interest rate.

[2] On this point, also see Mansoorian (1992).

Maximizing (1) subject to (3) and (4) yields the following optimality condition, amongst others:

$$\frac{u_m(c,m)}{u_c(c,m)} = r + \pi \tag{5}$$

which requires the marginal rate of substitution between c and m at any time is equal to the nominal interest rate.

This maximization problem also requires that in the steady state the rate of time preference be equal to the interest rate:

$$r^* = \theta(m^* + k^*) \tag{6}$$

where stars denote steady state values. Also, note that in the steady state we should have $\dot{a} = \dot{k} = 0$. The latter, with the goods market clearing condition, implies:

$$f(k^*) = c^* + \delta k^* \tag{7}$$

where δ is the depreciation rate for capital.

From (7) and (3), with $\dot{a} = 0$, we obtain:

$$x^* = \pi^* m^* \text{, or } \mu m^* = \pi^* m^* \tag{8}$$

where μ is the rate of growth of nominal money balances.

To obtain the effects of an increase in μ on m^*, k^* and c^*, set $\mu = \pi^*$ and use the fact that $r = f'(k^*) - \delta$ in order to obtain from equations (5) to (7):

$$\frac{dm^*}{d\mu} = \left[\frac{u_{mm}}{u_c} - (f'(k) - \delta + \mu)\frac{u_{cm}}{u_c} + \frac{(u_{cm} - u_{cc})(f'(k) - \delta) - f''(k)u_c}{\left[\frac{f''(k)}{\theta'} - 1\right]u_c} \right]^{-1} < 0 \tag{9}$$

$$\frac{dk^*}{d\mu} = \left[\frac{f''(k)}{\theta'} - 1\right]^{-1} \frac{dm^*}{d\mu} > 0 \tag{10}$$

Thus, with this formulation, there is no monetary superneutrality. It can also be shown that the assumption $\theta' > 0$ ensures that the model exhibits saddlepoint stability.

3. Conclusions

In this note, we have suggested a simple variant of Uzawa preferences which has the same predictions as Uzawa's, but is more in line with the traditional literature in which savings was viewed as a decreasing function of the total value of financial assets. Our formulation is not subject to the criticisms traditionally directed at the Uzawa formulation. We assumed that the rate of time preference is an increasing function of the total value of financial assets. We have shown that an increase in the rate of growth of money will increase the capital stock in this economy. The reason is that the increase in the rate of growth of money will initially reduce the real value of financial assets. The will reduce the rate of time preference, and increase savings. This provides a restatement of the Mundell-Tobin effect in an optimizing framework.

Appendix

Solution to the Representative Agent's Problem

The rate of time preference is increasing in financial wealth:

$$\theta = \theta(a) = \theta(k+m) \text{ and } \theta'(k+m) > 0 \tag{1}$$

The representative household maximizes:

$$MaxU_0 = \int_0^\infty [U(c_t, m_t) e^{-\int_0^t \theta_v dv} dt \tag{2}$$

subject to the constraints:

$$\dot{a} = f(k) + x - \pi m - \delta k - c \tag{3}$$

$$a = k + m \tag{4}$$

$$\lim_{t \to \infty} a_t e^{\int_0^t r_v dv} \geq 0 \tag{5}$$

We make the standard assumptions:

$$U_c > 0,\ U_{cc} < 0,\ U_m > 0,\ U_{mm} < 0,\ U_{cm} = U_{mc} > 0 \tag{6}$$

and adopt the following notation:

$$Q = U_{cc}U_{mm} - U_{cm}^2 > 0,\ Q_1 = U_{mm} - U_{cm}\left(\frac{U_m}{U_c}\right) < 0,\ Q_2 = U_{cc}\left(\frac{U_m}{U_c}\right) - U_{cm} < 0 \tag{7}$$

This produces the Hamiltonian:

$$H = U(c,m) + \lambda\left[f(a-m) + x - \pi m - \delta(a-m) - c\right] \tag{8}$$

and the optimality conditions:

$$U_c(c,m) - \lambda = 0 \tag{9}$$

$$U_m(c,m) - \lambda[f'(k) - \delta + \pi] = 0 \tag{10}$$

$$\dot{\lambda} = -\lambda[f'(k) - \delta - \theta(k+m)] \tag{11}$$

$$\lim_{t\to\infty} a_t \lambda_t e^{-\int_0^t \theta_v dv} = 0 \qquad (12)$$

From the first order conditions and assuming a steady state, we obtain that

$$x^* = m^* \pi^* = \mu m^* \qquad (13)$$

From (9) and (10) we obtain:

$$\frac{U_m(c,m)}{U_c(c,m)} = f'(k) - \delta + \pi = r + \pi \qquad (14)$$

The steady state levels of m, c, and k are given by (14), (11) with $\dot{\lambda} = 0$, and with the resource constraint with $\dot{k} = 0$. Differentiating these three equations at the steady state we obtain equations (9) and (10) from the main text

<u>Derivation of the Stability Condition</u>

Here we show that the differential equation system of m, c, and k exhibit saddle point stability when $\theta' = 0$. We begin with the evolution of the capital stock which is given by the resource constraint:

$$\dot{k} = f(k) - \delta k - c \qquad (15)$$

To solve for the evolution of real money balances, recall that:

$$\dot{m} = (\mu - \pi)m \qquad (16)$$

Solving (14) for π and substituting into (16) we obtain:

$$\dot{m} = \left(\mu - \frac{U_m}{U_c} + f'(k) - \delta \right) m \qquad (17)$$

To solve for the evolution of consumption, start first order condition (9) which implies:

$$U_{cc}\dot{c} + U_{cm}\dot{m} = \dot{\lambda} \qquad (18)$$

Substituting for $\dot{\lambda}$ and $\dot{m}$ from (11) and (17) into (20) and using (9) we obtain:

$$\dot{c} = \frac{1}{U_{cc}} \left[U_c(c,m)\big(\theta(k+m) - f'(k) + \delta\big) - U_{cm}\left(\mu - \frac{U_m}{U_c} + f'(k) - \delta \right) m \right] \qquad (19)$$

Equations (15), (19) and (21) provide a differential equation system in c, m, and k. Linearizing the equations around the steady state (c^*, m^*, k^*) we obtain:

$$\begin{bmatrix} \dot{c} \\ \dot{m} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} \frac{-U_{cm}Q_2 m^*}{U_{cc}U_c} & \frac{\theta' U_c^2 + U_{cm}Q_1 m^*}{U_{cc}U_c} & \frac{[-f''(k^*)(U_c + U_{cm}m^*)] + \theta' U_c}{U_{cc}} \\ \frac{Q_2 m^*}{U_c} & \frac{-Q_1 m^*}{U_c} & f''(k^*)m^* \\ -1 & 0 & \theta \end{bmatrix} \begin{bmatrix} c - c^* \\ m - m^* \\ k - k^* \end{bmatrix}$$

The determinant of the above matrix is:

$$|\Delta| = -1 \begin{vmatrix} \frac{U_{cm}Q_1 m^* + \theta' U_c^2}{U_{cc}} & \frac{[-f''(k^*)(U_{cm}m^* + U_c)] + \theta' U_c}{U_{cc}} \\ \frac{-Q_1 m^*}{U_c} & f''(k^*)m^* \end{vmatrix}$$

$$+\theta \begin{vmatrix} \frac{-U_{cm}Q_2 m^*}{U_{cc}U_c} & \frac{\theta' U_c^2 + U_{cm}Q_1 m^*}{U_{cc}} \\ \frac{Q_2 m^*}{U_c} & \frac{-Q_1 m^*}{U_c} \end{vmatrix} \qquad (22)$$

The first part of (22) is:

$$(-1)\left[\frac{f''m^{*2}U_{cm}Q_1 + f''m^*U_c^2\theta' - f''m^*Q_1(U_{cm}m^*) + Q_1 m^*\theta' U_c}{U_{cc}U_c}\right]$$

$$= \left[\frac{-f''m^{*2}U_{cm}Q_1 - \theta' U_c^2 f''m^* + f''m^{*2}U_{cm}Q_1 + f''Q_1 m^* U_c - Q_1 m^* \theta' U_c}{U_{cc}U_c}\right]$$

$$= \left[\frac{-\theta' U_c^2 f''m^* + f''Q_1 m^* U_c - Q_1 m^* \theta' U_c}{U_{cc}U_c}\right] = \left[\frac{-\theta' f''U_c + f''Q_1 - \theta' Q_1}{U_{cc}U_c}\right]$$

$$= \frac{Q_1 f'' - \theta'(U_c + Q_1)}{U_{cc}U_c} < 0$$

The second part of (22) is:

$$= \theta\left[\left(\frac{U_{cm}Q_2 m^{*2} Q_1}{U_c^2 U_{cc}}\right) - \left(\frac{Q_2 m^* U_c^2 \theta' + Q_2 m^{*2} U_{cm} Q_1}{U_c^2 U_{cc}}\right)\right]$$

$$= \theta\left[\frac{U_{cm}Q_2 m^{*2} Q_1 - Q_2 m^* U_c^2 \theta' - Q_2 m^{*2} U_{cm} Q_1}{U_c^2 U_{cc}}\right]$$

$$= \frac{-\theta\theta' Q_2 m^* U_c^2}{U_c^2 U_{cc}} = \frac{-\theta\theta' Q_2 m^*}{U_{cc}} < 0$$

The determinant is the sum of two negative parts; therefore $|\Delta| < 0$.

The trace of the matrix is:

$$trace = -\frac{U_{cm}Q_2 m^*}{U_{cc}U_c} - \frac{Q_1 m^*}{U_c} = -m^*\left[\frac{Q_1}{U_c} + \frac{U_{cm}Q_2}{U_{cc}U_c}\right] + \theta \tag{23}$$

Using our assumptions, (23) can be reduced:

$$trace = -m^*\left[\frac{U_{mm} - U_{cm}\left(\frac{U_m}{U_c}\right)}{U_c} + \frac{U_{cm}\left[U_{cc}\left(\frac{U_m}{U_c}\right) - U_{cm}\right]}{U_{cc}U_c}\right] + \theta$$

(24)

$$trace = -m^*\left[\frac{U_{mm}U_{cc} - U_{cc}U_{cm}\left(\frac{U_m}{U_c}\right) + U_{cm}U_{cc}\left(\frac{U_m}{U_c}\right) - U_{cm}^2}{U_{cc}U_c}\right] + \theta = -m^*\left[\frac{Q}{U_{cc}U_c}\right] + \theta > 0$$

Since the matrix has a negative determinant and a positive trace, only one of the three characteristic roots is negative. The system defined by $(\dot{c}, \dot{m}, \dot{k})$ is a stable saddle point.[3]

[3] Note that our assumption, $\theta' > 0$, ensures that the determinant is negative. Also, note that when $\theta' = 0$, the determinant reduces to that derived by Sidrauski.

Bibliography

Blanchard, O.J., and S. Fischer (1989) "*Lectures on macroeconomics*." (Cambridge: MIT Press)

Dornbusch, R (1973a) "Currency depreciation, hoarding and relative prices." *Journal of Political Economy* 81, 893-915

-----(1973b) "Money, devaluation and nontraded goods." *American Economic Review* 63, 871-80

Matsuyama, K. (1987) "Current account dynamics in a finite horizon model." *Journal of International Economics* 23, 299-313

-----(1988) "Terms-of-trade, factor intensities and the current account in a life-cycle model." *Review of Economic Studies* 55, 247-52

Mansoorian, A. (1992) "Income effect, wealth effect, and the terms of trade with finite horizons." *Canadian Journal of Economics* 25, 474-84

Mundell, R.A. (1963) "Inflation and real interest." *Journal of Political Economy* 71, 280-283

Obstfeld, M. (1982) "Aggregate Spending and the terms of trade: is there a Harberger-Laursen-Metzler effect?" *Quarterly Journal of Economics* 97, 251-70

Persson, T., and L.E.O. Svensson (1985) "Current account dynamics and the terms of trade: Harberger-Laursen-Metzler two generations later." *Journal of Political Economy* 93, 43-65

Sen, P., and S.J. Turnovsky (1989) "Deterioration of the terms of trade and capital accumulation: a reexamination of the Laursen-Metzler effect." *Journal of International Economics* 26, 227-50

Sidrauksi, M. (1967) "Rational choice and patterns of growth in a monetary economy." *American Economic Review Papers and Proceedings* 57, 534-544

Svensson, L.E.O., and A. Razin (1983) "The terms of trade and the current account: the Harberger-Laursen-Metzler effect." *Journal of Political Economy* 91, 97-125

Tobin, J. (1965) "Money and economic growth." *Econometrica* 33, 671-684

Uzawa, H. (1968) "Time preference, the consumption function, and optimal asset holdings." In *Value, Capital and Growth: Essays in Honor of Sir John Hicks,* ed. J.N Wolfe *(Chicago: Aldine)*, 485-504