Unit Roots, Nonlinear Cointegration and Purchasing Power Parity*

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Abstract

We test long–run PPP within a general model of cointegration of linear and nonlinear form. Nonlinear cointegration is tested with rank tests proposed by Breitung (2001). We start with determining the order of integration of each variable in the model, applying relatively powerful DF–GLS tests of Elliott, Rothenberg and Stock (1996). Using monthly data from the post–Bretton Woods era for G–10 countries, the evidence leads to a rejection of PPP for almost all countries. In several cases the price variables are driven by permanent shocks that differ from the ones that drive the exchange rate. Also, nonlinear cointegration cannot solve the PPP puzzle.

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1. Introduction

Empirical support for the theory of purchasing power parity (PPP) has been rather mixed. PPP has been tested extensively. Taylor and Taylor (2004) provided a recent survey. Basically, two alternative approaches have been followed. One approach tests for a unit root in the real exchange rates, which should not have a unit root but rather be a covariance stationary process if long–run PPP holds.[1] A recent example of this approach is Lopez, Murray and Papell (2005) who used state of the art unit root tests that we also apply in our paper. The other approach tests instead for cointegration among prices and the nominal exchange rate, which should form a stationary linear (or nonlinear) combination if long–run PPP holds. The advantage of this approach over the previous one is that it allows for a more general form of PPP where the adjustment of domestic and foreign prices need not be symmetric and proportional with the exchange rate.[2] A recent example of an analysis within the linear cointegration framework is Cheung, Lai and Bergman (2004). But, Michael, Nobay and Peel (1997) argued that conventional linear cointegration tests ignore nonlinearities and may therefore be biased against long–run PPP.[3] In summary, the empirical evidence with either one of the two approaches has, however, not been conclusive to date as to whether PPP holds or not.[4]

In this paper, we follow the approach based on cointegration methods. We start with the relationship between nominal exchange rates and domestic and foreign prices and test step–by–step the necessary assumptions for long–run PPP. We consider in turn each G–10 country over the post–Bretton Woods floating exchange rate period. First, the variables involved in the PPP cointegrating relation should all have the same order of integration. We apply the DF–GLS test of Elliott, Rothenberg and Stock (1996) along with the modified Akaike criterion of Ng and Perron (2001). We test each variable for two unit roots, or equivalently integration of order two, denoted

[1]Most authors considered here a mean–reverting process, however, if a Balassa–Samuelson effect is present, the real exchange rate may be stationary around a deterministic time trend instead.

[2]See, for example, the widely cited paper by Cheung and Lai (1993a) that tests for linear cointegration.

[3]Sercu, Uppal and Van Hulle (1995), and Dumas (1992) provided theoretical models for nonlinearities in PPP based on transaction costs. Also, Taylor, Peel and Sarno (2001), and Kilian and Taylor (2003), among others, applied nonlinear empirical models to real exchange rates instead.

[4]Other researchers attempted to resolve the issue in models of fractional cointegration (Cheung and Lai, 1993b) or in a panel cointegration framework (e.g., Pedroni, 2001) but the puzzle still remains, as Taylor and Taylor (2004) documented.

as I(2), and for one unit root or I(1). This issue, as it turns out, is of particular importance for the price variables. Second, we explicitly test for nonlinear cointegration of general form by applying a test suggested by Breitung (2001). We also test for linear cointegration and for symmetry and proportionality of the adjustment to PPP.

The extent to which PPP holds in the long run is a crucial question in the context of New Open Economy Macroeconomics.[5] In this literature, one class of models incorporates sticky prices or menu costs whereas another class of models is based on international product differentiation. Complete long–run exchange rate pass–through to import prices generally holds in models with sticky prices or menu costs but does usually not hold in models with product differentiation. Complete long–run exchange rate pass–through is a necessary condition for PPP to hold. Campa and Goldberg (2001) rejected complete long–run pass–through for 9 of 25 countries that they studied.[6] Of the countries that we consider in our paper, Canada, Sweden, the UK and the US are among the countries with incomplete long–run pass–through in the Campa and Goldberg study over the period 1975 to 1999.

The rest of the paper proceeds as follows. Section 2 briefly discusses the methodology of the various tests for the nonlinear model of cointegration. Section 3 presents the empirical model and motivation for nonlinear cointegration. It also provides the data description and the analysis of test results for one and two unit roots and for linear and nonlinear cointegration. Finally, Section 4 concludes.

2. Methodology

2.1 Rank Tests for Cointegration

In this section we will briefly discuss the method of rank tests for nonlinear cointegration of Breitung (2001). Consider two real–valued time series $\{x_t\}_1^T$ and $\{y_t\}_1^T$ that are nonlinearly related as $y_t = f(x_t) + u_t$, where $y_t \sim I(1)$ and $f(x_t) \sim I(1)$, i.e., each series is integrated of order one. Under the null hypothesis, u_t is $I(1)$ so that y_t and x_t are not cointegrated. Under the alternative hypothesis, u_t is $I(0)$ so that y_t and x_t are cointegrated. The standard assumption has been that $f(x_t)$ is a

[5]See, among others, the recent papers by Betts and Devereux (2001), Smets and Wouters (2002), and Monacelli (2005).

[6]See also Donnenfeld and Haug (2003).

linear function. However, economic theory often gives raise to nonlinear relationships so that $f(x_t)$ is assumed here to be a nonlinear function. Breitung showed that residual–based linear cointegration tests are inconsistent for some class of nonlinear functions.[7] To overcome this problem Breitung proposed tests based on the rank transformation of the time series.

Consider a slightly more general form with $u_t = g(y_t) - f(x_t)$, where $f(x_t) \sim I(1)$, $g(y_t) \sim I(1)$, and $u_t \sim I(0)$. Breitung defined a ranked series as $R_T(x_t) =$ Rank [of x_t among $x_1, \ldots, x_T$], and $R_T(y_t)$ accordingly. The rank statistics are constructed by replacing $f(x_t)$ and $g(y_t)$ with the ranked series, $R_T[f(x_t)] = R_T(x_t)$ and $R_T[g(y_t)] = R_T(y_t)$. The sequence of ranks is invariant to a monotonic transformation of the data.

In general it is not known whether the functions $g(y_t)$ and $f(x_t)$ are monotonically increasing or decreasing. For this situation, Breitung proposed a two–sided test:

$$\Xi_T^* = T^{-3} \sum_{t=1}^{T} (\tilde{u}_t^R)^2 / \{\tilde{\sigma}_{\Delta u}^2\}, \tag{1}$$

with $\tilde{u}_t^R$ the least squares residuals from a regression of $R_T(y_t)$ on $R_T(x_t)$. $\tilde{\sigma}_{\Delta u}^2$ is the variance of $\Delta \tilde{u}_t^R$. Critical values for this rank test are given in Table 1 in Breitung (p. 334). The null hypothesis is rejected when the test statistic is below the critical value. The Ξ_T^* test can be extended to models with three or more variables.

The cointegration rank test is designed to reject the null hypothesis of no cointegration when the residuals $\tilde{u}_t^R$ are $I(0)$. Cointegration, if it exists, may be of linear form or of nonlinear form. The Monte Carlo experiments in Breitung demonstrated that the rank test has good power properties not only in the nonlinear case but also in the linear case. To decide whether a cointegrating relation is linear or nonlinear, Breitung proposed a score statistic based on the rank transformation of the time series. This test is applied if the cointegration rank test indicates cointegration.

2.2 Score Statistic for a Rank Test of Neglected Nonlinear Cointegration

Consider the following nonlinear relationship between two time series: $y_t = \delta_0 + \delta_1 x_t + f^*(x_t) + u_t$, where $\delta_0 + \delta_1 x_t$ is the linear part. Under the null hypothe-

[7] Also, see Granger and Hallman (1991).

sis, $f^*(x_t) = 0$ and the u_t are $I(0)$ so that there is linear cointegration. Under the alternative hypothesis, $f^*(x_t) \neq 0$ and the u_t are $I(0)$ so that there is nonlinear cointegration. The score test statistic is given by TR^2 from a least squares regression of $\widehat{u}_t$ on $c_1 + c_2 x_t + c_3 R_T(x_t) + e_t$. The $\widehat{u}_t$ are the residuals under the null hypothesis, possibly corrected for serial correlation and endogeneity using for example the dynamic ordinary least squares method (DOLS) of Stock and Watson (1993).[8] Under the null hypothesis, the test statistic is distributed as χ^2 with one degree of freedom. The extension of this test to more than two variables is straightforward.

3. Empirical Analysis

3.1 The Empirical Model of PPP

In our paper we consider two linear and two nonlinear versions of the PPP relationship. The two linear versions are given by:

$$e_t = \alpha + \beta(p_t - p_t^*) + u_t \qquad \text{linear Model A} \tag{2}$$

$$e_t = \alpha + \beta_1 p_t - \beta_2 p_t^* + u_t \qquad \text{linear Model B.} \tag{3}$$

where e_t is the natural logarithm of the nominal exchange rate expressed in terms of the domestic price of foreign exchange. p_t and p_t^* are the natural logarithm of the domestic and foreign price, respectively; α is a constant reflecting differences in units of measurement; and u_t is a covariance–stationary mean–zero error term representing the deviations from PPP.

Linear cointegration in Model A or B ensures that the variables in the model move towards a long–run PPP equilibrium. If e_t and $p_t - p_t^*$ are each I(1) in Model A, and a linear combination of these variables exists that makes u_t covariance–stationary, then cointegration exists and PPP holds. For Model B, we need instead e_t, p_t and p_t^* to be each I(1) and a linear combination of these variables to be I(0). It is important to note that we test in our paper for relative PPP and not absolute PPP. The restricted version of PPP in Model A imposes the symmetry restriction that the nominal exchange rate responds equally in absolute value to changes in the domestic

[8] We apply the Schwarz Bayesian information criterion to select appropriate leads and lags for DOLS.

price level and in the foreign price level.[9] We follow here Cheung and Lai (1993a) and explicitly test for the restriction that Model A imposes relative to Model B. Cheung and Lai (1993a) argued that measurement errors make Model B a more appealing specification than Model A. Taylor(1988) provided further arguments why $\beta_1 = \beta_2$ may not hold.

Michael, Nobay and Peel (1997) postulated a linear cointegrating relation as in Model A and assumed a nonlinear adjustment process for the equilibrium– or error–correction term u_t.[10] This means that there is a nonlinear short–run adjustment process towards the log–run equilibrium. The long–run equilibrium is represented by a linear cointegrating relationship and only the short–run correction process is nonlinear.

The real exchange rate, q_t, studied in some of the PPP literature, follows from equation (2) with the additional assumption of long–run proportionality between exchange rates and prices so that $\beta = 1$ and $q_t \equiv e_t - (p_t - p_t^*)$. If this restriction is supported by the data and e_t and $p_t - p_t^*$ are each I(1) and cointegrated $\beta = 1$, then q_t will follow a covariance–stationary and mean–reverting process and long–run PPP will hold. Again, we will test the proportionality restriction rather than impose it.[11]

We consider in our paper nonlinear cointegration in addition to linear cointegration. However, it is now a nonlinear combination of the variables that renders u_t covariance–stationary if there nonlinear cointegration exists. An example of a nonlinear adjustment process to long–run PPP is the quadratic form (Model A is taken for convenience):

$$e_t = \alpha + \beta(p_t - p_t^*)^2 + u_t.$$

The reaction (in percent) of the exchange rate to changes in the price ratio is given by:

$$\frac{\partial e_t}{\partial(p_t - p_t^*)} = 2\beta(p_t - p_t^*).$$

The logarithm of the nominal exchange rate adjusts faster, the larger the deviation

[9]For a discussion on the proportionality and symmetry conditions related to long–run PPP, see Moosa (1994) and the references therein.

[10]Imposing the restrictions implied by Model A may be overly restrictive and bias results against linear cointegration in favor of nonlinear cointegration.

[11]See Cheung and Lai (1993a, pp. 189–190) for further discussion.

of relative prices from long–run PPP.[12] Sercu, Uppal and Van Hulle (1995) showed how shipping costs can lead to a band around the nominal exchange rate where no adjustment takes place when relative prices fluctuate across countries within a given range. They used a two country model with one traded good. In a multiple goods world where goods have different shipping costs and also non-traded goods are present, a scenario with adjustment speeds depending on the extent of the price differential seems more appropriate than threshold adjustment.[13] The larger the wedge between domestic and foreign prices in a given period, the larger is the number of goods with profitable arbitrage. Therefore, the more apart relative prices of two countries, the more arbitrage will take place and the higher is the speed of adjustment of the nominal exchange rate.

The general forms of the nonlinear versions of the above linear models are given by:

$$e_t = \alpha + f(p_t - p_t^*) + u_t \quad \text{nonlinear Model A} \tag{4}$$

$$e_t = \alpha + f(p_t, p_t^*) + u_t \quad \text{nonlinear Model B,} \tag{5}$$

With respect to cointegration, we carry out Breitung's tests to determine whether u_t is stationary when $f(\cdot)$ is of nonlinear form.

3.2 The Data and Sample Periods

Our data set is monthly and was extracted from the CD–ROM version of the IMF's *International Financial Statistics* (IFS). We use the end of period nominal exchange rate (IFS line ae), and the consumer price index (IFS line 64) for G–10 countries.[14] The sample period spans from 1973:5 to 2004:05 for Canada, Japan, Sweden, Switzerland, and the UK. The remaining five countries (Belgium, France, Germany, Italy, and the Netherlands) have joined the euro and hence the data are available from 1973:5 to 1998:12.[15]

[12]Cheung, Lai, and Bergman (2004), and Engel and Morley (2001) argued that nominal exchange rates, and not prices, are the "sticky" variable in the adjustment process to PPP, opposite to what had been commonly assumed previously. They also showed that the adjustment speed of exchange rates and of prices differs for adjustment towards their respective (unobservable) equilibrium values.

[13]See also the discussion in Taylor and Taylor (2004, pp. 146–149).

[14]G–10 actually consists of 11 countries.

[15]The start date allows for an adjustment period following the formal end of the Bretton Woods system of fixed exchange rates with the Smithsonian Agreement in February 1973.

We treated initially the United States as a numeraire country for both prices and exchange rates. Our empirical analysis is based on two sample windows. The first sample, which we refer to as “full sample”, starts from 1973:5 and ends in 1998:12 for euro countries, and in 2004:05 for the other five countries. We also test the PPP theory for the post–1982 sample period. Sims and Zha (2002), and Clarida, Gali and Gertler (2000) analyzed the post–1982 period separately and found a significant difference in US monetary policy. This sample spans from 1982:11 to 1998:12 for the euro countries included and from 1982:11 to 2004:05 for the other five countries. We refer to this as “sub Sample. In addition, we consider German Mark based exchange rates and PPP relations for the euro countries among the group of G–10 in order to see whether our results are sensitive to the choice of numeraire country.

3.3 Full Sample Analysis for US Dollar Based Exchange Rates

First, we test the order of integration of the variables that enter Models A and B. Following the suggestion of Dickey and Pantula (1987), we start with testing for two unit roots or I(2) because there is some empirical evidence in the literature suggesting that the natural logarithm of prices may be I(2) and inflation henceforth I(1). The null hypothesis is two unit roots (or one unit root in the first differenced variable) against the alternative hypothesis of I(1). If the null hypothesis of two unit roots is rejected, we test the null hypothesis of I(1) against the alternative of covariance–stationarity next.

We apply the DF–GLS test of Elliott, Rothenberg and Stock (1996) for our unit root tests. When testing for I(1), we allow for a constant but no deterministic time trends in the test regression. The DF–GLS test procedure applies the Dickey-Fuller τ–test to locally demeaned series. It has generally higher power than the standard Augmented Dickey–Fuller (ADF) unit root test. Ng and Perron (2001) studied the size and power properties of the DF–GLS test in typical finite samples and recommended using a modified Akaike criterion (MAIC) in order to select the lag length in the test regressions. We follow this suggestion. When testing for two unit roots, we do not put a constant term in the test regression as this would imply the presence of a deterministic time trend in the levels of the series. It is therefore unnecessary to locally demean the series and we apply the standard ADF test in this case, again in connection with MAIC. The DF–GLS test with a constant only has the same limiting distribution as the ADF test without a constant. We use the program of MacKinnon (1996) to calculate p–values.

Table 1 reports p–values for the tests for two roots and for one unit root. The results for $p_t - p_t^*$ reveal two unit roots for 6 countries because the null hypothesis of I(2) cannot be rejected at the 5% significance level. The null hypothesis of I(2) is rejected for the remaining 4 countries (as indicated by bold figures in Table 1). The null hypothesis of two unit roots for p_t is not rejected for 3 countries and is also not rejected for p_t^*, the US price level. It is rejected for the remaining 7 countries. Two unit roots are clearly rejected for all countries in the case of the nominal exchange rate and results are not reported to conserve space.

We proceed to testing of the I(1) hypothesis for the countries for which we were able to reject two unit roots.[16] We cannot reject the hypothesis of a unit root for all exchange rates, for $p_t - p_t^*$ for the 4 countries with rejections of I(2), and for p_t for the 7 countries with rejections of I(2).

The variables that enter (linear or nonlinear) Model A have for the majority of countries different orders of integration. For (linear or nonlinear) Model B the base country is the US and its price level, p_t^*, follows an I(2) process which is different from the majority of the processes for the price level of the other countries. This result implies that some price levels, those that are I(2), are driven by different permanent shocks than the exchange rate, which is for all countries I(1).

A long–run equilibrium in the form of cointegration can only exist if the variables have the same order of integration. This implies that we can only test for PPP in Model A for Canada, Japan, Sweden and the UK. Model B is ruled out because the US has a price level that is I(2) whereas all exchange rates are I(1).

We first consider linear and then nonlinear cointegration. The trace test of Johansen (1995) for linear cointegration, reported in Table 1, leads to a rejection of the null hypothesis of no cointegration for Japan and the UK. On the other hand, this hypothesis is not rejected for Canada and Sweden.[17] This leaves us with only two countries with linear cointegration, Japan and the UK. However, the cointegrating

[16]We report test results for one unit root for all countries for completeness only. The procedure of Dickey and Pantula (1987) is to stop further testing once the null hypothesis of I(2) is not rejected.

[17]The possibly more powerful maximum eigenvalue test of Johansen (1995) leads to the same results. We find one cointegrating vector with either test.

vector for Japan has the wrong sign so that we are left with only the UK.[18]

Would possibly nonlinear cointegration lead to more results in favor of cointegration? We first apply Breitung's (2001) test for linear or nonlinear cointegration, the Ξ_T^* test, for which the null hypothesis of no cointegration is rejected when the test statistic takes on a value below the critical value at a given significance level. As Table 2 shows, we reject the null hypothesis at the 1% level for all 4 countries that have the same order of integration for individual variables in nonlinear Model A. This test does not tell whether cointegration is of linear or of nonlinear form. We apply Breitung's nonlinear score test for this purpose. Results in Table 2 clearly indicate that all cointegration is of linear form. We therefore rely on the above results with the Johansen test that has a narrower alternative hypothesis and therefore more power. In summary, we have not uncovered any evidence for nonlinear cointegration up to this point, and neither much evidence for linear cointegration and therefore for long–run PPP.

3.4 Sub Sample Analysis for US Dollar Based Exchange Rates

We consider now the period from 1982:11 onwards, as motivated in Section 3.2. We follow numerous other empirical studies that documented a change in US monetary policy at that time. Such a structural change may have introduced a spurious second unit root in the time series over the full sample. If this is the case, we would expect less evidence for two unit roots in the sub sample than in the full sample.

We repeat the unit root tests for the shortened sample and report results in Table 3. The ADF test with MAIC suggests two unit roots in $p_t - p_t^*$ for 4 countries. This result is not much different from that for the full sample where we found evidence of I(2) for 6 countries. The ADF test indicates two unit roots for the price level, p_t, for 7 countries, which is a significant increase in the number of I(2) cases compared to the full sample. In addition, the US price level, p_t^*, has again two unit roots. Again, we clearly reject two unit roots for the nominal exchange rate for all countries (results are not reported). The DF–GLS tests cannot reject a unit root for all variables of all countries, at the 5% level of significance, except for the exchange rate of the UK.

[18] We tested the hypothesis that $\beta = 1$ for the UK but rejected this hypothesis with a p–value of zero for a likelihood ratio test.

The exchange rate of the UK therefore seems to be I(0).

The variables in Model A have the same order of integration for Belgium, Canada, France, Italy, and Sweden. Hence, it is possible to test for linear and nonlinear cointegration for these countries within Model A.[19] Model B is again ruled out because the price level of the US is I(2), as before over the full sample.

The Johansen tests in Table 3 clearly reject the null hypothesis of no linear cointegration for Belgium, France, Italy and Sweden but not for Canada. However, all estimated values of β have the incorrect sign for PPP, except for Sweden. Hence, we find linear cointegration that supports long–run PPP again only for one country, which is Sweden.[20]

Table 4 presents test results for Breitungs tests of nonlinear cointegration over the sub sample for Model A. We reject the null hypothesis of no cointegration in favor of cointegration of either linear or nonlinear form for all 5 countries considered. The nonlinear score test indicates linear cointegration, except for Italy, taking a 5% level of significance. Our analysis over the sub sample leaves us with two cases that support long–run PPP: a linear cointegrating relation for Sweden and a nonlinear cointegrating relation for Italy.

3.5 Full Sample Analysis for German Mark Based Exchange Rates for Euro Countries

If PPP is likely to hold for any set of countries, it should be for the countries that adopted the euro. We use the German Mark as the base currency for the exchange rate and apply the various tests to Belgium, France, Italy, and the Netherlands, which are the euro countries among the G–10.

Results are reported in Tables 5 and 6. Two unit roots in $p_t - p_t$ are rejected for Belgium and the Netherlands but not for France and Italy. Further, two unit roots in p_t can be rejected for all euro countries considered.[21] The same is true for the German Mark based exchange rates. Also, we reject two unit roots in the price level for Germany (p_t). Next, the tests for I(1) suggest that $p_t - p_t$ is I(1) for Belgium

[19] Compared to the full sample, we have to exclude here Japan and the UK but can add in Belgium, France and Italy.

[20] We test for Sweden the hypothesis that $\beta = 1$ and get a p–value of zero.

[21] Results from Table 1 are repeated in Table 5 for convenience.

and the Netherlands and that the price level is I(1) for all euro countries considered, including Germany. The German Mark based exchange rate, e_t, is I(1) as well for all countries.

We test for linear cointegration within Model A for Belgium and the Netherlands and within Model B for all 4 countries. The Johansen tests in Table 5 clearly reject the null hypothesis of no cointegration in all cases. However, we find two cointegrating vectors for the Netherlands for Models A and for Model B. This means in Model A that all variables should be I(0), which contradicts our unit root test results. We therefore dismiss Model A for the Netherlands. On the other hand, two cointegrating vectors in Model B are not a problem. However, none of the estimated cointegrating vectors for all Models A and B in Table 5 has the correct signs as required by PPP.

Next, we apply the tests for nonlinear cointegration. We find evidence for cointegration of linear or nonlinear form for all countries considered for Models A and B. The score test indicates only for Belgium that cointegration is of nonlinear form. The analysis of the euro countries therefore leaves us with again only one case that supports PPP: the nonlinear Model B of Belgium.

4. Concluding Remarks

We re–examined the PPP relation over the post–Bretton Woods floating exchange rate period for the G–10 countries. We considered US dollar based exchange rates for a sample of monthly observations starting in 1973:05 and for another sample starting in 1982:11 instead. We also considered German Mark based exchange rates for the G–10 countries that adopted the euro.

We applied more powerful unit root tests, the DF–GLS tests, than the previous literature in order to determine the order of integration of the variables involved in cointegrating relations implied by PPP. In particular, we tested for integration of order two and not only for integration of order one. Furthermore, we allowed for non–symmetric price adjustment and for non–proportional movements of prices and exchange rates in the long–run PPP model. Also, we considered the possibility of a nonlinear cointegrating relationship of general form, in addition to conventional linear cointegration. We applied for this purpose recently developed direct tests for nonlinear cointegration based on ranked time series.

We found evidence for integration of order two for the price levels and the domestic to foreign price ratios for around half the countries over two different samples and US dollar as well as German Mark based exchange rates. On the other hand, all nominal exchange rates are integrated of order one. The cases of different orders of integration for prices or price ratios and exchange rates imply that different permanent shocks are at work and that linear or nonlinear cointegration is ruled out. For the case where the order of integration is the same, we find mostly cointegrating vectors that do not fit the long–run PPP model, despite using a very general specification. In our analysis, we find only two countries for which nonlinear cointegration is supported and only two country for which linear cointegration is supported, among a total of 15 cointegrating regressions. Furthermore, two of these results in favor of PPP are not robust: the UK result holds only over the full sample and not over the sub sample; the result for Belgium holds only for German Mark based exchange rates and not for US based exchange rates.

In summary, our results add to the empirical evidence against the validity of long–run PPP. This in turn lends indirect support to the New Open Economy Macroeconomics theories that are based on international product differentiation with incomplete long-run exchange rate pass–through to import prices. Also, we find that nonlinear cointegration cannot solve the PPP puzzle.

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Table 1. US Dollar Based Analysis: P-Values of Unit Root and Johansen Cointegration Tests for the Full Sample Starting in 1973:05

Countries	Unit Root Tests†					Johansen (Trace) Cointegration Test‡
	$\Delta (p_t - p^*_t)$	Δp_t	$p_t - p^*_t$	p_t	e_t	Model A
Belgium	0.082	**0.014**	**0.036**	0.778	0.132	-
Canada	**0.000**	0.080	0.366	0.832	0.692	0.789
France	0.055	**0.029**	0.424	0.611	0.340	-
Germany	0.062	**0.047**	0.792	0.866	0.540	-
Italy	0.093	**0.025**	0.738	0.761	0.807	-
Japan	**0.000**	**0.000**	0.935	0.943	0.748	0.000
Netherlands	0.102	**0.029**	0.703	0.842	0.447	-
Sweden	**0.004**	0.157	0.576	0.690	0.557	0.232
Switzerland	0.085	**0.009**	0.788	0.945	0.760	-
UK	**0.025**	0.057	0.767	0.842	0.431	0.000

Notes: † For the unit root test of the first differenced data, we apply the ADF test with no constant in the test regression. The lag length has been chosen using the modified Akaike's criterion (MAIC) suggested by Ng and Perron (2001). We test for a unit root in the levels data with the DF-GLS test of Elliot, Rothenberg and Stock (1996) that includes a constant in the test regression but no deterministic time trends. Lags are chosen again with MAIC. The maximum number of lags has been fixed at 16 (15) for G-5 (euro-area) countries. A value of .000 indicates a p-value of less than .0004. P-values are calculated with the program of MacKinnon (1996). The p-value for two unit roots in the US price level, p^*_t, is 0.058 for the sample of the G-5 countries and 0.073 for the sample period of the euro- area countries. For the test of one unit root, it is 0.915 and 0.777, respectively. The p-values for the logarithm of the nominal exchange rate, e_t, for two unit roots reject the null hypothesis for all countries. Bold figures indicate rejection at the 5% level of significance.

‡ For Johansen cointegration tests we allow for a constant but no time trend in the cointegrating regression and VAR. The number of lags is chosen using the Schwarz Bayesian criterion. The p-values for Johansen tests are computed using the program of MacKinnon, Haug, and Michelis (1999).

Table 2. US Dollar Based Analysis: Rank Tests of Nonlinear Cointegration for the Full Sample Starting in 1973:05

Countries	**Model A**	
	Ξ^*_T	Nonlinear Score Test
Belgium	-	-
Canada	0.0027^{***}	0.071
France	-	-
Germany	-	-
Italy	-	-
Japan	0.0028^{***}	0.042
Netherlands	-	-
Sweden	0.0027^{***}	0.615
Switzerland	-	-
UK	0.0028^{***}	0.003

Notes: : Significance at the 1%, 5%, and 10% level is indicated by ***, **, *. Critical values for the Ξ^*_T test statistic are from Breitung (2001, Table 1). The 1% critical value is 0.0136. The null hypothesis of no nonlinear cointegration is rejected for a test statistic value smaller than the critical value. The nonlinear-score test follows a χ^2 distribution with one degree of freedom.

Table 3. US Dollar Based Analysis: P-Values of Unit Root and Johansen Cointegration Tests for the Sub Sample Starting in 1982:11

Countries	Unit Root Tests†					Johansen (Trace) Cointegration Test‡
	$\Delta\,(p_t - p^*_t)$	$\Delta\, p_t$	$p_t - p^*_t$	p_t	e_t	Model A
Belgium	**0.031**	**0.019**	0.364	0.884	0.498	0.001
Canada	**0.000**	0.150	0.563	0.918	0.328	0.474
France	**0.003**	**0.002**	0.300	0.893	0.324	0.000
Germany	0.053	0.112	0.757	0.819	0.548	-
Italy	**0.000**	**0.019**	0.873	0.909	0.157	0.000
Japan	0.284	0.069	0.976	0.664	0.720	-
Netherlands	0.164	0.202	0.632	0.876	0.523	-
Sweden	**0.010**	0.075	0.485	0.745	0.087	0.007
Switzerland	0.106	0.219	0.961	0.704	0.562	-
UK	**0.017**	0.225	0.667	0.816	**0.018**	-

Notes: See Table 1. The maximum number of lags has been fixed at 15 (14) for G-5 (euro-area) countries. For p^*_t , the p-value for the ADF test for two unit roots is 0.267 for the sample period of the G-5 countries and 0.249 for the sample period of the euro-area countries. The p-value for the DF-GLS unit root test is 0.931 and 0.730, respectively.

Table 4. US Dollar Based Analysis: Rank Tests of Nonlinear Cointegration for the Full Sample Starting in 1982:11

Countries	**Model A**	
	Ξ^*_T	Nonlinear Score Test
Belgium	0.0054^{***}	1.787
Canada	0.0039^{***}	2.902^{*}
France	0.0055^{***}	0.099
Germany	-	-
Italy	0.0053^{***}	5.874^{**}
Japan	-	-
Netherlands	-	-
Sweden	0.0039^{***}	0.779
Switzerland	-	-
UK	-	-

Notes: See Table 2.

Table 5. German Mark Based Analysis: P-Values of Unit Root and Johansen Cointegration Tests for the Euro Countries 1973:05 – 1998:12

Countries	Unit Root Tests†					Johansen (Trace) Cointegration Test‡	
	$\Delta (p_t - p'_t)$	Δp_t	$p_t - p'_t$	p_t	e'_t	Model A	Model B
Belgium	**0.010**	**0.014**	0.718	0.778	0.896	0.000	0.000
France	0.099	**0.029**	0.650	0.611	0.834	-	0.000
Italy	0.103	**0.025**	0.767	0.761	0.981	-	0.000
Netherlands	**0.016**	**0.029**	0.653	0.842	0.798	0.000$^{\Diamond}$	0.000$^{\Diamond}$

Notes: See Table 1. The hypothesis of two unit roots is rejected for all exchange rates. A "◊" indicates that two cointegrating vectors are found. The p-value for the German price level, p'_t, is 0.048 for the test for two unit roots and 0.699 for the test for one root.

Table 6. German Mark Based Analysis: Rank Tests of Nonlinear Cointegration for the Euro Countries 1973:05 – 1998:12

Countries	**Model A**		**Model B**	
	Ξ_T^*	Nonlinear Score Test	Ξ_T^*	Nonlinear Score Test
Belgium	0.0038***	0.503	0.0035***	13.350***
France	-	-	0.0036***	3.089
Italy	-	-	0.0036***	0.133
Netherlands	0.0036***	1.891	0.0038***	0.165

Notes: See Table 2. The 1% critical value for the Ξ_T^* test for Model B is 0.0119 (Breitung, 2001, Table 1). The nonlinear-score test for Model B follows a χ^2 distribution with two degrees of freedom.