# The Term Spread:
# International Evidence of Non-Linear Adjustment*

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**Abstract**

This study tests whether changes in the short-term interest rate can best be modelled in a non-linear fashion. We argue that there are good theoretical and empirical reasons for adopting this strategy. Using monthly data from several industrialized countries, namely Canada, Germany, Sweden, Switzerland, UK, and US, we show that the short-term interest rate movements are better explained, usually via the exponential smooth transition autoregression (ESTR). Unlike the existing literature on non-linear estimation, we consider a number of candidates for the transition variable. These include: an error correction term, estimated from an underlying cointegrating relationship predicted by the expectations hypothesis, the US spread, the domestic spread, inflation and output growth forecasts, and deviations from an inflation target in the case of Canada, the UK and Sweden.

The sample spans the period from 1960-1998. We cannot reject non-linearity in the behavior of interest rate changes most often when the (lagged) domestic spread serves as the transition variable. In the case of the inflation targeting countries in our sample, the most appropriate transition variable can be the deviation from the publicly announced inflation target. We supplement estimates with extensive diagnostic testing to ensure that we can reject the linear alternative with reasonable confidence. We believe that changes in central bank policies and in the reaction of market participants over time to such changes argue in favor of the non-linear estimation approach. We would also argue that any model of the term spread over a fairly long span of time necessitates resort to non-linear estimation methods.

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## Introduction

Among the most widely examined series in finance is the term spread. It is easy to see why. Several studies (e.g., Jorion and Mishkin 1991, Hu 1993, Bernard and Gerlach 1996, Barr and Campbell 1997, to name just a few) have empirically demonstrated that future inflation and economic growth are each significantly explained by the slope of the yield curve. While many theoretical models of the term structure are inherently non-linear, they are usually linear in empirical work either for ease of exposition or estimation. (e.g., see Cochrane 2001, chapter 19).[1]

Campbell, Lo, and MacKinlay (1997, chapter 12) point out that the behavior of financial time series is inherently non-linear (also see Taylor 1986). Ellingsen and Söderström (1999) argue that the manner in which monetary policy is carried out is also likely to induce non-linear behavior in the terms spread. Lanne (2002) suggests, at least for US data, that neglecting non-linearity in interest rates (and inflation) can have policy implications because linear models provide misleading inferences about the behavior of real interest rates in particular. Figure 1 plots a short-term and a long-term interest rate for the US at the monthly frequency since 1982 together with the resulting term spread. These time series have been used in several studies of the kind conducted here. The data reveal sharp changes in the term spread that illustrate vividly why non-linear methods may be suitable for such a time series.

Despite recent advances in estimating non-linear time series models (e.g., see Granger and Teräsvirta 1993, Teräsvirta 1998, van Dijk, Teräsvirta and Franses 2001) there is as yet no general consensus on the usefulness of non-linear models for the term spread. Hence there is a need for additional research. First, by adopting the usual theoretical approach of using a one

[1] Aït-Sahailia (1996) suggests that it is preferable to model interest rates with the property of non-linear mean reversion. Nevertheless, the finite sample properties of Aït-Sahailia's test have been criticized (e.g., Pritsker 1998, and Jones 2003).

period lag of the variable of interest to explain the (smooth) transition from one regime to another, the role of economic theory is effectively downplayed in the standard empirical modeling strategy. Second, many studies (see below) are either content to estimate the relationship of interest in linear form, or rely on a minimal set of tests to reject linearity in favor of some non-linear alternative.

The principal contribution of this paper is to explore empirically the relative contribution of several candidates for regime shifts in the behavior of interest rates suggested by economic analysis. Consequently, we do not solely rely on an ad hoc specification for the transition variable used to model potential non-linearity in interest rate behavior. We also address the need to rigorously test for non-linearity before abandoning the linear alternative. In doing so, we extend the linear error-correction model of the term structure of interest rates (see, e.g., Campbell and Shiller 1987) by allowing for non-linear adjustment to the equilibrium and for changes in regime, with smooth transition adjustment. We also allow for non-linearity in the adjustment process for the error-correction term, in addition to non-linearity in the short run dynamics of the term spread.

Briefly, our findings are as follows. While the (lagged) domestic spread is sometimes the preferred variable to use in the transition function for the non-linear model, an error correction term and the US spread are generally better candidates. In the case of UK data, deviations from the targeted inflation rate provide the trigger for non-linear dynamics. This is a notable result since the Bank of England formally targets inflation. Other forward-looking variables such as inflation, the output gap, or real GDP growth, also figure as potentially useful candidates for the transition function. Hence, economic theory is helpful if one is to conclude in favor of the widely held view that interest rate behavior is non-linear.

The rest of the paper is organized as follows. First, we briefly set out the case for non-linear interest rate behavior. Since theories that link short and long rates, such as the expectations hypothesis, have been the subject of a considerable amount of empirical research, it seems sensible to make our points via a focus on the term spread. Next, we outline the econometric specification and the strategy used to search for non-linearity in the data. Following a description of the data we present empirical evidence for a sample covering almost four decades of data for a group of industrial countries, namely Canada, Germany, Sweden, Switzerland, the US, and the UK.[2] The paper concludes with a summary and issues to be taken up in future research.

## **The Term Spread and Non-Linear Adjustment**

The Expectations Hypothesis (EH) of the term structure predicts that in an unfettered market differences in yields between government bonds that mature at different dates reflect expectations about the future course of interest rates. Hence, one can think of long-term and short-term interest rates as being ‘attracted’ to each other, in the sense of cointegration, whereas short-term deviations represent error corrections (Campbell and Shiller 1987, Engle and Granger 1987). There is, of course, a large literature that documents this property for a wide variety of countries (e.g., Hall, Anderson, and Granger 1992, Fuhrer 1996, and Siklos and Wohar 1997). Cointegration tests, therefore, provide a natural way to determine whether interest rates across the term structure have a common stochastic trend. The error correction process, by contrast, is one vehicle for introducing non-linearities in the adjustment to equilibrium. Cointegration also presupposes that the series individually possess a unit root. Alternatively, the expectations hypothesis requires the presence of a large autoregressive root and, in that case, standard inference is not asymptotically valid (Elliott and Stock 1994). Lanne (1999) reports that this may

[2] In an earlier draft of the paper we also used data from Australia and New Zealand. However, as CPI data, a key component in the determination of nominal interest rates, are available only at the quarterly frequency we opted to

be an important reason for the rejection of the expectations hypothesis over periods covering more than one policy regime. However, once the EH hypothesis is tested for separate sub-samples, it is no longer rejected. Also, Lanne's (1999) results assume that the location of the break is known with certainty.

There are several reasons to expect non-linearities in the equilibrium adjustment process between long-term and short-term interest rates. Transactions costs may differ according to the maturity of the bond considered and may well change over time (Anderson 1997). Risk-premia may also be time-varying and exhibit non-linear behavior (Fama 1990). Policy regimes aimed at influencing the yield spread may also be sufficiently important as to create structural breaks in the underlying relationships of interest, and the adjustments from such regime changes may also be non-linear.[3] Bekaert, Hodrick and Marshall (1997), using a more restrictive framework than ours and data from the US, the UK and Germany, use a regime-switching approach to argue that the failure of the expectations hypothesis is due to fewer high interest rate regimes than expected. Favero and Giavazzi (2002), building on the theoretical and empirical work of others, find that financial shocks are propagated internationally in a non-linear fashion. It is conceivable, therefore, that for some of the countries in our sample the foreign term spread may trigger non-linearities in the domestic spread.

Finally, several countries have recently adopted inflation targets (Bernanke et al. 1999, Siklos 1999). Theoretically, the spread between long-term and short-term interest rates, while subject to the formal constraint of the expectations hypothesis, could conceivably drift for a time. However, given a credible inflation target, inflation and inflation expectations are bounded by

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exclude these two countries from further analysis.

[3] A separate issue is whether linearity fails due to a structural break or is the result of an inherently non-linear process. Although our interest in this paper is more focused on linear versus non-linear alternatives we do consider structural stability questions. Also see Koop and Potter (1998).

the target band and should be attracted toward the middle of the target band. As with the target zone literature on exchange rates, the spread is stationary, though possibly in a non-linear fashion, mean reverting within the target zone and non-stationary when no target zone is specified (Amano, Black and Kasumovich 1997, and Siklos and Granger 1997). One potential problem in estimating the impact of inflation targets is that the span of the sample since their introduction is relatively short. Nevertheless, to the extent that most countries had an implicit inflation target, it is conceivable then that while the term spread is non-linear, there is a linear component that might be well captured via a linear error correction type mechanism.

In contrast, our approach is to ask a somewhat different question: can a non-linear model explain the behavior of the term spread relatively better than a linear model across policy regimes? Which economic variables could possibly explain the transition from one regime to another? Clearly, while our strategy avoids some of the problems with existing analyses of the term spread, it leaves others unresolved, such as the convenient assumption that unit roots in the short and long term interest rate are a reasonable approximation to the time-series behavior over the sample period considered.

Existing empirical evidence tends to be heavily influenced by the US experience, and is not overwhelmingly favorable to the expectations hypothesis as stated above. Hardouvelis (1994) is one oft-cited study that finds evidence against the EH for US data while the evidence is more favorable for the remaining G7 countries considered in the sample. Dotsey and Otrok (1995), and Wallace and Warner (1996), respectively, reject and accept the EH, although the latter study notes that the results are sample sensitive. Gerlach and Smets (1997) find that countries that participated in the European exchange rate mechanism (ERM) are more likely to produce spreads consistent with the EH. Hall et al. (1991), and Siklos and Wohar (1997), find evidence of cointegration across the term structure consistent with the EH for a sample consisting of data from the US and other industrialized countries.

Finally, more recent work on the EH and the role of the monetary authorities in influencing the spread finds that such considerations are important in the US and in other industrial countries. See, for example, Kugler (1996), Fuhrer and Moore (1995), Haubrich and Dombrosky (1996), Dillén (1997), Smets and Tsatsaronis (1997). With the exception of Siklos (2000), there have been no studies of the impact of inflation targeting in tests of the expectations hypothesis.

## Econometric Specification

As is well known, cointegration requires that long-term and short-term yields display unit root behavior. Hence, we follow the usual practice of generating the Augmented Dickey-Fuller (ADF) test statistic to examine the unit root hypothesis. The lag augmentation term in the test equation is estimated by applying Akaike's information criterion (AIC), following the suggestion of Agiakloglou and Newbold (1996) who found that this method performed well in Monte Carlo studies. We imposed a maximum of 24 lags for the lagged differences in the interest rates for the full sample, which seemed reasonable based on an examination of the time series properties of the data. We also tested for two unit roots and rejected this hypothesis in all cases. Next, we proceed to test for cointegration using Johansen's method, perhaps the most widely used technique under the circumstances.[4] The technique is based on maximum likelihood estimation of the relationship between short and long rates in the form of a VECM (vector error-correction model). The VECM is specified with a constant term only in the cointegrating vector so that

[4] We retain the linear cointegration testing procedure. While there exist tests for non-linear cointegration (e.g., Breitung 2001), these require additional strong assumptions and are idiosyncratic to the particular form of the hypothesized non-linearity. Corradi, Swanson, and White (1998) explore the issue of testing for specific forms of non-linear cointegration.

there are no deterministic time trends allowed for in the underlying data, as an unrestricted constant term in the VECM would imply. The lag lengths were selected using the Bayesian Schwarz criterion following Reimers (1993). The details of unit root and cointegration testing are well known and readers are referred to Johansen (1995) and Maddala and Kim (1998), among many excellent references on the subject, for additional details.

In addition, given the well-known sensitivity to the assumptions required to obtain the likelihood ratio test statistics of interest in Johansen tests for cointegration, we repeat all cointegration tests using the Engle-Granger (1987) formulation.[5] We next estimate a linear error correction model (ECM) of the change in yield

$$\Delta r_{1,t} = \mathrm{A}(L)\Delta r_{1,t-1} + B(L)\Delta r_{n,t-1} + \alpha(r_{1,t-1} - \beta_0 - \beta_1 r_{n,t-1}) + \varsigma_{0t}\,, \tag{1}$$

where $\Delta r_{1,t}$ is the one period yield change ($\Delta r_{n,t}$ is the n period yield change), $\varsigma_{0t}$ is a mean-zero white noise error term, the last term in parentheses is the error correction term, A (L) and B(L) are distributed lag functions of order p, and the other terms were previously defined. Note that, to permit comparability, we follow past practice and model the relationships of interest in terms of changes in the short-run interest rate (e.g, Anderson 1997).[6] There is considerable cross-country evidence that long-term interest rates are weakly exogenous and we therefore specified (1) as a single equation ECM instead of a VECM. This allows us to specify meaningful and parsimoneous non-linear models below.

For the non-linear model we estimate a variety of smooth transition regressions (STR; see Granger and Teräsvirta 1993, Teräsvirta 1998). Such models presume that there are regime shifts

[5] See also Chao and Phillips (1999).

[6] Corradi, Swanson and White (2000) consider the possibility of non-linear cointegration and find US evidence supportive of the hypothesis for US data covering the 1970-1988 period (Hall, Anderson and Granger (1992) used the same sample). This excludes the most recent policy regime in the US. Anderson (1997) finds evidence of non-linear error correction in the same data set as do Enders and Siklos (2001), who used a different sample of US data. We do not consider such extensions in the present paper.

in the underlying data and that the transition from one regime to the next is a locally linear one. Among the many useful properties of STR models is that their structure permits specification, estimation and diagnostic testing to follow more or less the usual approach used to estimate linear ARIMA models of the Box-Jenkins variety (Teräsvirta 1994). Under the present circumstances this means estimating a non-linear error-correction model of the form

$$\Delta r_{1,t} = \beta' X_t + (\beta' X_t) F(\tau_t; \gamma, k) + \eta_t, \qquad (2)$$

where F(·) is generally bounded between 0 and 1, F(·) is the transition function, $\gamma$ is a positive slope parameter to indicate how rapidly the transition from one regime to another takes place, k locates where the transition occurs, while $\tau_t$ is the transition variable. $X_t$ is a vector of regressors that is defined as

$$X'_t = (\Delta r_{1,t-1},\ \Delta r_{1,t-2},\ \ldots,\ \Delta r_{1,t-p},\ \Delta r_{n,t-1},\ \Delta r_{n,t-2},\ \ldots,\ \Delta r_{n,t-p},\ (r_{1,t-1} - \hat{\beta}_0 - \hat{\beta}_1 r_{n,t-1}))\,.$$

In what follows, denote the last term, which is the estimated linear error-correction term, by $ec_{t-1}$. The transition variable is key to the identification of a non-linear relationship since it is one of the economic variable that is believed to trigger the smooth transition from one regime to another. A variety of formulations for F(·) have been suggested in the literature (e.g., see Granger and Teräsvirta 1993, Teräsvirta 1998, Potter 1999). Granger and Teräsvirta (1993, Chapter 7), Teräsvirta (1998), and others, assume that the functional form for F(·) is either of the *logistic* (LSTR) or *exponential* (ESTR) variety. In the LSTR model, the transition function is monotonically increasing in $\tau_t$ and allows for asymmetric transition:

$$F(\tau_t;\gamma,k) = \{1 + \exp[-\gamma(\tau_t - k)]\}^{-1}$$

The ESTR model is a non-monotonic alternative that is symmetric around k:

$$F(\tau_t;\gamma,k) = 1 - \exp[-\gamma(\tau_t - k)^2]$$

The adjustment speed $\gamma$ and location parameters k can be estimated via non-linear least squares. We follow Lütkepohl et al. (1999) and others (e.g., Hendry 1995, chapter 16, and Baba, Hendry and Starr (1992), van Dijk and Franses 2000) in focusing on non-linearities in the error correction mechanism. Note also that the linear ECM is a special case of (1). Hence, rejection of the null that $\gamma$=0 implies that the data support the non-linearity hypothesis. Unlike Lütkepohl et. al. (1999), however, we separately estimate the linear cointegrating vectors prior to estimating the linear ECM in equation (1), as opposed to performing the estimations of (1) in one step as they did. Equation (2) is not identified under the null hypothesis of linearity. Teräsvirta (1998) suggests estimating an auxiliary regression, consisting of the regressors multiplied by the transition variable, with the addition of non-linear terms, in order to test for nonlinearity. In the present case the auxiliary regression has the form

$$\Delta r_{1,t} = \beta' X_t + \delta_0' X_t \tau_t + \delta_1' X_t \tau_t^2 + \delta_2' X_t \tau_t^3 + \eta_t . \qquad (3)$$

where the null hypothesis of linearity is $\delta'_0=\delta'_1=\delta'_2=0$. Typically, the practice has been to let each regressor in $X_t$ in turn be the transition variable. It is important that $\tau_t$ is moment stationary up to a certain order, except when it is dominated by a polynomial in time (see Lin and Teräsvirta 1994). In many applications, there is little theoretical or empirical guidance for the choice of a transition variable. However, in tests of the behavior of the spread, there is guidance from both sources as noted in the previous section. Hence, we examine in turn the error correction term, the lagged spreads, and forecasts for inflation and real GDP growth, as candidates for the transition variable. Therefore, $\tau_t$ is in turn represented by

$$\tau_t = (\, ec_{t-1}, S_{nt-1}{}^{US}, S_{nt-1}, \pi_t^e, y_t^e, (\pi_{t-1} - \pi^*)) \qquad (4)$$

where $\pi^e_t$ and $y^e_t$ are, respectively, private sector forecasts of inflation and real GDP growth for time t conditional on information at time *t-1*. All series are entered in their I(0) formulations in the regressions. Since one of the objectives of the study is to investigate differences in short term interest rate behavior as between inflation and non-inflation targeting countries we also consider deviations from the announced inflation target. Note that if the target is credible, ($\pi_{t-1}$ - $\pi^*$) is expected to be I(0). Analog to the linear error-correction process in equation (1), where the error-correction term enters in lagged form, we choose for $\tau_t$ a variable from the information set available at time t-1.[7]

We also believe, but cannot prove, that there are at least two other candidates for non-linearities in the term spread that have not been sufficiently emphasized so far in the literature. First, if, as has been pointed out (e.g., Bernanke at. al 1999), central bank policies have changed considerably over the past few decades, then this could possibly induce non-linearities in the term spread in the form of a structural break. The term spread is, after all, an important economic indicator monitored by these same monetary authorities. Alternatively, if the researcher's objective is to model the spread over a fairly long span of time, it is likely that non-linearities will be present in the time series, again because of changes in regimes that are difficult to identify via linear econometric methods (again, see Figure 1).

**Data**

We use monthly data for a sample spanning the period 1960-1998. We use a Treasury bill rate to proxy the short-term interest rate while the yield on long-term government bonds, generally maturing in 10 years or more, represents our measure of long-term yields. Our data set consists of 6 industrialized countries. They are: Canada, Germany, Sweden, Switzerland, the UK and the US. Three of the countries, namely Canada, the UK and Sweden, adopted inflation targets in the 1990s (Siklos 1999 provides the precise dates and institutional details). The data

[7] See also Michael et al. (1997) on institutional factors in the case of purchasing power parity that may make longer delays in adjustment theoretically plausible.

were obtained from the International Financial Statistics CD-ROM (see the data appendix for more details). We considered specifications in interest rate levels as well as in logs of the levels but the results reported in the next section are all based on the levels specifications. Data on inflation forecasts and output gaps were obtained from various issues of the OECD *Economic Outlook* and from various issues of *The Economist's* Poll of Forecasters. The OECD forecasts are for the annual percent change in the current year GDP deflator published twice a year. Similarly, forecasts of the output gap were used to proxy expectations of future economic activity. The raw data were converted into monthly data by fitting a cubic spline function to fill in the missing observations with the last observation matching the raw data. The main advantage of these series is their availability over a long span of time. *The Economist* Poll of Forecasters is an average of a dozen or more forecasts for current year annual CPI inflation and real GDP growth made by private banks (Siklos 1999 provides more details). The data usually appear in the second monthly issue of *The Economist*. Data from *The Economist* are, however, available only since August 1990. Therefore, wherever possible, we relied on the OECD data.

## Empirical Evidence

Tables 1 and 2 show unit root and cointegration tests. Using the AIC criterion, it is difficult to reject the null of a unit root for any of the nominal interest rate series in our data set. This is consistent with the existing evidence about interest rate behavior. The results are more mixed for the term spread. Thus, although we can easily reject at the 5% level of significance the null of a unit root in the spread, the same null cannot be rejected for Germany and Switzerland. The selection criterion picks a zero lag in two of the three cases of non-rejection of the null of a unit root. In these cases, the rejections are somewhat sensitive to the chosen lag length in the augmentation term of the ADF test equation. In addition, as noted earlier, these results are also likely to be sensitive to the presence of breaks in the data which we model by estimating a non-linear approximation to the model of short term interest rate behavior. Turning to the forecast data, we find that one cannot reject the null of a unit root in inflation forecasts and in several output gap/GDP growth forecasts. For the countries considered, it is likely that breaks in the data such as the end of the Bretton Woods system of pegged exchange rates, or the introduction of a new monetary policy framework as in the adoption of inflation targets, may also explain apparent unit root behavior where one would not expect it to emerge. We therefore apply several Lagrange multiplier (LM) tests for breaks.

Although estimation in this paper focuses on the full available sample, it is important to consider the possibility that, even if the linear model is rejected, diagnostic checking of non-linear models may reveal statistical inadequacies along one or more dimensions (see below). Indeed, extensive testing led us to re-estimate all of the non-linear models for US data for the post 1980 period due to a major change in the operating framework of monetary policy.

The cointegration tests shown in Table 2 reveal some evidence of a linear equilibrium type relationship between long and short rates. However, these tests are neither necessary nor sufficient for non-linear cointegration to exist or not to exist. Michael at al. (1997) point out that non-linearities could lead to the failure to reject the null hypothesis of no cointegration. We analyze non-linearity within the error-correction model. Table 2 shows that the null of no linear cointegration can be rejected for some countries, but not for others. Also, results are sensitive to the test used.[8]

Table 3 presents tests of the null hypothesis of linearity against a smooth transition alternative (either of the ESTR or LSTR variety). The *p*-values are shown for each of the transition variables considered in (4). The preferred specification is shown in boldface, that is, the transition variable that yielded the *smallest* p-value across the various cases considered. In some cases, there is really little difference across *p*-values and it is, therefore, possible that there is more than one candidate for the transition variable.[9] Note that not all transition variables are relevant in all cases, either because they are inapplicable (e.g., as in deviations from an inflation target for a non-inflation targeting country), or are unlikely to be directly relevant in explaining the spread (e.g., the US spread in the case of Sweden).

The null hypothesis for the linearity tests is that all of the cross products and higher order terms are jointly insignificant resulting in an F-test with (3p, T-4p) degrees of freedom (Teräsvirta 1998, p. 516, outlines step-by-step the calculation details), where p is the number of parameters and T the number of observations. Both the logistic STR (LSTR) and the exponential STR (ESTR) variants were estimated and the model that fits the data best was selected. All reported STR models lead to positive estimates of the adjustment speed $\gamma$ and the estimates of the location parameter k were well within the bounds of the respective transition variable. Further, the chosen STR model was subjected to a variety of diagnostic tests as recommended by Teräsvirta (1998, p. 531 ff), namely no remaining non-linearity, no autocorrelation in the error term, the absence of ARCH, as well as parameter constancy. These are discussed later.

The null of linearity is strongly rejected in Table 3 when the lagged domestic interest rate spread is the transition variable for four countries. In one case, the rejection is not that strong (p=.033). However, in the case of Canada, Sweden and the UK, the error correction term and in

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[8] The Engle-Granger test is based on a regression of the short rate on the long rate. All tests were also run with the long rate as the dependent variable, as recommended by Engle and Granger (1987). As the chosen dependent variable did not affect the findings of cointegration only one set of results is shown in Table 2.

[9] Clearly, the search proceeds in a manner likely to yield some evidence of non-linearity in the term spread. Nevertheless, our interest is motivated to a greater extent by the search for economically sensible candidates to explain a preference of non-linear over linear models. Also, see Corradi, Swanson and White (2000).

the case of Canada and Sweden macroeconomic forecasts are also good transition variable candidates. Rejection of linearity using the inflation target variable occurs only for UK data. This result is interesting since it suggests that deviations from the UK inflation target, at least for the sample considered, are explaining the (smooth) transition from a policy of no stable nominal anchor to one where the inflation rate becomes the (credible) nominal anchor. In the case of the other two inflation targeting countries in the sample, namely Sweden and Canada, one is unable to reject linearity when deviations from the target represent the transition variable. Note, however, that the design of the targets differs significantly in these three countries. In Canada, the mid-point of ($\pi_{t-1}$ - $\pi^*$) has been falling gradually over time. In Sweden, the target was initially a point (2%) and then became a range (1-3%) after 1995. Finally, in the UK, the inflation control measures were first specified as a range (1-4%, until 1997) and then as an inflation rate below 2.5%. Of course, the fact that these targets were introduced in the early 1990s only undoubtedly represents another factor in the results obtained in Table 3. Finally, it is worthwhile noting that the ESTR model outperforms the LSTR model in all cases. This suggests that smooth transition tends to be symmetric around k.

When the US spread is the transition variable, the null of linearity for Canada is strongly rejected. This confirms the role of US disturbances in explaining movements in Canadian short-term interest rates. Nevertheless, the fact that forward looking variables are relevant in the case of Canada and Sweden (as well as the US), as is the error correction term, is important since these variables have not typically been used as transition variables in the estimation of non-linear models.

For the US, the full sample period leads to a rejection of the linear models with all possible transition variables considered. However, the non-linear models over the full sample period are not stable based on LM stability tests applied to the non-linear models. Due to structural change, only the sample form 1982 onwards leads to a stable non-linear model. In Table 3, this period corresponds to the second row for the US, where only the domestic spread leads to a rejection of linearity.

As noted earlier, it is not sufficient to simply reject linearity. The non-linear model must also pass a number of model adequacy tests. These are displayed in Table 4. Four sets of tests were considered. The application of these tests to STR models requires regressions involving the gradient vectors from the non-linear maximum likelihood function (see Teräsvirta, 1998, pp. 518-525; we implemented step 1' on p. 520). The test for ARCH-like behavior in the residuals and the degree of autocorrelation in these same residuals are commonly used diagnostic tests in both linear and non-linear models. Four of the seven cases showed some evidence of ARCH effects: Canada, Sweden, Switzerland, and the error-correction case of the UK. We report for these the maximum likelihood estimates in EViews which fits an ARCH(1). The row "no remaining ARCH" in Table 4 lists for these cases the test of additional ARCH after an ARCH(1) has been fitted. Moreover, three is no evidence of autocorrelation in the residuals for all six countries. Next, the test for remaining non-linearity is considered. It is obtained by asking whether non-linear terms are statistically significant when added to the baseline STR model in (4) (see Teräsvirta 1998, pp. 520-22). There is no strong evidence in favor of remaining non-linearity at the 5% significance level. Canada is a borderline case.

The test denoted $LM_1$ in Table 4 is a test for smooth monotonic changes in the parameters of the STR model and, as the speed of adjustment goes to infinity, the limiting case is a single structural break The $LM_2$ test is a test for a smooth non-monotonic change (in the STR parameters) symmetric about t-k, with the limiting case being two structural breaks. $LM_3$

modifies the transition function to permit non-monotonic, as well as monotonic, and non-symmetric changes in the STR model parameters (see Eitrheim and Teräsvirta 1996, and Teräsvirta 1998, pp. 522-24). As noted earlier, although it was found that while linearity could easily be rejected for the full sample using US data, the estimated model could not pass either the no remaining non-linearity or the parameter constancy tests for the full sample. However, upon re-estimation for the post-1982 sample, that is, following the "regime" change initiated by then Fed Chairman Paul Volcker, the non-linear US models pass all the diagnostic tests. The same problem was not encountered in any of the other countries' data sets. While it is possible that the US experience represents a "shock" that was not replicated elsewhere, our results do appear to indirectly reinforce the existing literature's view that the US spread has behaved significantly differently than the spread of other countries. For the UK we provide diagnostic tests for two cases, owing to the relatively brief sample since inflation targets were introduced. Nevertheless, whether the deviations from the inflation target or the error correction term are the transition variables, both models pass all of the model adequacy tests. Of the other countries, Sweden's evidence with respect to stability for the $LM_3$ test is a borderline case.

Figure 2 plots values of the transition function against the chosen transition variable for each country in the data set. All have the typical U or V shape that one would expect. It shows what values the transition function $F(\tau_t;\gamma,k) = 1 - \exp[-\gamma(\tau_t - k)^2]$ takes on for various values of the transition variable $\tau_t$ on the horizontal axis. The adjustment is around k, which is zero for Canada and Switzerland, negative for the UK(ec) and positive for all others. The values of the adjustment speed $\gamma$ are quite large for Switzerland, the UK(ec) and the US. Canada and Germany show somewhat lower speeds, and Sweden and the UK (IT band) rank last. However, even the last ranked countries have still a relatively fast adjustment speed. Overall, there is reasonably strong evidence pointing to the non-linear estimation approach to the spread in an international data set.

**Conclusions**

The premise of the paper is that there are good theoretical and empirical reasons to prefer non-linear estimation over a linear approximation when modeling the term spread. In particular, important changes in policy regimes and financial markets reactions to them argue for such an approach. Moreover, linear models are not as flexible as non-linear ones when the spread is modeled over a fairly long span of time. Finally, the non-linear approach allows the highlighting of economic variables likely to trigger the transition from one "regime" to another.

We considered in our study a linear versus a non-linear, smooth transition, error-correction model for the adjustment of short-term interest rates. We used monthly data since 1960 to show that linearity is rejected in six countries. While the (lagged) domestic spread is the most likely candidate for the variable used in the transition function for the non-linear model, it is noteworthy that an error correction term and the US spread are possibly just as important transition variables. Deviations from the targeted inflation rate are important in the UK case. In addition, forward looking variables such as inflation, output gap or real GDP growth also figure as important candidates for the transition function. Given the relevant literature's emphasis on the role of the spread as a forward-looking indicator of inflation and economic activity, these findings are also significant. Finally, there are some differences in the speed with which the spread signals a change in regime with US and UK data, in particular, indicating that a threshold may exist such that it signals a rapid transition from one regime to another (also see Enders and Siklos 2001). Consistent with other recent evidence (e.g., Lanne 1999, Bekaert, Hodrick and Marshall 1997a) we also reject the EH for the full sample in all countries. It is, therefore, possible that finding non-linearities in the term spread is the price we pay for rejecting the EH hypothesis which is, however, fundamentally a linear relationship.

Nevertheless, a number of issues remain to be addressed. Although there are good theoretical reasons to expect non-linearity in the term structure none of the existing theories have, so far, proved decisive. Consequently, if the empirical results of this paper are taken seriously, they point to the need for more theoretical work with explicit non-linear models to better isolate the source(s) of departures from linearity. Perhaps most important is the fact that the linear and non-linear models have not been subjected to a comparison of forecasting performance. Unfortunately, the literature is rather sparse on the subject (see, however, Granger and Teräsvirta 1993, ch. 8) and the technical difficulties of forecasting non-linear models beyond one period are considerable. However, it is important to point out that while several variables led us to reject the linear model, clarity in the choice of variables for the transition function was readily obtained when the various non-linear models were put through various diagnostic tests. The latter prove to be more important and more weight should be placed on these than has heretofore been the case.

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**Table 1**

***p*-Values for Unit Root Tests**

| **Country** | **Series** | | | | |
|---|---|---|---|---|---|
| | **Short rate** $r_{n,t}$ | **Long rate** $r_{1,t}$ | **Spread** $S_{n,t}$ | **Inflation Forecast** $\pi^{e}_{t}$ | **Output Gap, or Real GDP Growth Forecast** $y^{e}_{t}$ |
| Canada | .23 (10) | .45 (11) | .01 (10) | .12 (23) | .04 (23) |
| Germany | .47 (2) | .56 (1) | .14 (0) | .49 (20) | .11 (20) |
| Sweden | .05 (0) | .45 (7) | .0001(0) | .89 (24)[1] | .61 (0)[1] |
| Switzerland | .38 (22) | .40 (23) | .15 (0) | .80 (0)[1] | .06 (12)[1] |
| UK | .06 (1) | .62 (3) | .04 (1) | .37 (20) | .03 (20) |
| US (full) | .06 (16) | .43 (6) | .003 (16) | .49 (20) | .002 (23) |
| US (sub-sample)[2] | .42 (5) | .44 (12) | .13 (9) | .45 (12) | .23 (12) |

Note: Data used are monthly (1960-1998) from the source listed in the text, except for GDP forecasts for Canada (1971:06-1998:12), the UK (1968:06-1998:12), the US (1964:06-1998:12) and Sweden and Switzerland (1991:01-1998:12), and inflation forecasts for Canada (1960:12-1998:12), and Sweden and Switzerland (1991:01-1998:12). Interest rates and inflation rates are in percent. The test is the Augmented Dickey-Fuller t-ratio with the number of lags (in months) in parentheses selected according to Akaike's criterion. Generally, a time trend was not included in the test regressions. *p*-values have been calculated with the program from MacKinnon (1996).

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Inflation rates for Canada (.52 at 24 lags), the UK (.49 at 24 lags) and Sweden (.52 at 12 lags) exhibit unit root behavior. These rates will be used to calculate the deviations from the inflation target mid-range $\pi^*$ in Table 3.

1. CPI inflation and real GDP growth forecasts from the Economist were used.
2. 1982.11-1998.12.

**Table 2**

***p*-Values for Cointegration Tests**

| | Test Statistics | |
|---|---|---|
| **Country** | **Johansen** | **Engle-Granger** |
| Canada | .004 [1] | .14 [14] |
| Germany | .55 [1] | .09 [13] |
| Sweden | .007 [0] | .21 [23] |
| Switzerland | .25 [1] | .05 [24] |
| UK | .34 [1] | .46 [23] |
| US (full) | .02 [2] | .02 [14] |
| US (sub-sample) | .86 [1] | .19 [9] |

Note: The Johansen test is the likelihood ratio statistic for the trace version of the test with lags in the VECM shown in brackets using Schwarz's information criterion. A constant is present only in the cointegrating vector of the VECM and there are no deterministic trends. The Engle-Granger test is the Augmented Dickey-Fuller test statistic for the residuals from the cointegrating equation of the short rate on a constant and the long rate with the lag augmentation given in brackets. A maximum of 14 lags was assumed in the test equation with the lag chosen according to the longest lag augmentation with a t-value greater than 1.6, in absolute value. *p*-values calculated with programs from MacKinnon, Haug, and Michelis (1999) and from MacKinnon (1996). See Table 1 for sample details and the appendix for variable definitions.

**Table 3**

***p*-Values for the Test of Linearity Against a STR**

| | **Transition Variable** | | | | | |
|---|---|---|---|---|---|---|
| **Country** | $\mathbf{ec_{t-1}}$ | $\mathbf{S_{n,t-1}^{US}}$ | $\mathbf{S_{n,t-1}}$ | $\boldsymbol{\pi^e_t}$ | $\mathbf{y^e_t}$ | $\boldsymbol{\pi_{t-1} - \pi^*}$ |
| Canada | .004 | $\mathbf{4.54 \times 10^{-12}}$ | .033 | $5.97 \times 10^{-6}$ | $2.55 \times 10^{-5}$ | .280 |
| Germany | .408 | **.014** | .53 | .044 | .797 | n.a. |
| Sweden | $7.86 \times 10^{-9}$ | n.a. | $\mathbf{4.66 \times 10^{-9}}$ | .196 | $1.14 \times 10^{-6}$ | .427 |
| Switzerland | .135 | n.a. | **.008** | .352 | .129 | n.a. |
| UK | .024 | n.a. | .342 | .845 | .758 | **.022** |
| US (full) | 0 | n.a. | **0** | $5.48 \times 10^{-7}$ | .001 | n.a. |
| US (subsample) | .147 | n.a. | **.016** | .523 | .273 | n.a. |

Note: The transition variables are in levels or first differences, depending on the unit root test results reported in Table 1. Also, see Table 1 for definitions and sources for the time series, as well as sample length information. The smallest p-values in each row are shaded. Other transition variable candidates with *p*-values not much different from the lowest are in bold face. The first difference was used for the spread of Germany and Switzerland, for all inflation forecasts, and for the GDP forecast of Germany, Sweden and Switzerland. $S_{n,t-1}^{US}$ and $S_{n,t-1}$ are the yield spreads as defined in equation (2). The minimum and maximum values for the inflation target used to calculate the mid range of the inflation target $\pi^*$ are available as follows: for Canada 1991:02-1998:12, for Sweden 1993:03-1998:12, and for the UK 1992:10-1998:12. Otherwise samples are as shown in Table 1. n.a. means not applicable.

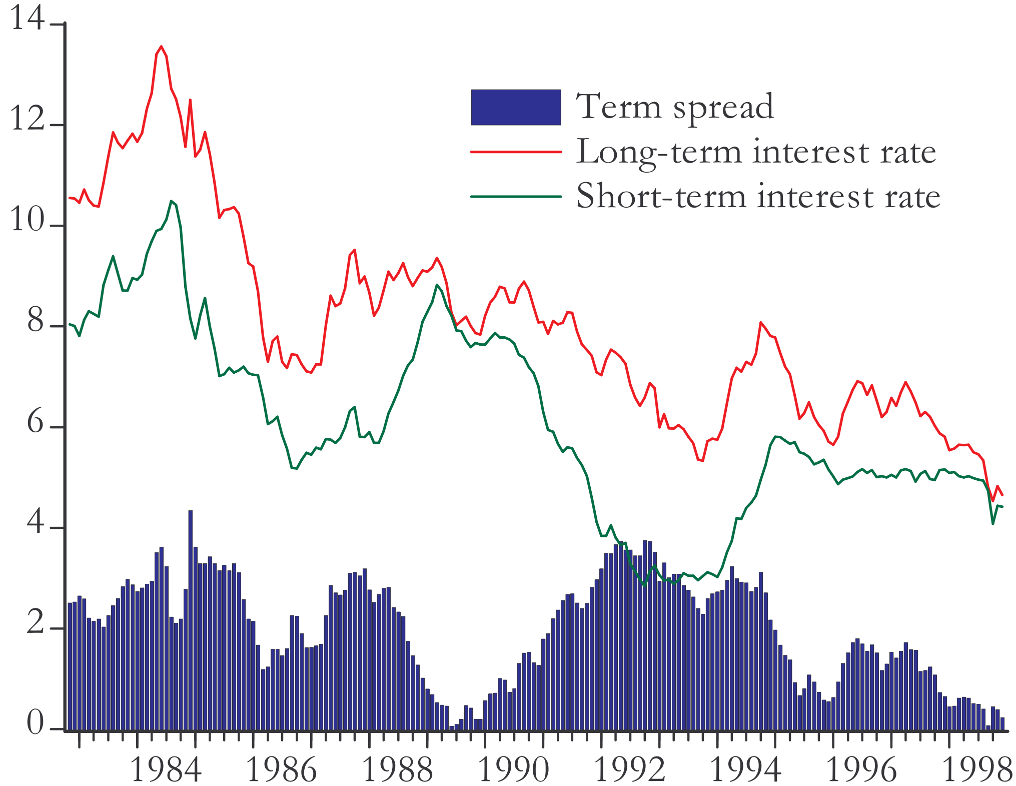
**Table 4**

***p*-Values for Diagnostic Tests of STR Models**

| | **Country** | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Canada** | **Germany** | **Sweden** | **Switzerland** | **UK** | | **US** |
| | **Transition Variable** | | | | | | |
| **Type of Non-linear Model** | ESTR | ESTR | ESTR | ESTR | ESTR | | ESTR |
| **Transition Variable** | *US Spread* | *US Spread* | *Domestic Spread* | *Domestic Spread* | *IT band* | *ec* | *Domestic Spread* |
| **Type of Test** | | | | | | | |
| *No Remaining ARCH* | .95 | .51 | .64 | .41 | .39 | .71 | .39 |
| *No Error Auto-correlation* | (3) .72<br>(6) .50<br>(12) .14 | (3) .99<br>(6) .77<br>(12) .32 | (3) .42<br>(6) .79<br>(12) .73 | (3) .37<br>(6) .85<br>(12) .30 | (3) .99<br>(6) .98<br>(12) .99 | (3) .98<br>(6) .98<br>(12) .96 | (3) .99<br>(6) .99<br>(12) .99 |
| No *Remaining Non-linearity* | .045 | .07 | .64 | .35 | .99 | .97 | .73 |
| *Parameter Constancy* | $LM_1$ =.40<br>$LM_2$ =.62<br>$LM_3$ = .69 | $LM_1$= .98<br>$LM_2$ =.98<br>$LM_3$ =.62 | $LM_1$ =.08<br>$LM_2$ =.11<br>$LM_3$ =.05 | $LM_1$=.06<br>$LM_2$=.10<br>$LM_3$=.08 | $LM_1$ =.99<br>$LM_2$ =.89<br>$LM_3$ =.58 | $LM_1$ =.99<br>$LM_2$ =.99<br>$LM_3$ =.99 | $LM_1$=.99<br>$LM_2$=.87<br>$LM_3$=.89 |

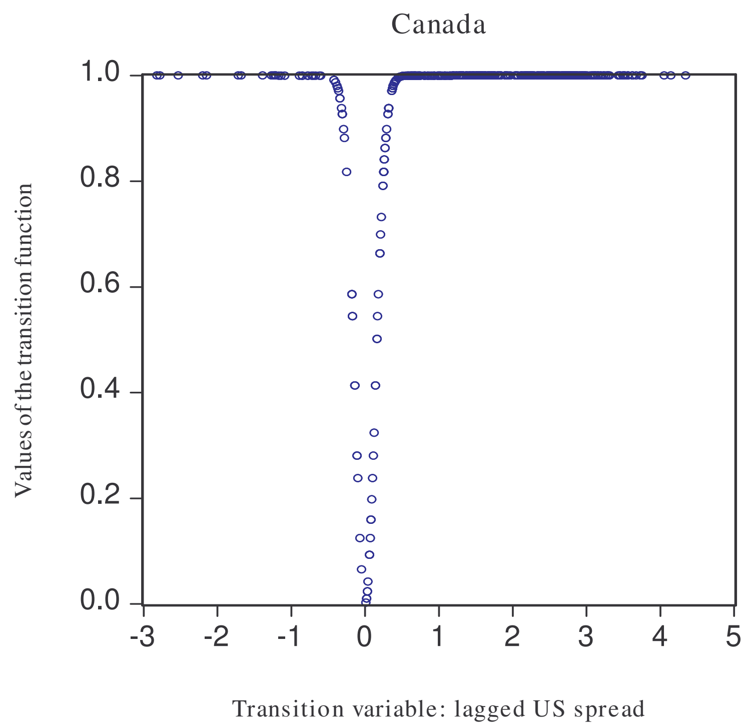
Note: Samples are given in Table 1. For the US, results are for the 1982-98 sample. No remaining ARCH is an LM test for ARCH order q, where q=1. Higher order ARCH tests were also considered but did not affect the conclusions. No remaining autocorrelation gives the p-value for autocorrelations at lags shown in parenthesis. No remaining non-linearity gives the p-value of the test proposed by Eitrheim and Teräsvirta (1996). The parameter constancy tests are a generalization of Lin and Teräsvirta (1994), as described in Teräsvirta (1998, pp. 522-24), that test for the constancy of all parameters in the estimated equation. *ec* is the lagged error correction term derived from the underlying cointegrating relationship between long and short rates.

**Figure 1. US Long and Short-Term Interest Rates**




Note: The short-term interest rate is the 3 month treasury bill rate; the long-term interest rate is the long-term government bond yield. The spread is the long rate less the short rate.

**Figure 2. Transition Functions**



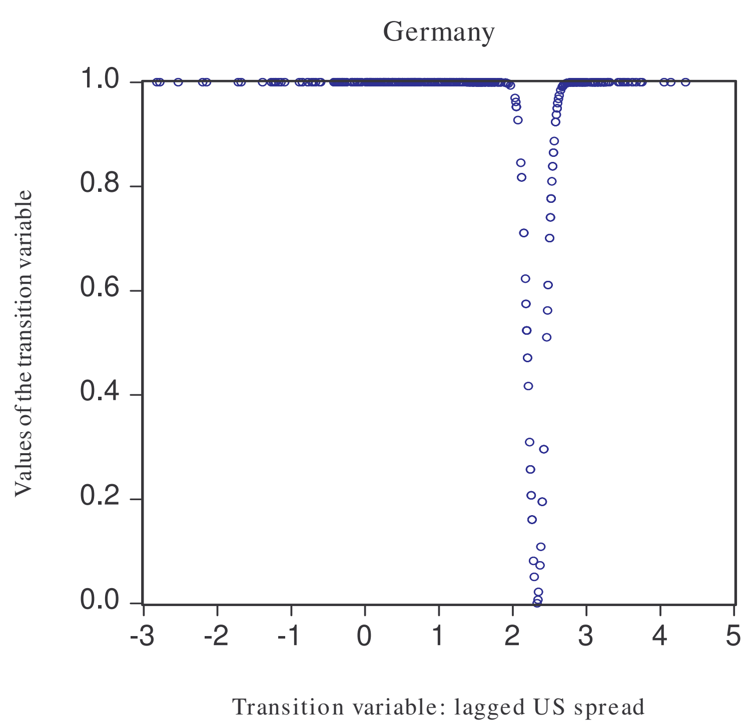
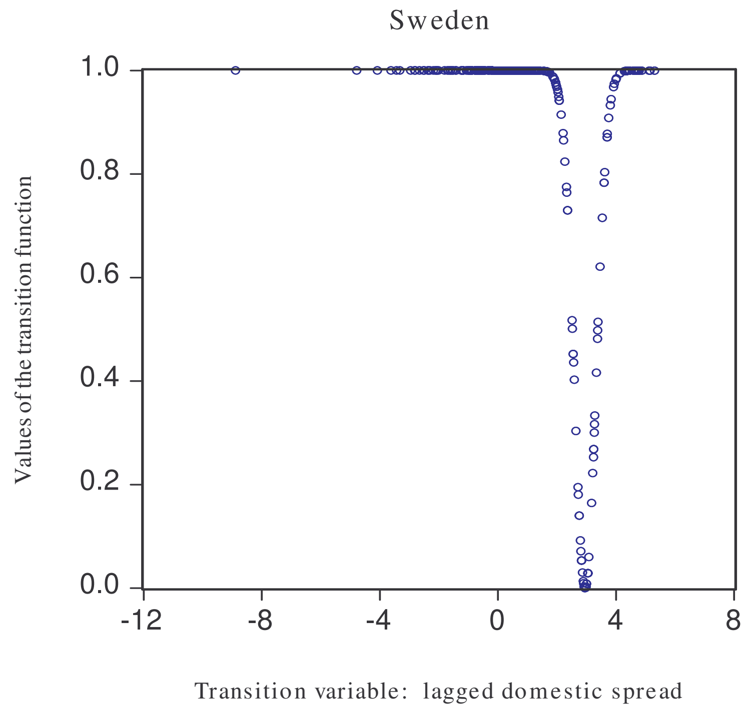



**Figure 2 cont'd**



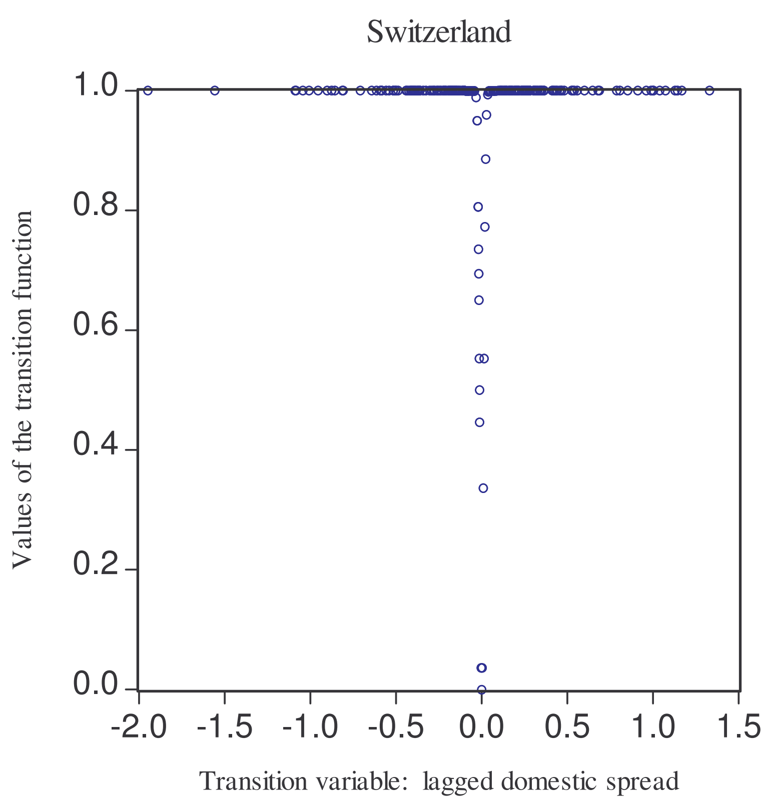
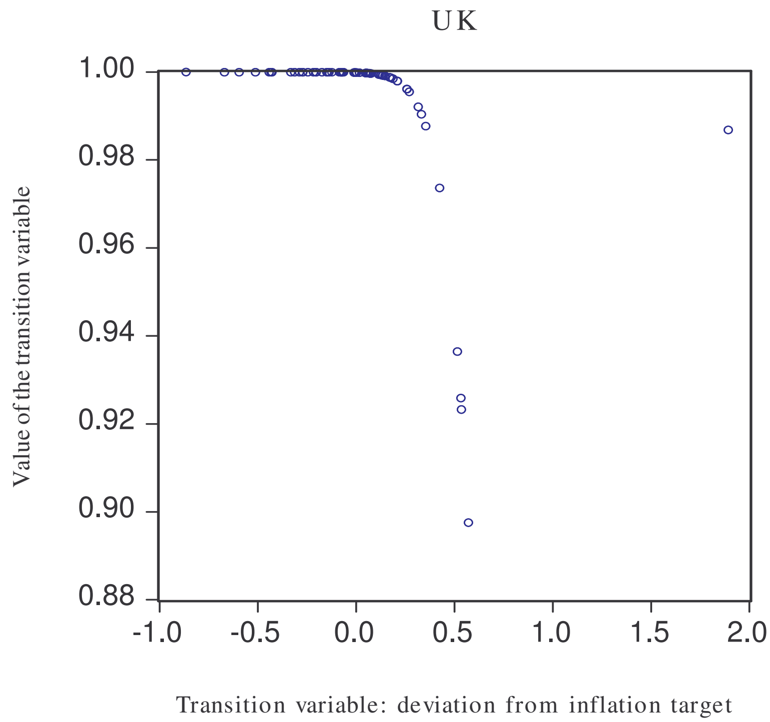



**Figure 2 cont'd**



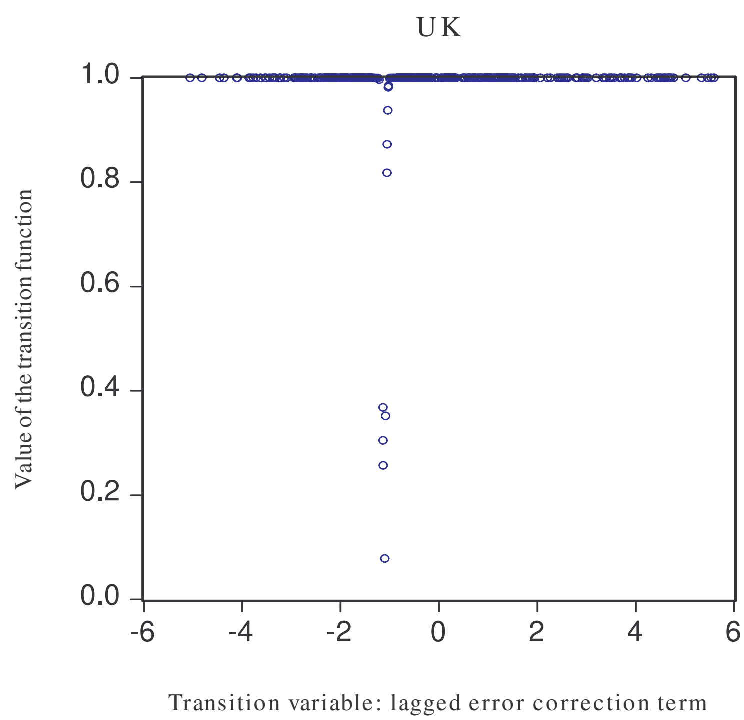
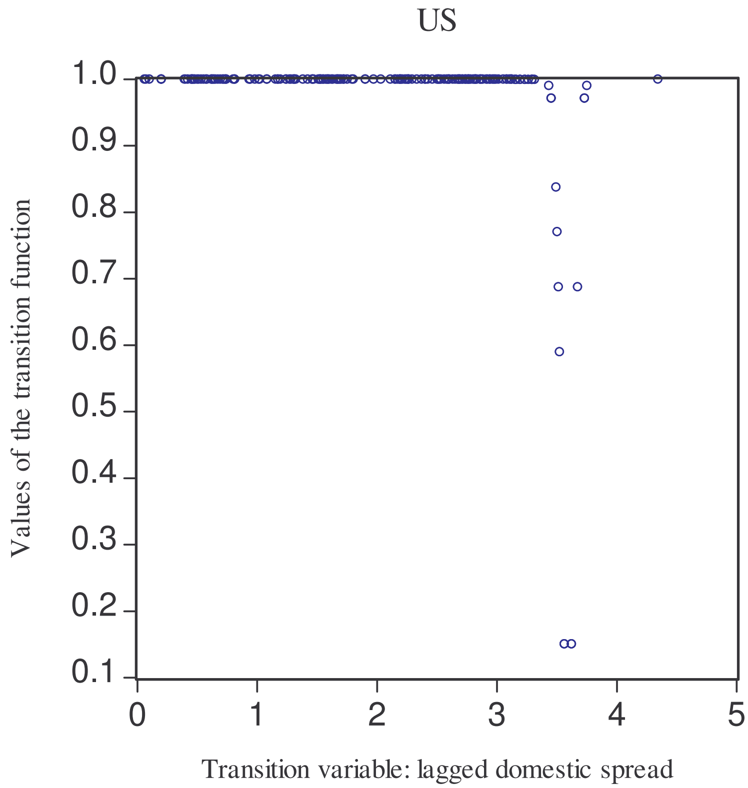



**Figure 2 cont'd**



## Data Appendix: IFS series definitions

| Country | Short-term rate | Long-term rate |
|---|---|---|
| **Canada** | 90 day Treasury bills | 10 years and over government bonds |
| **Germany** | Treasury bill rate | Long-term government bonds |
| **Sweden** | 3 month treasury discount notes | Long-term government bonds |
| **Switzerland** | Treasury bill rate | Long-term government bonds |
| **UK** | Treasury bill rate | Long-term government bonds |
| **US** | Treasury bill rate | 10 years government bonds |