Monetary Non-Superneutrality and Endogenous Time Preference in an Infinitely Lived, Representative Agent Model

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Abstract

This model demonstrates a restatement of the Mundell-Tobin effect and monetary non-superneutrality using an infinitely lived, representative agent model. The rate of time preference is assumed to be an increasing function of the total value of current financial wealth. An increase in the monetary growth rate reduces the value of real assets and the rate of time preference, which raises savings, consumption and the capital stock. This model offers an optimizing equivalent to descriptive models that assume savings are a decreasing function of wealth. This confirms Epstein and Hynes' intuition without being prone to the counterintuitive assumptions of Uzawa.

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1. Introduction

The Mundell-Tobin effect, which describes the causality underlying monetary non-superneutrality, has previously been demonstrated only in descriptive, non-optimizing models (Begg 1980) or representative agent models based on unpalatable assumptions (Uzawa 1968). This paper provides a restatement of the Mundell-Tobin effect in an optimizing model where the rate of time preference is an increasing function of real financial assets. The critical outcome is that monetary superneutrality is *not* the inevitable result of optimizing agent models. Rather, it results from the assumption of exogenous time preference. Endogenous time preference generates monetary non-superneutrality, where the real interest rate is a decreasing function of monetary growth and can be targeted as a policy tool by the central monetary authority.

Monetary growth is generally studied using the optimizing framework of an infinitely lived, representative agent with an exogenous rate of time preference. Sidrauski (1967a,b) assumes exogenous time preference and proves equality and invariance between the steady state rate of time preference, the real interest rate and the marginal product of capital. Exogenous time preference generates monetary superneutrality by severing the link between the real and monetary sectors. Monetary growth affects inflation and real balance holdings, but real sector variables such as consumption and the capital stock are unaffected.

Mundell (1963) and Tobin (1965) were the first to prove that if agents are able to hold wealth as capital or real balances, the real interest rate is transformed into an endogenous variable, which is determined by an interaction of monetary and portfolio decisions. Monetary growth decreases the initial value of financial assets and raises the real interest rate. As the opportunity cost of holding real balances increases, the equilibrium level of savings rises. Since savings are assumed to be a decreasing function of financial wealth, this results in higher levels of consumption and capital stock. However, their results were derived in a non-optimizing, descriptive model.

Begg (1980) also uses a descriptive, non-optimizing model that assumes perfect foresight to demonstrate the Mundell-Tobin effect. Monetary non-superneutrality is attained by arbitrarily placing a wealth effect in the consumption function and assuming that the demand for real balances is a decreasing function of the nominal interest rate. Satisfying these conditions make the real interest rate and the marginal product of capital endogenous, which links the real and monetary sectors. Placing a wealth effect in the consumption function replicates a savings function that is decreasing in the level of financial wealth. Monetary growth restores equilibrium in the goods market by raising the steady state levels of consumption and capital.

Uzawa (1968) was the first to replicate a Mundell-Tobin effect in an infinitely lived, representative agent model that implements endogenous time preference. He assumes that the rate of time preference depends positively on the level of current utility, which itself is an increasing function of consumption. Monetary growth raises the opportunity cost of holding real balances and renders the initial equilibrium too costly. This increases the real interest rate and decreases the demand for real balances, which increases savings and the capital stock. Endogenous time preference restores the link between the two sectors; thus ensuring that monetary growth is non-superneutral. The real interest rate and the marginal product of capital equal the rate of time preference, but are all endogenous as the discount rate varies positively with fluctuations in consumption and utility.[2]

Uzawa's procedure for transforming the rate of time preference into an endogenous function received considerable criticism. His results of monetary non-superneutrality and steady state stability depend critically on the assumption that the rate of time preference is an increasing function of instantaneous utility and consumption. This assumption is a highly questionable, counterintuitive description of behavior. It implies that savings are an *increasing* function of wealth. Increasing present consumption raises the rate of time preference, which increases present consumption and diminishes future consumption. This contradicts the accepted intuition that savings are a *decreasing* function of financial wealth as described by the Mundell-Tobin effect.[3]

This paper transforms the rate of time preference into an endogenous function by instead assuming that it depends positively on the level of real financial assets, which are defined as capital plus real balances. The result is an infinitely lived, representative agent model that demonstrates monetary non-superneutrality as described in the Mundell-Tobin effect. This assumption is consistent with descriptive models that regard savings as a *decreasing* function of wealth. Monetary growth lowers the real value of financial assets and the rate of time preference. Inflation raises the real interest rate and the opportunity

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[2] Uzawa's formulation of endogenous time preference has received considerable attention in the open-economy literature. See Dornbusch (1973a,b), Obstfeld (1981a,b, 1982), Svensson and Razin (1983), Persson and Svensson (1985), Matsuyama (1987,1988), Sen and Turnovsky (1989) and Mansoorian (1992).

[3] Persson and Svensson (1985,45) refer to this underlying assumption as being, "…arbitrary, and even counterintuitive." Blanchard and Fisher (1989,71) conclude, "…the Uzawa function…is not particularly attractive as a description of preferences and is not recommended for general use."

cost of holding real balances. The result is increased savings and higher levels of steady state capital and consumption.[4]

Section two develops the representative agent model. Section three demonstrates that the equilibrium solution is steady state stable along a saddle path, assuming that the rate of time preference is an increasing function of real financial assets. Section four provides a summary of the results.

2. The Representative Agent Problem

This endogenous time preference model is a synthesis that extends the contributions of Sidrauski, Uzawa and Begg. Like Sidrauski, real balances are assumed to offer utility and are assumed exist as an argument in the utility function. Like Uzawa, time preference is endogenous, but is instead an *increasing* function of real financial assets. Like Begg, wealth effects contribute to monetary non-superneutrality, but they are not arbitrarily placed in the consumption function. The rate of time preference interacts with real financial assets to provide optimizing foundations to the use of a wealth effect.[5]

The rate of time preference (q) is an increasing function of real financial assets (a), which are defined as the level of real balances (m) plus the real capital stock (k):

$$q = q(a) = q(k+m) \tag{1}$$

$$q'(k+m) > 0 \tag{2}$$

[4] Monetary non-superneutrality has also been demonstrated in models that assume overlapping generations (Drazen 1981) and a transactions technology or cash-in-advance (Clower) constraint (Stockman 1981, Abel 1985,1987). Those models abandoned the infinitely lived, representative agent model, which, following Sidrauski (1967a,b) was believed to imply monetary superneutrality. This paper maintains the general form of the infinitely lived, representative agent framework. Together with the assumption of endogenous time preference as an increasing function of total financial assets, this model is functionally equivalent and less restrictive than those assuming overlapping generations or a cash-in-advance constraint.

[5] Epstein and Hynes (1983) first offered the intuition for using wealth effects to transform time preference into an endogenous function, but it received only a footnote. They argue that monetary growth raises the opportunity cost of holding real balances, which shifts a positively sloped rate of time preference function

This assumption implies monetary non-superneutrality and the Mundell-Tobin effect in an optimizing model that is not prone to the criticism that weakens Uzawa (1968). Raising the level of real financial assets increases the rate of time preference and future consumption. This does not contradict the accepted intuition that savings are a *decreasing* function of financial wealth as described by the Mundell-Tobin effect.

The representative agent maximizes the intertemporal utility function:

$$Max U_0 = \int_0^\infty [u(c_t, m_t) e^{-\int_0^t q_v dv} dt \qquad (3)$$

Subject to the constraints:

$$\dot{a} = f(k) + x - pm - dk - c \qquad (4)$$ [6]

$$a = k + m \qquad (5)$$

$$\lim_{t \to \infty} a_t e^{\int_0^t r_v dv} \geq 0 \qquad (6)$$

(c) is the consumption level, (l) is a co-state variable, (p) is the inflation rate, (d) is the constant, positive depreciation rate on the existing capital stock, (r) is the real interest rate and (x) is the real balances transferred from the public to the private sector. The standard concavity assumptions hold with respect to the utility function in (6):

$$u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0 \qquad (7)$$

$$u_{cm} = u_{mc} > 0 \qquad (8)$$

down along a negatively sloped marginal product of capital locus. This reduces the real interest rate and increases steady state capital according to the Mundell-Tobin effect.

[6] This is a dynamic budget constraint that combines both stock and flow constraints. The derivation of this constraint is found in the Appendix.

and the following notation is adopted:

$$q = u_{cc} u_{mm} - u_{cm}^2 > 0 \tag{9}$$

$$q_1 = u_{mm} - \left(\frac{u_{cm} u_m}{u_c} \right) < 0 \tag{10}$$

$$q_2 = \left(\frac{u_{cc} u_m}{u_c} \right) - u_{cm} < 0 \tag{11}$$ [7]

This produces the current valued Hamiltonian:

$$H = u(c,m) + l\,\{ f(a-m) + x - pm - d(a-m) - c \} \tag{12}$$

and the optimality conditions:

$$u_c(c,m) - l = 0 \tag{13}$$

$$u_m(c,m) - l\,[f'(k) - d + p] = 0 \tag{14}$$

$$\dot{l} = -l\,[f'(k) - d - q(k+m)] \tag{15}$$

$$\lim_{t \to \infty} a_t\, l_t\, e^{-\int_0^t q_v dv} = 0 \tag{16}$$ [8]

The steady state is defined by three equations. From (13) and (14), the result is:

$$\frac{u_m(c,m)}{u_c(c,m)} = f'(k) - d + p \tag{17}$$

From the resource constraint:

$$\dot{k} = f(k) - dk - c \tag{18}$$ [9]

From (18) with $\dot{k} = 0$ provides:

$$c^* = f(k^*) - dk^* \tag{19}$$

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[7] (8) implies that both assets, capital and real balances, are complementary commodities in the accumulation process. This assumption is necessary to ensure that (10) and (11) are negative, which guarantees that both assets are normal goods.

[8] (16) is a "No Ponzi Game" condition that makes it impossible to borrow capital infinitely.

From (15) with $\dot{l} = 0$ gives:

$$q(k^* + m^*) = f'(k^*) - d = r \tag{20}$$

From (19) and (4) using $\dot{a} = 0$, the result is:

$$x^* = m^* p^* = mm^* \tag{21}$$

which implies:

$$p^* = m \tag{22}$$

Equilibrium in the capital market equilibrium implies that the real interest rate equals the marginal product of capital less depreciation:

$$r = f'(k^*) - d \tag{23}$$

To determine the effect of monetary growth on the steady state levels of consumption, real balances and the capital stock, denoted as (c^*, m^*, k^*), linearize (17), (19) and (20) around (c^*, m^*, k^*) using (22) and (23) to obtain:

$$(f'(k^*) - d)dk^* = dc^* \tag{24}$$

$$q'(dm^* + dk^*) = f''(k^*)dk^* \tag{25}$$

$$u_{mm}dm^* + u_{mc}dc^* = f''(k^*)u_c dk^* + u_c dm + (f'(k^*) + m)[u_{cc}dc^* + u_{cm}dm^*] \tag{26}$$

Adopt the notation:

$$u_{mc} - [f'(k) + m]u_{cc} \equiv j > 0 \tag{27}$$

$$u_{mm} - [f'(k^*) + m]u_{cm} \equiv y < 0 \tag{28}$$

$$[q' - f''] \equiv r > 0 \tag{29}$$

[9] The derivation of this resource constraint is found in the Appendix.

Use (27), (28) and (29) to rewrite (24), (25) and (26):

$$\begin{bmatrix} -1 & 0 & q \\ 0 & q' & r \\ j & y & -f''u_c \end{bmatrix}\begin{bmatrix} dc^* \\ dm^* \\ dk^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ U_c \end{bmatrix} \tag{30}$$

The determinant of (30) is:

$$|B| = (-1)\begin{vmatrix} q' & r \\ y & -f''u_c \end{vmatrix} + q\begin{vmatrix} 0 & q' \\ j & y \end{vmatrix} = (-1)[-f''u_c q' - r\,y] + q[-j\,q']$$
$$= [u_c f''q' + y\,r] - qq'j < 0 \tag{31}$$

As the determinant is non-zero, the effects of money growth can be obtained by applying Cramer's Rule to (30). With respect to consumption:

$$\frac{dc^*}{dm} = \frac{\begin{vmatrix} 0 & 0 & q \\ 0 & q' & r \\ u_c & y & -f''u_c \end{vmatrix}}{|B|} = \frac{q\begin{vmatrix} 0 & q' \\ u_c & y \end{vmatrix}}{|B|} = \frac{-qq'u_c}{|B|} > 0 \tag{32}$$

with respect to real balances:

$$\frac{dm^*}{dm} = \frac{\begin{vmatrix} -1 & 0 & q \\ 0 & 0 & r \\ j & u_c & -f''u_c \end{vmatrix}}{|B|} = \frac{-u_c\begin{vmatrix} -1 & q \\ 0 & r \end{vmatrix}}{|B|} = \frac{u_c\,r}{|B|} < 0 \tag{33}$$

and with respect to the capital stock:

$$\frac{dk^*}{dm} = \frac{\begin{vmatrix} -1 & 0 & 0 \\ 0 & q' & 0 \\ j & y & u_c \end{vmatrix}}{|B|} = \frac{-1\begin{vmatrix} q' & 0 \\ y & u_c \end{vmatrix}}{|B|} = \frac{-q'u_c}{|B|} > 0 \tag{34}$$

Monetary growth rate positively affects the steady state level of consumption and capital, and decreases the holdings of real balances. When time preference is assumed to be an increasing function of real financial assets, the real interest rate and the marginal product of capital are endogenous. This flexibility ensures that monetary fluctuations are transferred throughout the real economy. Monetary growth increases the opportunity cost of holding real balances and the rate of time preference. As the demand for real balances falls, increased savings results in higher steady state levels of consumption and capital.

3. Stability Analysis

The differential equation system defined by $(\dot{c}, \dot{m}, \dot{k})$ exhibits saddle point stability if the rate of time preference is an increasing function of financial wealth, or $q'(k+m) > 0$. Begin with the evolution of the capital stock, which is given by the resource constraint:

$$\dot{k} = f(k) - dk - c \tag{35}$$

To solve for the evolution of real balances, note that:

$$\dot{m} = (m - p)m \tag{36}$$

Solve for p from (17) and substitute the result into (36) to obtain:

$$\dot{m} = \left(m - \frac{u_m(c,m)}{u_c(c,m)} + f'(k) - d \right) m \tag{37}$$

To solve for the evolution of consumption, differentiate (13) with respect to time to get:

$$u_{cc}\dot{c} + u_{cm}\dot{m} = \dot{l} \tag{38}$$

Substitute $\dot{l}$ from (15) and $\dot{m}$ from (37) into (38) using (13) to get:

$$\dot{c} = \frac{1}{u_{cc}} \left[u_c(c,m)\big(q(k+m) - f'(k) + d\big) - u_{cm}\left(m - \frac{u_m(m,c)}{u_c(m,c)} + f'(k) - d \right) m \right] \tag{39}$$

(35), (37) and (39) constitute the reduced form model in (c,m,k). Set $\dot{c}=\dot{m}=\dot{k}=0$ and denote the solution of the steady state system by (c^*,m^*,k^*). The steady state values of (c,m,k) are such that:

$$q(k^*+m^*)-f'(k^*)+d=0 \tag{40}$$

$$m-\frac{u_m}{u_c}+f'(k^*)-d=0 \tag{41}$$

$$f(k^*)-dk^*-c^*=0 \tag{42}$$

Linearize (35), (37) and (39) around (c^*,m^*,k^*) using (40), (41) and (42) to obtain:

$$\begin{bmatrix}\dot{c}\\ \dot{m}\\ \dot{k}\end{bmatrix}=\begin{bmatrix}\frac{-u_{cm}q_2m^*}{u_{cc}u_c} & \frac{q'u_c^2+u_{cm}q_1m^*}{u_{cc}u_c} & \frac{[-f''(k^*)(u_c+u_{cm}m^*)]+q'u_c}{u_{cc}}\\ \frac{q_2m^*}{u_c} & \frac{-q_1m^*}{u_c} & f''(k^*)m^*\\ -1 & 0 & q\end{bmatrix}\begin{bmatrix}c-c^*\\ m-m^*\\ k-k^*\end{bmatrix} \tag{43}$$

The determinant of the coefficient matrix in (43) is:

$$|\Delta|=-1\begin{vmatrix}\frac{u_{cm}q_1m^*+q'u_c^2}{u_{cc}u_c} & \frac{[-f''(k^*)(u_{cm}m^*+u_c)]+q'u_c}{u_{cc}}\\ \frac{-q_1m^*}{u_c} & f''(k^*)m^*\end{vmatrix}$$
$$+q\begin{vmatrix}\frac{-u_{cm}q_2m^*}{u_{cc}u_c} & \frac{q'u_c^2+u_{cm}q_1m^*}{u_{cc}u_c}\\ \frac{q_2m^*}{u_c} & \frac{-q_1m^*}{u_c}\end{vmatrix} \tag{44}$$

The first part is:

$$(-1)\left[\frac{\left(u_{cm}q_1m^* + q'u_c^2\right)\left(f''m^*\right)}{u_{cc}u_c} - \frac{\left(-q_1m^*\right)\left[-f''\left(u_{cm}m^* + u_c\right) + q'u_c\right]}{u_{cc}u_c}\right]$$

$$= \left[\frac{-f''m^{*2}u_{cm}q_1 - q'u_c^2 f''m^* + f''m^{*2}u_{cm}q_1 + f''q_1m^*u_c - q_1m^*q'u_c}{u_{cc}u_c}\right]$$

$$= \left[\frac{-q'u_c^2 f''m^* + f''q_1m^*u_c - q_1m^*q'u_c}{u_{cc}u_c}\right] = \left[\frac{-q'f''u_c + f''q_1 - q'q_1}{u_{cc}u_c}\right] \tag{45}$$

$$= \frac{q_1 f'' - q'\left(u_c f'' + q_1\right)}{u_{cc}u_c} < 0$$

The second part is:

$$= q\left[\left(\frac{u_{cm}q_2m^{*2}q_1}{u_c^2u_{cc}}\right) - \left(\frac{q_2m^*u_c^2q' + q_2m^{*2}u_{cm}q_1}{u_c^2u_{cc}}\right)\right]$$

$$= q\left[\frac{u_{cm}q_2m^{*2}q_1 - q_2m^*u_c^2q' - q_2m^{*2}u_{cm}q_1}{u_c^2u_{cc}}\right] \tag{46}$$

$$= \frac{-qq'q_2m^*u_c^2}{u_c^2u_{cc}} = \frac{-qq'q_2m^*}{u_{cc}} < 0$$

The determinant in (43) is the sum of two negative parts and is itself negative, $|\Delta| < 0$.

The trace of the matrix in (43) is:

$$trace = -\left[\frac{u_{cm}q_2m^*}{u_{cc}u_c}\right] - \left[\frac{q_1m^*}{u_c}\right] = -m^*\left[\frac{q_1}{u_c} + \frac{u_{cm}q_2}{u_{cc}u_c}\right] + q \tag{47}$$

Using (9), (10) and (11), (47) is reduced to:

$$trace = -m^*\left[\frac{u_{mm} - u_{cm}\left(\dfrac{u_m}{u_c}\right)}{u_c} + \frac{u_{cm}\left[u_{cc}\left(\dfrac{u_m}{u_c}\right) - u_{cm}\right]}{u_{cc}u_c}\right] + q$$

$$= -m^*\left[\frac{u_{mm}u_{cc} - u_{cc}u_{cm}\left(\dfrac{u_m}{u_c}\right) + u_{cm}u_{cc}\left(\dfrac{u_m}{u_c}\right) - u_{cm}^2}{u_{cc}u_c}\right] + q \tag{48}$$

$$= -m^*\left[\frac{q}{u_{cc}u_c}\right] + q > 0$$

The coefficient matrix in (43) has three characteristic roots. As the trace is positive and the determinant is negative, only one of the three characteristic roots is negative. The system defined by $(\dot{c}, \dot{m}, \dot{k})$ is steady state stable at a saddle point.

4. Summary and Conclusions

This paper strongly rejects the position that monetary superneutrality is a stylized fact of monetary growth models with optimizing, infinitely lived, representative agents. It is the assumption of exogenous time preference that generates monetary superneutrality. Exogenous time preference fixes the real interest rate and the marginal product of capital so that real variables such as capital and consumption are unaffected by monetary growth. The only appreciable effect of increasing the monetary growth rate is to raise the opportunity cost, and lower the steady state demand, for holding real balances.

By assuming that the rate of time preference is endogenous and an increasing function of real financial assets, the optimizing representative agent model in this paper generates monetary non-superneutrality, as described by the Mundell-Tobin effect. Monetary growth affects real variables since the real interest rate and the marginal product of capital vary with changes in the level of financial wealth and the rate of time preference. This method of transforming the rate of time preference into an endogenous function provides the first optimizing model that yields the same results as the descriptive models of Mundell, Tobin and Begg, while avoiding the criticism of counterintuitive preferences that hinders Uzawa's optimizing model.

Endogenous time preference results in monetary non-superneutrality in an infinitely lived, representative agent model that is consistent with the underlying causality of the Mundell-Tobin effect. Assuming that time preference depends positively on the level of real financial assets presents an optimizing equivalent to the assumption that savings are a *decreasing* function of financial wealth. Monetary growth decreases the rate of time preference and raises future consumption. The real interest rate fluctuates with monetary growth and can be a policy tool that is targeted by the central monetary authority.

Mathematical Appendix

1. Derivation of the Dynamic Budget Constraint

The dynamic budget constraint is a one-equation representation of both a stock and flow constraint. Deriving the stock constraint is based on the assumption that the stock of non-human or physical wealth at any time t is allocated between the two assets, capital and real balances. In per capita terms the stock constraint is represented by:

$$a_t = k_t + m_t \tag{A1}$$

a_t is the per capita stock of non-human wealth, k_t is the capital labor ratio and m_t is the stock of real balances.

Deriving the flow constraint assumes that disposable income is allocated between saving and consumption at time t:

$$y_d = c_t + s_t \tag{A2}$$

y_d is disposable income, c_t is consumption and s_t is the level of saving.

Per capita disposable income is the sum of per capita output and per capita real transfers from the government, all evaluated at time t:

$$y_d = f(k_t) + x_t \tag{A3}$$

$f(k_t)$ is a constant returns to scale production function, x_t is the value of real government transfers. The transfers are financed completely through money creation. The production function satisfies the usual concavity assumptions:

$$f'(k) > 0,\ f''(k) < 0 \tag{A4}$$

and the Inada conditions:

$$f(0) = 0,\ f'(0) = \infty,\ f'(\infty) = \infty,\ f'(\infty) = 0 \tag{A5}$$

Real saving is the sum of capital accumulation and additions to real balances at time t:

$$s_t = i_t + b_t \tag{A6}$$

i_t is the accumulation of capital and b_t is the addition to real balances.

To ensure that production may begin without a lag, there is an initial endowment of real balances and capital at time t:

$$k_t, m_t > 0 \tag{A7}$$

Capital accumulation is the net change in the capital labor ratio plus the replacement of depreciated capital at time t:

$$i_t = \dot{k}_t + dk_t \tag{A8}$$

where d is a constant but positive rate of capital depreciation and:

$$\dot{k}_t = \frac{dk_t}{dt} \tag{A9}$$

The additions to real balances are the net increases in real balances plus the monetary growth required to hold the expected value of existing real balances at a constant level at time t:

$$b_t = \dot{m}_t + p_t m_t \tag{A10}$$

p is the expected inflation rate and:

$$\dot{m} = \frac{dm}{dt} \tag{A11}$$

Using (A2), (A3), (A6) and (A8) to (A11), the flow budget constraint is represented as:

$$f(k) + x = c + \dot{k} + dk_t + \dot{m} + p\dot{m} \tag{A12}$$

Suppress the time subscripts and rewrite (A12) to obtain:

$$\dot{m} + \dot{k} = f(k) + x - pm - dk - c \tag{A13}$$

Differentiate the stock budget constraint in (A1) with respect to time:

$$\dot{a} = \dot{k} + \dot{m} \tag{A14}$$

Substitute (A14) into (A13) to find the dynamic budget constraint (4) from the main text:

$$\dot{a} = f(k) + x - pm - dk - c \tag{A15}$$

2. Derivation of the Resource Constraint

The resource constraint is derived with the stock and flow constraints and the assumption that agents have perfect foresight. Substitute (A15) into (A14) to obtain:

$$\dot{k} = f(k) + x - pm - dk - c - \dot{m} \tag{A16}$$

By the assumption of perfect foresight:

$$x = pm = mm \tag{A17}$$

and:

$$\dot{m} = (m - p)m \tag{A18}$$

Substitute (A18) into (A16) to obtain the resource constraint (19) from the main text:

$$\dot{k} = f(k) - dk - c \tag{A19}$$

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