Optimal Incentives in Dynamic Multiple Project Contracts *

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Abstract

We design a multiple project-funding contract that provides optimal incentives to recipients, in a setting where externalities exist among the multiple projects and where donors and recipients may differ in their valuation of the projects.

To do so, we study optimal incentive payments in a dynamic principal-agent framework with focus on two-project contracts. The principal cannot observe the agent's investment, but only completed projects. We consider principals that cannot commit to contract termination before completion of the projects; we assume that the contract does not end until both projects are accomplished.

We derive the optimal contract for each possible combination of principal-agent-project characteristics to find that projects should be undertaken simultaneously when value externalities among them are large, i.e. when completing both projects gives the recipient significantly more utility than the sum of the projects' independent values. The principal's utility maximizing strategy, when technical externalities among projects are important, is a sequential contract that starts with the project that generates the externality.

We find that differences in project valuation between agents and recipients may, in some cases, lead to inefficient contracts, when in other situations the ability of the principal to choose the timing of the project competition may be a safety clause for him.

Keywords: Dynamic contracts, Multitask, Foreign Aid.

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1 Introduction

What should multiple project foreign aid contracts look like? How should incentives be provided to managers when multiple projects must be completed and managers are specific to these projects? Principals in both settings share a commitment problem: due to their Samaritan's dilemma and specificities in the relationship, whatever the outcome of investment in each period, they cannot commit to abandoning the contract. Moreover, agents in these settings share a special characteristic: they value the projects to be completed.

The questions we attempt to answer are as follows. First, what is the optimal timing to complete the projects? Second, what is the optimal transfer scheme between the principal and agent in the framework of multiple, related projects? And third, what is the impact of the principal's and agent's preferences over the projects, in equilibrium, on the efficiency of the contract provided?

We study, in a dynamic principal-agent framework, the optimal incentive payments and timing of two projects, and how both technical characteristics of the projects and the preferences of principals and agents affect the optimal contract.

Both foreign aid donors and shareholders face a moral hazard problem: the use of funds by the recipient country/manager is not observable by the donor/shareholder. In extant foreign aid literature, conditionality of aid is the most commonly proposed solution to the misallocation of resources problem[1]. However, conditionality introduces a commitment problem: *even if the conditions are not met, the donors want to alleviate the lot of the poor, and so they give the aid anyway; the recipients will anticipate this behavior*[2]. For example, Dreher (2002)[3] states that the World Bank has almost never cancelled a program, even if non-compliance is evident. By further example, when firms expand to new markets, local knowledge may require managers to be utilized specifically for the relationship, and the threat of firing the manager would imply the end of business in that area, beyond the projects involved in the contract.

We present a contract that accounts for the principal's commitment problem by not allowing for cancellation of the contract before the projects are completed. This requires that the contract proposed have an indeterminate length. We assume that the principal

[1] Drazen (1999), Svenson (2000, 2003), and Azam and Laffont (2003), among others, present models that "condition" aid flows on a given performance, a degree of political and economic change, or on a defined consumption level for the poorest people in the recipient country, respectively.

[2] Easterly (2001).

[3] Killick (1998), Dreher (2002) and The World Bank (2005) present reviews of the literature and examples of time inconsistency involving conditional aid contracts.

can commit to a sequence of transfers for each possible history of successes and failures of the projects, but not to cut the flow of funds before all projects are accomplished.

Given that multiple actions must be undertaken, it is necessary to establish appropriate timing for the completion of each project: either simultaneous or sequential, in any possible order. This intervention timing is a key element in the contract design. On the one hand, the projects may influence one another, and the timing of each project determines these influences. On the other hand, donor and principal priorities may not be aligned. This misalignment of preferences may have both positive and negative effects on the efficacy of the contract provided, depending on how welfare is defined.

In the foreign aid example, pressures for global vertical financing and strong donor preference towards certain projects create a scenario in which, though aid is required, the aid package offered by the donor may not be optimal from the perspective of the recipient country. Both the Rome Declaration on Aid Harmonization (2003) and the Paris Declaration on Aid Effectiveness (2005) note that in order to enhance effectiveness, it is necessary to ensure that development assistance is delivered in accordance with recipient country priorities.

In the corporate finance example, shareholders' optimal timing may not coincide with managers'. In this case, the ability to choose the structure of the contract allows shareholders to impose their utility maximizing timing in an incentive-compatible way for managers.

To account for the multiple related actions that the contract requires for implementation, we consider a "multitask" contract that involves two projects, which require separate investments. In the existing literature, Holmstrom and Milgrom (1991) propose a static multitask model where the agent performs multiple tasks simultaneously. By contrast, we propose a dynamic setting where the principal can choose the timing of each task to be either simultaneous or sequential. When investment in a project is successful, it produces an observable outcome: the completed project. Moreover, the implementation of the tasks to complete it has a direct effect on both the principal's and agent's utility, since both value completed projects. We propose a results-oriented approach: the optimal contract is given by a sequence of transfers after each feasible history of successes and failures in the accomplishment of the projects that are covered by the contract.

Relative to prior literature, the novelties of this model are found in its introduction of a multitask structure on an indeterminate-length contract, which is flexible in the timing of tasks and in the unique principal and agent preferences over the projects involved in the contract. The agent's valuation of the projects, together with the particular dynamics of each timing structure, allow for high-powered incentives in each task.

When making the investment decision, the agent compares the utility of consumption from the transfer received with the cost of investment and the promised utilities, after each possible realization of the investment. To provide the agent incentives to invest, we allow for two types of "bonuses." The first arises when he signs the contract (the participation bonus): it is given by the difference between the utility the contract provides to the agent and his reservation value (the agent's utility if he does not sign the contract). This participation bonus is derived from the agent's valuation of the projects: even if the transfer is the minimum feasible one, and the agent invests it, the agent gets an expected lifetime utility greater than he would had he not signed the contract, since there is a positive probability that the projects will be completed in the future. The agent may also receive a subsequent project bonus when one of the projects is completed. The project bonus is the difference between the promised utility to the agent and the minimum lifetime utility the agent can receive on his own when only one of the projects has been completed.

To derive the optimal contract, the principal compares costs and benefits for each feasible timing structure (simultaneous or sequential completion). We find that the cheapest contract, for either simultaneous or sequential investment, is a stationary contract that provides the agent with the same transfer for all attempts until one of the projects is successful. If a project rewards the agent with a project bonus, the agent receives a decreasing sequence of promised utilities (and transfers) for successive attempts, until the remaining project is completed successfully. This decreasing sequence converges to the cheapest stationary contract.

For the agent, investment entails giving up a certain amount of consumption today against a positive probability that the project(s) will be completed successfully tomorrow, since from the transfer received, he decides whether to invest in one or both projects, or to consume all of the received funds. More risk averse agents require better rewards, in the form of project bonuses, in order to undertake the risk of investing in the projects in the first place. Time discounting also plays a role in determining the amount of the optimal project bonus: the more the agent discounts the future, the more likely he is to require project bonuses to compensate him for the delay between investment and outcome.

In considering the sequential contract, we are interested in the optimal project sequencing. When the two projects are symmetric in their costs and probabilities of success, it is optimal to begin the sequence with the project that the agent values the most. However, when there is a positive technical externality among projects (i.e. one project's completion increases the probability of success, or decreases the cost of the remaining project), it may be optimal to begin with the less valued project, depending on parameter values.

On the other hand, when we examine the simultaneous contract, we find that project bonuses are typically awarded for the project that the agent values the least. This rule could be reversed for projects with different investment costs or different probabilities of success, if these differences are significant enough.

Comparing the two timing structures, we find that the simultaneous contract is cheaper than the sequential one whenever the latter would provide a transfer large enough to cover investment in both projects. In fact, when the simultaneous investment cost is smaller than the sum of the investment costs in each of the single projects, and when the valuation of the last project in the sequence has little effect on the joint valuation of the projects, the principal can always design an alternative simultaneous contract that is cheaper and that includes project bonuses. Even if the sequential contract is the cheapest alternative when investment is observable, we show that the simultaneous contract is likely to become the cheapest alternative when moral hazard enters the equation.

Comparing the costs and benefits of the two timing alternatives, we find that simultaneous is the optimal timing for the principal when valuation interactions among projects are large, i.e., when completing both projects gives the agent significantly more utility than the sum of the projects' independent values. The principal's utility maximizing strategy, when technical externalities are important, is a sequential contract that begins with the project that generates that technical externality.

The structure of the paper is as follows. We begin with a description of the structure of the model, which includes the projects' characteristics as well as principal and agent preferences, and we present the steps in the choice between optimal contracts. In Section 3, we present the cost minimization problem, and compare the cost minimizing contract for both timing structures. We continue in Section 4 with a comparison of the expected benefits of each timing alternative. Finally, in Section 5, we compare sequential and simultaneous costs and benefits to derive qualitative results on the optimal contract choice for different projects and agents. In Section 6, we present conclusions.

2 Structure of the model

We present a dynamic moral hazard model with an **indeterminate time horizon**: no cancellation clauses are allowed until both projects are completed. The principal faces a Samaritan's Dilemma[4]: he cannot commit to stop the flow of funds to the agent until both

[4] Term introduced by Buchanan (1975)

projects are accomplished. A risk neutral "altruistic" principal signs a contract with a risk averse agent who makes an unobservable investment decision.

We name the projects A and B. Let $H_t = \{\emptyset, A, B, AB\}$ be the set of possible combinations of projects completed at period t, where $\emptyset$ denotes neither of the projects being completed, and let $H^t = H_0 \times H_1 \times H_2 \times .. \times H_t$ be the $(t+1)$ product set of H. Every element $h^t \in H^t$ describes the history of successes and failures in the accomplishment of the projects up to period t. Let $T \leqslant \infty$ be the (indeterminate) end period of the contract.

The **timing of events** is as follows: after a given history h^t, the principal transfers to the agent $\tau_t(h^t)$. Once he receives the transfer, the agent makes an unobservable (discrete) investment choice $I_t(h^t)$. Each possible combination of projects completed, $i = \emptyset, A, B, AB$, is realized with probability $\pi_i(I_t \mid h^t)$. In the next period, the outcome of the investment is realized and is observed by the principal and the agent. In case there is still one (or both) projects to be completed, the principal transfers $\tau_{t+1}(h^{t+1})$ to the agent, and the contract continues until both projects are completed.

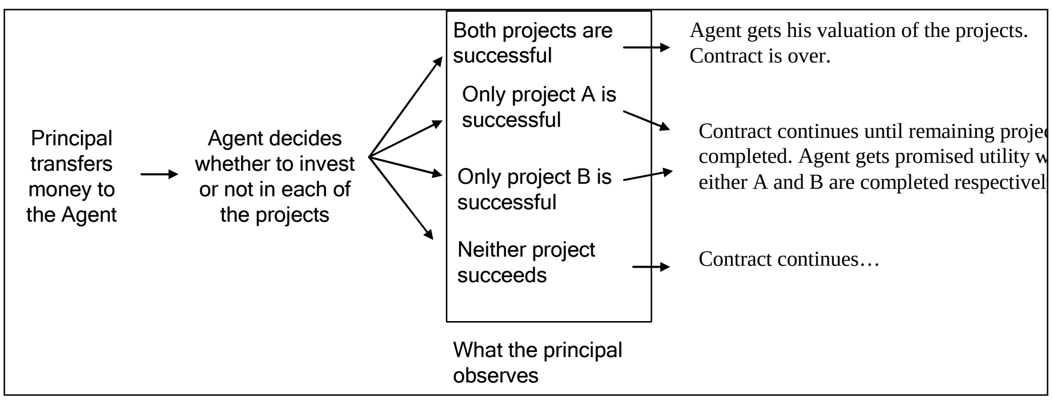

Figure 1: Timing of events

A **contract** in this framework is given by a sequence of transfers for every possible history of successes and failures in the accomplishment of the projects covered by the contract. The principal offers a sequence of transfers $\{\tau_t(h^t)\}_{t=0}^{T}$ conditional on the history h^t of projects completed, given that this is the only information available to the principal. In every period, the agent chooses the investment he wants to perform, $I_t \in \{0, \Psi_A, \Psi_B, \Psi_{AB}\}$, where Ψ_x denotes cost of investment for $x = A, B, AB$. The agent decides whether not to invest, invest in project A, in project B, or in both projects simultaneously. Given investment $I_t(h^t)$, the probability of $i = \{\emptyset, A, B, AB\}$ being realized is given by $\pi_i(I_t \mid h^t)$. Probability of success depends on the projects completed and on the agent's investment choice.

We assume the principal cannot commit to abandon the contract before both projects have been completed, but he can commit to a sequence of payments after each possible history of success and failure in the accomplishment of the projects. In the case of managers and shareholders, we can interpret the commitment to a sequence of transfers as a reputation device. We present in the appendix the cheapest contract when the donor cannot commit to a sequence of transfers, and show how the main results of the optimal contract are not affected by the commitment to disbursements assumption.

2.1 Project characteristics

All technical characteristics of the projects are common information for principal and agent. We consider two projects, project A and project B, with value for the agent[5] W_A and W_B respectively when only one of them is completed, and value for the agent W_{AB} when both are completed. The physical characteristics of the projects are such that they can be completed either sequentially or simultaneously.

We define **value interaction** as the difference $(W_{AB} - W_A - W_B)$, the extra utility that completing both projects provides over the sum of their independent values. To be completed, each combination of projects, $x = A, B, AB$, requires an investment of cost Ψ_x that is successful with probability π_x, that depends on the history of success and failure of the projects.

We account for two types of **externalities among projects**: in probability of success and in cost of investment[6]. We allow probabilities of success and cost of investment to depend on the projects already completed and on the projects in which the agent is investing simultaneously. In our notation, we have $\Psi_i(h^t)$ and $\pi_i(I_t \mid h^t)$ for $i = \{\emptyset, A, B, AB\}$.

2.2 Principal's and agent's preferences

Next we introduce the principal's and the agent's preferences and their recursive formulation. The contract's time horizon is indeterminate: the contract is over once both projects have been completed. Let $T \leqslant \infty$ be the expected end period of the contract. We assume principal and agent have the same discount factor β.

The **principal** can only observe, period after period, whether the projects have been successfully completed or not. He uses the history of successes and failures in the accomplishment of the projects to condition the sequence of transfers offered to the agent. The

[5] The assumption that valuations of the projects are common knowledge avoids the adverse selection problem studied in entrepreneur financing models, like Bergeman and Hege (2002)

[6] We denote Ψ_{AB} as the cost of investing in the two projects simultaneously. We allow for $\Psi_{AB} \gtrless \Psi_A + \Psi_B$.

principal chooses the contract $\left\{\tau_t(h^t)\right\}_{t=0}^{T}$, a sequence of transfers for each possible history of success and failure of the projects, that maximizes his utility.

Let $Z_i(W_i)$ be the utility that completed project i provides to the principal, that is an increasing function of the agent's valuation of project i. We allow this function to differ among projects to reflect the preferences of the principal over the projects[7]. The lifetime utility of the principal is given by:

$$U_0(h^0) = -E[\sum_{t=0}^{T}\beta^t\tau_t(h^t)]+$$
$$+E[\sum_{t=0}^{T}\beta^{t+1}\left[\pi_A(I_t \mid h^t)\frac{Z_A(W_A)}{(1-\beta)} + \pi_B(I_t \mid h^t)\frac{Z_B(W_B)}{(1-\beta)} + \pi_{AB}(I_t \mid h^t)Z_{AB}(W_{AB})\right]]$$

where the first term is the expected cost and the second them is the expected utility from the completed projects.

The cost of a transfer scheme (that starts after history h^0, where none of the projects has been completed) is defined by the expected discounted sum of the transfers, and is given by

$$\begin{aligned} C_\phi(h^0) = E\left[\sum_{t=0}^{T}\beta^t\tau_t(h^t)\right] &= \tau_0(h^0) + \beta\pi_A(I_0 \mid h^0)E\left[\sum_{t=1}^{T}\beta^t\tau_t(h^t) \mid A\ completed\right] \\ &+\pi_B(I_0 \mid h^0)E\left[\sum_{t=1}^{T}\beta^t\tau_t(h^t) \mid B\ completed\right] \\ &+(1-\pi_A(I_0 \mid h^0) - \pi_B(I_0 \mid h^0) - \pi_{AB}(I_0 \mid h^0))E\left[\sum_{t=1}^{T}\beta^t\tau_t(h^t) \mid none\ completed\right] \end{aligned}$$

$$\begin{aligned} C_\phi(h^0) &= \tau_0(h^0) + \beta\pi_A(I_0 \mid h^0)C_A(h_A^1) + \beta\pi_B(I_0 \mid h^0)C_B(h_B^1) \\ &+\beta(1-\pi_A(I_0 \mid h^0) - \pi_B(I_0 \mid h^0) - \pi_{AB}(I_0 \mid h^0))C_\emptyset(h_\emptyset^1) \end{aligned}$$

where $C_i(h_i^t)$ represents the present discounted cost for the principal of the sequence of transfers that starts after history h^t when project i has already been completed (for $i = A, B$,or neither project completed).

[7]In the development setting, we need to clarify that we consider a "purely altruistic" principal, in the sense that no warm-glow giving/impure altruism (Andreoni 1990) is considered.

The Rome Declaration on Aid Harmonization (2003) points out the need to ensure that development assistance is delivered in accordance with recipient country priorities. Flexibility on the $Z_i(W_i)$ function for each of the projects allows us to show how strong donor preferences towards certain projects may lead to inefficiencies in the contracts provided.

Given the contract he is offered, the **agent** makes the unobservable (discrete) decision whether or not to invest in each of the projects. Each period, the agent consumes the part of the transfer he does not invest, what makes consumption and investment decisions non-separable. We assume the agent has no additional funding to invest in the project apart from the donor's transfers. Savings from the received funds are not allowed[8]. The utility function of the agent is an increasing, concave and differentiable function of consumption, with $u(0) = 0$. The agent's reservation utility, the value of the alternative opportunities if he does not sign the contract, is zero.

The agent gets a flow utility w_x every period once project x has been completed, $w_x = W_x(1-\beta)$.

The agent chooses $\left\{I_t(h^t)\right\}_{t=0}^{T}$, the projects in which to invest, given $\left\{\tau_t(h^t)\right\}_{t=0}^{T}$, the sequence of transfers he is offered by the principal. Let $\Psi(I_t \mid h^t)$ be the cost for the agent of his investment choice, and let $V(h^t)$ denote the agent's lifetime utility after history h^t. The agent's present value of the contract after initial history h^0 (where none of the projects has still been completed) is given by

$$V_\phi(h^0) = E\left[\sum_{t=0}^{T}\beta^t u(\tau(h^t) - \Psi(I_t \mid h^t))\right]$$
$$+\sum_{t=0}^{T}\left[\beta^{t+1}\pi_A(I_t \mid h^t)w_A + \beta^{t+1}\pi_B(I_t \mid h^t)w_B + \beta^T\pi_{AB}(I_t \mid h^t)W_{AB}\right]$$

Let us define

$$P_{1x} = \sum_{t=1}^{T}\left[\beta^{t+1}w_x + \beta^{t+1}\pi_y(I_t \mid h_x^t)w_y + \beta^T\pi_{XY}(I_t \mid h_x^t)W_{XY}\right]$$

as the expected present value of utility flows from the completed projects when project X is already completed, and let $V_x(h_x^1)$ denote present value for the agent of the continuation of the contract once project X has been completed. The agent's preferences and can be

[8] Allowing the agent to save from the transfers would impose an additional constraint on the feasible contracts. Agent would compare expected returns from investment with expected returns from savings, since both alternatives have the same marginal cost for the agent in his non-separable utility function. Werning (2000 and 2002) and Chiappori at al.(1994) focus on the access to credit markets in repeated moral hazard models when effort decision is separable from investment decision.

written recursively as

$$V_{\phi}(h^0) = u(\tau(h^0) - \Psi(I_0 \mid h^0))$$
$$+\beta\pi_A(I_0 \mid h^0)\left[w_A + E\left[\sum_{t=1}^{T}\beta^t u(\tau(h_A^1) - \Psi(I_1 \mid h_A^1)) + P_{1A}\right]\right]$$
$$+\beta\pi_B(I_0 \mid h^0)\left[w_B + E\left[\sum_{t=1}^{T}\beta^t u(\tau(h_B^1) - \Psi(I_1 \mid h_B^1)) + P_{1B}\right]\right]$$
$$+\beta(1 - \pi_A(I_0 \mid h^0) - \pi_B(I_0 \mid h^0) - \pi_{AB}(I_0 \mid h^0))$$
$$*E\left[\sum_{t=1}^{T}\beta^t u(\tau(h_\emptyset^1) - \Psi(I_1 \mid h_\emptyset^1)) + P_{1\emptyset}\right] + \beta\pi_{AB}(I_0 \mid h^0)W_{AB}$$

$$V_{\phi}(h^0) = u(\tau(h^0) - \Psi(I_0 \mid h^0)) + \beta\pi_A(I_0 \mid h^0)V_A(h_A^1) + \beta\pi_B(I_0 \mid h^0)V_B(h_B^1)$$
$$+\beta(1 - \pi_A(I_0 \mid h^0) - \pi_B(I_0 \mid h^0) - \pi_{AB}(I_0 \mid h^0))V_\emptyset(h_\emptyset^1) + \beta\pi_{AB}(I_0 \mid h^0)W_{AB}$$

2.3 Optimal contract

A contract in this model is given by the sequence of transfers $\{\tau_t(h^t)\}_{t=0}^{T}$, which specifies transfers after each possible history of success and failure of the projects until both projects are completed.

The contract offered needs to be incentive compatible (agent performs the desired investment) and the recursive formulation should be consistent (the promise keeping constraint should be satisfied). The investment that the contract wants to implement has to be feasible, the transfers should be at least as big as the cost of the investment the contract aims to implement, i.e. $\tau(h^t) \geqslant \Psi(I_t \mid h^t)$. And the principal should also account for the agent's participation constraint: the agent will walk out of the contract if it does not provide him at least the utility he had before signing the contract.

The optimal contract (that starts at initial history h^0) can be characterized recursively by $(\tau_0(h^0)$, $V_A(h_A^1)$, $V_B(h_B^1)$, $V_\emptyset(h_\phi^1))$, which are the transfer and promised utilities in case project A, B or neither of the projects are completed, respectively. When both projects are completed, the agent gets W_{AB}, his valuation of both projects completed, and the contract is over.

The contract may involve two types of "bonuses" for the agent. When he signs the contract, the agent gets a **Participation Bonus** which is calculated by the difference between the utility the contract provides him when he signs and his reservation value. This participation bonus is driven by the agent's valuation of the projects: even if the transfer is the minimum feasible one and the agent invests it, the agent gets an expected lifetime utility greater than what he would obtain if he had not signed the contract, due to the positive probability that the projects are completed in the future.

The agent may also get a subsequent **Project Bonus** when only one of the projects has been successful. The project bonus is the difference between promised utility to the agent and the minimum lifetime utility the agent can receive when only one of the projects has been completed.

The **principal's utility** has two parts: the negative effect of the present discounted value of the transfers and the positive effect of the completed projects. To choose the timing of the contract he wants to implement, the principal compares the cost and benefits of the two feasible timing alternatives: simultaneous or sequential completion of the projects. We derive in Section 3 the cheapest contract for each timing structure, and in Section 4 we study the expected benefits of each timing structure. Once costs and benefits are defined, we compare the principal's utility for two timing structures (in Section 5) and we describe how this comparison varies with agent's and project's characteristics.

3 Cost minimizing contract

We solve for the cheapest contract for each timing alternative by using the recursive formulation introduced by Spear and Srivastava (1987). We differ from the standard dynamic moral hazard models[9] on several points. On the one hand, the *length* of our contract is indeterminate, since due to the principal's commitment problem, our contract does not allow for any cancellation clause before both projects are accomplished. On the other hand, the agent's effort-investment decision is *not separable* from his consumption decision (he decides how to use the received funds between investment and consumption), and it involves *several tasks*, since two projects are candidates to receive investment. Moreover, the outcome of the investment, the completed projects, is *valued* by both principal and agent.

The proposed contract is a dynamic multitask contract, which involves two projects that require separate investments. In the literature, Holmstrom and Milgrom (1991) propose a static multitask model where the agent performs multiple tasks simultaneously. Sinclair-Desgagne (1999) obtains also in a static framework higher incentives by linking audits on the task to outcomes. By contrast, in our dynamic setting, the principal can choose the timing of the tasks as either simultaneous or sequential. When investment in a project is

[9] Among other applications in the literature, Hopenhayn and Nicolini (1997) and Pavoni (2006) adapt the recursive dynamic moral hazard to Unemployment Compensation Schemes.

successful, it produces an observable outcome, which is the completed project. Moreover, the implementation of the tasks has a direct effect on the principal's and agent's utilities since completed projects are valued by both.

Our contribution to the contract theory literature comes from the introduction of a multitask structure in an indeterminate length contract where the principal can choose initially the timing to implement the tasks depending on their characteristics. Our contract does not allow for any termination clause before the completion of the projects. The agent's valuation of the projects, together with the special dynamics of each timing structure, allow for *high-powered* incentives in each task.

After any feasible history of success and failure of the two projects, the contract can be summarized (recursively) by the following elements: transfer, and promised utilities for each possible realization of completion or incompletion of the projects. The principal needs to give incentives to the agent to give up some consumption today and invest that money to have tomorrow, with some probability, a completed project. The principal has two tools to provide incentives to the agent: the transfer, which determines consumption the agent can have today even if he invests, and the promises, the well-being promised to the agent when outcome of the investment is observed. We study, in a world where the principal can commit to a sequence of transfers for each possible history of success and failure of the projects, how the principal uses these instruments in each timing structure, and how his use of each of them depends on the projects' and agents' characteristics.

To obtain the cheapest contract for each timing alternative, we proceed backwards. We start with the cost of the contract when only one of the projects has been completed (section 3.1), and we use this information to compute the cost of the simultaneous (section 3.2) and sequential (section 3.3) contract costs. Once we have the costs of each timing structure, we compare them (section 3.4) to obtain the cheapest contract for each project's and agent's characteristics.

3.1 One project optimal contract

We start with the cost minimization problem when only one of the projects has been completed. The objective of the principal is to provide incentives to the agent to complete the remaining project at the cheapest cost, since the contract continues until the remaining project is completed.

Define $C_x^*(V)$ as the expected discounted cost for the principal associated with the cheapest feasible incentive compatible contract that provides the agent a lifetime utility of

V when project X has already been completed (and project Y still needs to be completed). The elements of the recursive contract in this situation are (τ, V'): the transfer and promised utility in case investment on the remaining project is not successful. The principal's cost minimization problem has the form:

$$C_x^*(V) = \min_{\tau, V'} \left[\tau + \beta(1-\pi_y)C_x^*(V')\right]$$

$$s.t.\ V = u(\tau - \Psi_y) + w_x + \beta\left[\pi_y W_{XY} + (1-\pi_y)V'\right] \tag{1a}$$

$$u(\tau) + \beta V' + w_x \leqslant u(\tau - \Psi_y) + \beta\left[\pi_y W_{XY} + (1-\pi_y)V'\right] + w_x \tag{1b}$$

$$V' \geqslant W_x \tag{1c}$$

$$\tau \geqslant \Psi_y \tag{1d}$$

where W_x denotes the agent's lifetime utility from completed project X, and w_x denotes the instantaneous flow project X completed provides to the agent. W_{XY} denotes the agent's lifetime utility when both projects are completed. Equation $(1a)$ is the promise keeping constraint, $(1b)$ the incentive compatibility constraint, $(1c)$ the Participation constraint and $(1d)$ the feasibility constraint (the agent should be transferred at least the cost of investment).

The function C_x^* is the fixed point of the T operator defined by

$$TC_x(V) = \min_{\tau, V'} \left[\tau + \beta(1-\pi_y)C_x(V')\right]$$

$$s.t.\ \ (1a), (1b), (1c)\ \text{ and }\ (1d)$$

T is an operator on the space of continuous, increasing and convex functions. T is a contraction on a complete metric space since Blackwell (1965) sufficient conditions for a contraction are satisfied. It has a unique fixed point C^*. The cost function C^* is increasing, convex and differentiable. The proof is provided in the Appendix B .

Let us start with the **cheapest stationary contract**. This contract provides the same transfer for all attempts until investment in the remaining project is successful. Derivation is shown at the Appendix A.1. At the cheapest stationary contract, the promise keeping constraint binds. Suppose it does not bind. Then we can propose an alternative contract with smaller transfer that still satisfies the incentive compatibility constraint $(1b)$, that gives the agent at least the required lifetime utility so that $(1a)$ is satisfied, and that is cheaper, what leads to a contradiction.

Let $\tilde{\tau}$ be the smaller transfer that satisfies the promise keeping and incentive compatibility constraints at the stationary contract when project X has already been completed. We define $\underline{V}_X$ as the utility that the cheapest stationary contract provides to the agent,

$$\underline{V}_X = \frac{u(\tilde{\tau} - \Psi_y) + w_x + \beta \pi_y W_{XY}}{1 - \beta(1 - \pi_y)} > W_x \tag{2}$$

We solve now for the **principal's (non-stationary) cost minimization problem,** when different transfers are allowed for different attempts until the remaining project is completed. Let μ, λ and γ be the Lagrange multipliers for promise keeping $(1a)$, incentive compatibility $(1b)$ and feasibility constraints $(1c)$ and $(1d)$ respectively. The First Order Conditions are:

$$\begin{aligned} 1 - \mu u'(\tau - \Psi_y) + \lambda\left(u'(\tau) - u'(\tau - \Psi_y)\right) - \eta &= 0 \\ \beta(1 - \pi_y)\left[\frac{dC_x^*(V')}{dV'} - \mu\right] + \lambda \pi_y - \gamma &= 0 \end{aligned} \tag{3}$$

And the Envelope Condition

$$\frac{dC_x^*(V)}{dV} = \mu \tag{4}$$

and since C_x^* is an increasing function, $\mu > 0$.

Intuitively, the sequence of promised utilities (and correspondent transfers) for the successive attempts until the remaining project is completed should be a non-increasing function. Suppose not: then it would be optimal for the agent not to invest and collect the increasing promised utilities (and correspondent transfers) that the contract provides him, but that would contradict incentive compatibility of the contract.

Given the non-increasing sequence of promised utilities, the participation constraint $V' \geqslant W_x$ can not bind since, in case of failure, the decreasing set of promised utilities would converge to $V = W_x$ that cannot be provided in an incentive compatible and promise keeping contract. We need to check the feasible levels of utility that can be provided in an incentive compatible and promise keeping contract.

Lemma 1 (Feasible utilities) *The set of lifetime utilities the principal provides at the cheapest incentive compatible and promise keeping contract is the interval* $[\underline{V}_X, W_{AB})$, *where* $\underline{V}_x$ *is the lifetime utility provided to the agent at the cheapest stationary contract.*

Proof. Suppose $V > W_{AB}$. The utility the contract provides to the agent is greater than the agent's valuation of the completed projects; the incentive compatibility constraint cannot be satisfied in this situation.

Suppose $V < \underline{V}_X$. If $V' < V$ at the optimal contract, we would reach $V' = W_x = V$, since sequence of promised utilities proposed is decreasing for the successive attempts. But this level cannot be provided by an incentive compatible and promise keeping contract. If $V' = V > W_x$ we would have a stationary contract at $V < \underline{V}_X$, but this would not be the cost minimizing alternative since $\underline{V}_X$ is the cheapest level of utility that can be provided in a promise keeping and incentive compatible stationary contract. ■

Once the set of utilities the principal is willing to provide is determined, we can rewrite the participation constraint ($1c$) as

$$V' \geqslant \underline{V}_X > W_x$$

and let γ be the multiplier for this constraint. In the Appendix A.1 we show that the cost function is non-increasing in the agent's valuation of the completed projects, increasing in the investment cost and decreasing in the probability of success.

Proposition 1 (Optimal contract) *The optimal sequence of transfers (and promised utilities) is decreasing for the successive failed attempts, and converges to $\tilde{\tau}$ (and $\underline{V}_X$), the cost minimizing stationary contract.*

Proof. From (3) and (4)we get that $V' \leqslant V$, the sequence of promised utilities (and transfers) is non-increasing for the successive attempts until remaining project is completed.

Suppose the sequence of promised utilities converges to $\bar{V} > \underline{V}_X$ (since $\bar{V} < \underline{V}_X$ would not be chosen as shown in Lemma 1). We can propose an alternative contract that provides $\hat{V}$ such that $\bar{V} > \hat{V} > \underline{V}_X$ that is cheaper than the contract that converges to $\bar{V}$, so we reach a contradiction.

Let (τ, V') be the optimal contract that provides utility V to the agent. In case of failure, the utility to be provided to the agent is $V' \leqslant V$. Let $(\hat{\tau}, \hat{V}')$ be the contract provided in case of failure. We know that $\hat{V}' \leqslant V' \leqslant V$, and we want to show that $\tau \geqslant \hat{\tau}$. Suppose not. Given convexity of the cost function, we know that the marginal cost of promise in case of failure is increasing with the utility to be provided. For smaller utilities to be provided, promise in case of failure is relatively cheaper, so the optimal contract provides a decreasing sequence of transfers for the successive attempts that converges to the cheapest stationary contract transfer $\tilde{\tau}$. ■

3.2 Sequential contract

We present the principal's cost minimization problem when he wants to give the agent incentives to invest in both projects sequentially: first project X is completed and subsequently project Y is completed. We start with symmetric projects, and without loss of generality we present the principal's problem when project B is completed first. In Section 3.2.1 we study the optimal order of the sequence of projects with respect to the agent's valuation of the projects and the externalities among them.

Define $C^*_{B\_A}(V)$ as the expected discounted cost for the principal of the cheapest sequential contract that starts with project B and delivers the agent a level of lifetime utility V. A contract in this situation is characterized by the triplet $(\tau, V_B^{'}, V')$: transfer, promised utility when project B is completed, and promised utility when investment is not successful. The principal's cost minimization problem has the form:

$$C^*_{B\_A}(V) = \min_{\tau, V_B^{'}, V'} \left[\tau + \beta\left[\pi_B C^*_B(V_B^{'}) + (1-\pi_B)C^*_{B\_A}(V')\right]\right]$$

$$s.t.\ V = u(\tau - \Psi_B) + \beta\left[\pi_B V_B^{'} + (1-\pi_B)V'\right] \tag{5a}$$

$$u(\tau) + \beta V' \leqslant u(\tau - \Psi_B) + \beta\left[\pi_B V_B^{'} + (1-\pi_B)V'\right] \tag{5b}$$

$$V_B^{'} \geqslant \underline{V}_B \tag{5c}$$

$$V' \geqslant 0 \tag{5d}$$

$$\tau \geqslant \Psi_B \tag{5e}$$

where the function $C^*_{B\_A}$ is the fixed point of the T operator on the complete metric space of increasing, convex and differentiable functions defined by

$$TC_{B\_A}(V) = \min_{\tau, V_B^{'}, V'} \left[\tau + \beta\left[\pi_B C^*_B(V_B^{'}) + (1-\pi_B)C_{B\_A}(V')\right]\right]$$
$$s.t.\ \ (5a)\,,(5b)\,,(5d)\,,(5c)\ \text{ and }\ (5e)$$

We prove in the Appendix A.1 that $C^*_{B\_A}$ is an increasing, convex and differentiable function.

The principal minimizes the cost of the contract subject to the promise keeping $(5a)$, the incentive compatibility $(5b)$ and the participation constraints, that in this case involve promise when B is completed $(5c)$[10] and when neither project is completed $(5d)$. Moreover, there is a feasibility constraint $(5e)$ since the transfer has to be at least as big as required

[10] When project B has been completed, participation constraint of the agent is given by $V_B^{'} \geqslant W_B$. But from the optimal contract when project B has been completed we know that $V_B^{'} \geqslant \underline{V}_B$ is needed for consistency with the principal's optimal choice once B is completed.

investment. The cost function C_B^* and the level of utility $\underline{V}_B$ represent the cost function and the minimum utility to be provided when project B is already completed, as previously derived. Whether $(5c)$ constraint binds or not determines if the project receives a project bonus, if utility provided when completed is greater than the minimum the principal should provide.

We need to take into account an additional "incentive compatibility" constraint for consistency of the sequential formulation: it has to be in the agent's best interest to invest first in the project the contract prescribes rather than to deviate and invest in the alternative project. When projects are technically symmetric, i.e. they have the same cost of investment and probabilities of success, for the sequential contract that starts with project B, this constraint is given by

$$V_B' \geqslant \underline{V}_A \tag{6}$$

since otherwise the agent would prefer to invest in A rather than B, given symmetry in costs of investment and probabilities of success. This constraint results from the assumption that the contract does not end until both projects are completed, and because of that we need to assume that, in case of deviation, the contract continues with the remaining project at the cheapest feasible contract, that is the stationary one.

Let $\mu, \lambda, \gamma_1, \gamma_2, \eta$ be the Lagrange multipliers for the $(5a)\,,(5b)\,,(5d)\,,(5c)$ and $(5e)$ constraints respectively. The First Order Conditions of the principal's problem are:

$$\begin{aligned}
1 - \mu u'(\tau - \Psi_B) + \lambda \left(u'(\tau) - u'(\tau - \Psi_B)\right) - \eta &= 0 \\
\beta \pi_B \left[\frac{dC_B^*(V_B')}{dV_B'} - \mu - \lambda \right] + \gamma_2 &= 0 \\
\beta(1 - \pi_B) \left[\frac{dC_{B_A}^*(V')}{dV'} - \mu \right] + \lambda \beta \pi_B + \gamma_1 &= 0
\end{aligned}$$

And the Envelope Condition

$$\frac{dC_{B\_A}^*(V)}{dV} = \mu$$

From the First Order Conditions and the convexity of the cost function we get

$$(1 - \pi_B) \left[\frac{dC_{B\_A}^*(V')}{dV'} - \frac{dC_{B\_A}^*(V)}{dV} \right] = -\lambda \beta \pi_B - \gamma_1$$

$$V' \leqslant V \tag{7}$$

The sequence of promised utilities is non-increasing for successive attempts until the first project in the sequence is completed.

Let us present the **stationary sequential contract,** the contract that provides same transfer (and same promised utility) for all attempts until the first project in the sequence is completed. The principal's objective is to choose the transfer and promised utility in case of success of the first project in the sequence that minimizes his cost. The principal's stationary cost minimization problem is given by

$$C(V) = \min_{\tau, V'_B} \frac{\left[\tau + \beta \pi_B C^*_B(V'_B)\right]}{[1-\beta(1-\pi_B)]}$$

$$s.t. \quad u(\tau)(1-\beta(1-\pi_B)) - u(\tau - \Psi_B)(1-\beta) \leqslant \beta\pi_B(1-\beta)V'_B \tag{8a}$$

$$V'_B \geqslant \underline{V}_B \tag{8b}$$

$$\tau \geqslant \Psi_B \tag{8c}$$

where $(8a)$ gives the set of pairs (τ, V'_B) that satisfy the incentive compatibility and (binding) promise keeping constraints, $(8b)$ gives the constraint on promise when project B is completed, and $(8c)$ is the investment feasibility constraint.

Let λ be the Lagrange multiplier for $(21a)$, γ_B be the Lagrange multiplier for $(8b)$ and γ_Ψ be the Lagrange multiplier for $(8c)$. The First Order Condition of the stationary problem with respect to τ is

$$1 + \lambda\left[[1-\beta(1-\pi_B)]\, u'(\tau) - (1-\beta)u'(\tau - \Psi_B)\right] - \gamma_\Psi = 0 \tag{9}$$

Suppose $(8a)$ does not bind, $\lambda = 0$. For (9) to be satisfied, we need $\gamma_\Psi > 0$, which implies $\tau = \Psi_B$. Since the promise keeping constraint binds, the utility provided to the agent is

$$V = \frac{\beta\pi_B V'_B}{(1-\beta(1-\pi_B))}$$

that when plugged into the incentive compatibility constraint gives

$$u(\Psi_B) \leqslant \beta\pi\left[V'_B - V\right] = \frac{\beta\pi_B(1-\beta)V'_B}{(1-\beta(1-\pi_B))}$$

$$\frac{u(\Psi_B)}{(1-\beta)}\frac{(1-\beta(1-\pi_B))}{\beta\pi_B} < V'_B$$

and either $V'_B = \underline{V}_B$ or the incentive compatibility constraint binds at the optimal contract. Suppose not, then we can propose an alternative contract $(\Psi_B, V'_B - \varepsilon)$ with $\varepsilon < V'_B - \underline{V}_B$ that is cheaper than the original contract with project bonus, which is a contradiction.

Let $(\tau^*, V_B'^*)$ be the cheapest stationary contract. It delivers the agent a lifetime utility

$$\underline{V}^{\text{seq}} = \frac{u(\tau^* - \Psi_B) + \beta\pi_B V_B'^*}{1 - \beta(1 - \pi_B)} \tag{10}$$

In the cost minimization problem, the feasibility constraint can not bind, since there is no incentive compatible and promise keeping contract that can provide $V = 0$. We need to define the set of feasible promised utilities that the principal may offer in the sequential cost minimization problem.

Lemma 2 (Feasible utilities) *The set of feasible utilities that may be provided in an incentive compatible and promise keeping sequential contract is in the interval of* $V \epsilon \, [\underline{V}^{\text{seq}}, W_{AB})$, *where* $\underline{V}^{\text{seq}}$ *denotes the utility provided at the cheapest sequential stationary contract.*

Proof. If $V > W_{AB}$ the agent gets more utility than what he would have gotten if both projects were completed, what contradicts the incentive compatibility constraint.

If $V < \underline{V}^{\text{seq}}$, by (7) we would have $V' \leqslant V < \underline{V}^{\text{seq}}$, and the sequence of provided utilities would converge to the stationary contract $\bar{V}$, $0 \leqslant \bar{V} < \underline{V}^{\text{seq}}$. If $\bar{V} = 0$, there is no feasible incentive compatible and promise keeping contract that can provide this utility. And if $\bar{V} > 0$, since $\underline{V}^{\text{seq}}$ is the cheapest incentive compatible and promise keeping utility that can be provided in a stationary contract, we reach a contradiction. ■

Once we have the set of feasible utilities, we can derive the optimal contract that allows for **different transfers for successive attempts** until a project is completed. The principal chooses a level of utility to provide to the agent that minimizes the cost of the contract subject to $(5a)$, $(5b)$, $(5c)$, $(5e)$ and the new feasibility constraint given by the set of feasible utilities.

Proposition 2 (Cheapest sequential contract) *The cheapest sequential contract is a stationary contract at* $V = \underline{V}^{\text{seq}}$ *until the first project is completed. If the second project in the sequence gets a project bonus, the agent then receives a decreasing sequence of transfers that converges to the stationary contract when this project is completed.*

Proof. The sequential cost function is increasing. Suppose we look for the optimal contract that provides an initial level of lifetime utility to the agent $V > \underline{V}^{\text{seq}}$. Condition (7) tells us that it is optimal to provide $V' \leqslant V$, and so the optimal contract converges to a stationary contract at $V = \underline{V}^{\text{seq}}$. But the principal can propose from the beginning of the relationship a stationary contract at $V = \underline{V}^{\text{seq}} > 0$ that is feasible and cheaper.

When a project receives a project bonus, the sequence of transfers is decreasing and converges to the cheapest stationary contract when the first project has been completed. Proof is provided in Proposition 1. ■

The optimally of the stationary contract is intuitive. The cheapest stationary contract is the cheapest contract that gives incentives to the agent to invest. Investment is a discrete choice and probability of success does not vary with amount invested (as long as it is at least the required level). Then, it is optimal for the principal to keep providing the cheapest stationary contract that gives the agent incentives to undertake the required given investment for all attempts up to the first project in the sequence is completed.

At the cheapest sequential contract, the agent gets a **Participation Bonus** when he signs the contract: he is provided a utility $\underline{V}^{\text{seq}}$ that is greater than his reservation utility. Even if the agent's transfer equals investment cost, the expected value of the completed projects provides him positive expected utility. This result, as opposed to the standard moral hazard models where agent's utility is driven down to the reservation level, is given by the fact that completed projects are valued by the agent.

3.2.1 Optimal sequence of projects

Once we have the optimal contract for symmetric projects, we look at the optimal sequential order of projects for non-symmetric projects.

We need to ensure that the agent prefers to invest in the project the principal wants him to complete first instead of investing in the alternative project. As done in (6) for technically symmetric projects, when the sequential contract starts with project X, the contract should be such that

$$V'_X \geqslant \underline{V}_Y$$

since otherwise the agent would rather invest in Y than in X, given symmetry in costs of investment and probabilities of success. Since the effect of technical externalities among the projects appears once one of the projects has been completed, this constraint still holds for the first project in the sequence, since the externality effect would be reflected on $\underline{V}_Y$.

We start by comparing projects that only differ in their valuation by the agent, and we find that it is optimal to start with the more valued project. We continue with projects for which there exist externalities: once one is completed, the remaining project is more likely to succeed or requires smaller investment than before. We find that when the less valued project is the one that generates the externalities, it is cheaper to start with this project when the externalities are large enough.

Lemma 3 (Cheapest timing for different valuations) *When projects only differ in their valuation by the agent, the cheapest sequential contract starts with the more valued project.*

Proof. Since projects only differ in their valuation by the agent, let $\pi = \pi_A = \pi_B$. Without loss of generality suppose $W_A < W_B$. Consider the sequential contract starting with project A, (τ, V_A'). For this contract to be incentive compatible we need (6) to be satisfied. From the cost of this contract and the properties of the cost function when only one project remains to be completed we find

$$\begin{aligned} C^*(V) &= \frac{\tau + \beta\pi C_A^*(V_A')}{[1-\beta(1-\pi)]} \geqslant \\ \frac{\tau + \beta\pi C_A^*(\underline{V}_B)}{[1-\beta(1-\pi)]} &> \frac{\tau + \beta\pi C_B^*(\underline{V}_B)}{[1-\beta(1-\pi)]} \end{aligned}$$

that shows that the cost of the stationary contract that starts with B is smaller than the cost of the original timing, what contradicts optimally of the sequence that starts with the less valued project. ■

Lemma 4 (Cheapest timing with externalities) *When one of the projects positively affects the probability of success of the other project, i.e.* $\pi_x = \pi_y = \pi$ *when neither of the projects is completed but* $\pi_x > \pi$ *once project* Y *has been completed, it is optimal to start with the less valued project that produces the positive externality when the externality is large enough.*

Proof. Without loss of generality assume $W_A < W_B$ and assume that it is project A that produces the externality. Consider the sequential contract that starts with project B, the more valued project. For this contract to be incentive compatible we need

$$V_B' \geqslant \underline{V}_A$$

to be satisfied. The cost of this contract is

$$C^*(V) = \frac{\tau + \beta\pi C_B^*(V_B')}{[1-\beta(1-\pi)]} \geqslant \frac{\tau + \beta\pi C_B^*(\underline{V}_A)}{[1-\beta(1-\pi)]}$$

which follows from $V_B' \geqslant \underline{V}_A$ and C_B^* being an increasing function. The cost function is decreasing in the valuation of the completed project and also decreasing in the probability of success. Given the characteristics of projects A and B, when the externality is large enough,

$$C_B^*(V_B') > C_A^*(V_B')$$

We could propose an alternative contract with the reverse order of projects that would be cheaper, what contradicts optimally of the original ordering. ■

3.2.2 Comparative statics: agents and projects characteristics

Our objective is to see how the cost minimizing contract adapts to different agent and project characteristics.

The agent is forced to undertake a risk with the investment. The **Project bonus** is the extra utility the contract provides to the agent once a project is completed. It is, together with the transfers, an instrument the principal has to give the agent incentives to invest and to compensate him for the risk he is undertaking. We want to know how the agent's risk aversion and time preferences affect the Project Bonuses provided in the cost minimizing contract. We obtain that agents with more concave utility functions and the ones that discount the future the most are the candidates to obtain project bonuses.

Proposition 3 (Risk aversion and project bonus) *At the cheapest stationary contract, a project bonus upon completion of the first project may be provided to agents with Arrow-Pratt absolute risk aversion*

$$r(\tau) > \frac{\beta\pi_B}{(1-\beta)\Psi_B} \tag{11}$$

Proof is provided at Appendix C. The intuition is as follows: agents with more concave utility functions are more likely to obtain project bonuses, since they need greater compensation for the risk of the investment they are induced to take. From (11) we find that for greater costs of investment more agents are likely to receive positive bonus. For projects with greater probability of success, the result is the opposite. These comparative statics are intuitive: greater cost of investment implies higher cost to take a risk, and agents need to be compensated for this fact. Greater probability of success for same investment makes the project less risky so there are fewer agents that need to be compensated.

With respect to time preferences, we find that greater discounting increases the set of candidate agents to receive project bonuses. The more the agents discount the future, the less they value today the project that may be completed next period, and the principal needs to provide extra compensation to induce the agents to invest.

Claim 1 (Sequential transfers change with projects externalities) *When one of the projects positively affects the probability of success of the other project, (i.e. $\pi_x = \pi_y = \pi$ when neither of the projects is completed but $\pi_{x|y} > \pi$ once project y has been completed), transfers are non-increasing and promises of success are non-decreasing with the size of the externality.*

Proof. We want to show that transfers are non-increasing in the technical externalities. Suppose not. Let (τ, V_B') be the cheapest contract in the absence of externalities. This contract is feasible when first project in the sequence generates a technical externality on the second project of the sequence (incentive compatibility and promise keeping constraints do not change). But since the promised utility in case of success becomes cheaper in the presence of externalities, we can propose an alternative incentive compatible and promise keeping contract with smaller transfer and greater promised utility that provides the agent same lifetime utility and is cheaper, what contradicts increasing transfers with externalities. ■

Claim 2 (Sequential transfers with respect to investment costs) *When the cost of investment decreases, the transfer and promised utility in case of success decrease.*

Proof. Suppose transfers and promised utilities were greater for smaller investment costs. Let (τ, V_B') be the cheapest (and by Proposition 2 stationary) contract for investment cost Ψ that provides the agent a utility $\underline{V}^{\text{seq}}$. For $\hat{\Psi} < \Psi$, (τ, V_B') is a feasible stationary contract that provides $V > \underline{V}^{\text{seq}}$ and satisfies incentive compatibility and promise keeping constraints, since

$$u(\tau) - u(\tau - \hat{\Psi}) + \left[u(\tau - \hat{\Psi}) - u(\tau - \Psi)\right] - \beta\pi\left[V_B' - \underline{V}^{\text{seq}}\right] =$$
$$= \left[u(\tau) - u(\tau - \hat{\Psi})\right] - \beta\pi\left[V_B' - V\right] + (1 - \beta\pi)\left[u(\tau - \hat{\Psi}) - u(\tau - \Psi)\right](\Psi - \hat{\Psi})$$

what implies

$$\left[u(\tau) - u(\tau - \hat{\Psi})\right] < \beta\pi\left[V_B' - V\right]$$

We can propose an alternative contract $(\tilde{\tau}, V_B')$ with $\tilde{\tau} = \tau - \varepsilon_\tau$,

$$\varepsilon_\tau = -\frac{u'(\tau - \hat{\Psi})\left[\Psi - \hat{\Psi}\right]}{u''(\tau)} > 0$$

that is cheaper than a contract that provides greater transfers for smaller probabilities of success, what leads to a contradiction. ■

Claim 3 (Sequential transfers with respect to value interactions) *As the value interaction among projects increases, the optimal contract provides non-increasing transfers.*

Proof is provided at Appendix C. Intuitively, as value interaction increases, increases the value of the completion of the projects for the agent, so less incentives through transfers need to be provided. Moreover, promise when one of the projects has already been completed becomes relatively cheaper, making promises relatively cheaper with respect to transfers as an instrument to provide incentives to the agent.

3.3 Simultaneous contract

The alternative to sequential investment in the projects is to induce the agents to invest in both projects simultaneously. We present here the cost minimization problem for the simultaneous timing alternative. We start with technically symmetric projects equally valued by the agent (to simplify notation we denote $\pi = \pi_A = \pi_B$), and we relax this assumption to see how the contract adapts to different technical characteristics of the projects and agent's preferences.

Let $C^*_{ab}(V)$ be the expected discounted cost for the principal of the cheapest contract where investment is induced on projects A and B simultaneously that provides lifetime utility V to the agent. The cheapest contract that induces the agent to invest in both projects simultaneously is the solution to the following minimization problem:

$$C^*_{ab}(V) = \min_{\tau, V'_A, V'_B, V'} \left[\tau + \beta\left[\pi(1-\pi)\left[C^*_A(V'_A) + C^*_B(V'_B)\right] + (1-\pi)^2 C^*_{ab}(V')\right]\right]$$

$$s.t. V = u(\tau - \Psi_{AB}) + \beta\left[\pi^2 W_{AB} + (1-\pi)^2 V' + \pi(1-\pi)V'_A + \pi(1-\pi)V'_B\right] \quad (12a)$$

$$u(\tau) + \beta V' \leqslant V \quad (12b)$$

$$u(\tau - \Psi_A) + \beta\left[\pi V'_A + (1-\pi)V'\right] \leqslant V \quad (12c)$$

$$u(\tau - \Psi_B) + \beta\left[\pi V'_B + (1-\pi)V'\right] \leqslant V \quad (12d)$$

$$V' \geqslant 0 \quad (12e)$$

$$V'_A \geqslant \underline{V}_A \quad (12f)$$

$$V'_B \geqslant \underline{V}_B \quad (12g)$$

$$\tau \geqslant \Psi_{AB} \quad (12h)$$

where λ_0, λ_A and λ_B are the Lagrange multipliers for the incentive compatibility constraints $(12b)$, $(12c)$ and $(12d)$, and μ is the multiplier for the promise keeping constraint $(12a)$. Constraints $(12g)$ and $(12f)$ are the participation constraints when one of the projects has been completed. C^*_j and $\underline{V}_j$ represent the cost function and the minimum utility to be provided when project j is already completed, for $j = A, B$. The function $C^*_{ab}(V)$ is the fixed point of the T operator on the metric space of increasing, convex and differentiable functions defined by

$$TC_{ab}(V) = \min_{\tau, V_A', V_B', V'} \left[\tau + \beta \left[\pi(1-\pi)\left[C_A^*(V_A') + C_B^*(V_B')\right] + (1-\pi)^2 C_{ab}(V')\right]\right]$$

$$\text{subject to } (12a), (12b), (12c), (12d), (12e), (12g), (12f) \text{ and } (12h)$$

C_{ab}^* is an increasing, differentiable and convex function. The proof is provided in the Appendix B.

To derive the optimal simultaneous contract we follow a reasoning parallel to the sequential contract. Let $\underline{V}^{sim}$ be the utility provided at the cheapest **stationary simultaneous contract**.

When we allow **transfers to vary for the successive attempts** until one (or both) projects are completed, the First Order Conditions of the principal's problem are

$$1 - \mu u'(\tau - \Psi_{AB}) = \eta - \lambda_A(u'(\tau - \Psi_A) - u'(\tau - \Psi_{AB})) - \lambda_B(u'(\tau - \Psi_B) - $$
$$-u'(\tau - \Psi_{AB})) - \lambda_0(u'(\tau) - u'(\tau - \Psi_{AB}))$$

$$(1-\pi)^2\beta\left[\frac{dC_{ab}^*(V')}{dV'} - \mu\right] = -\lambda_0\pi(2-\pi) - \pi(1-\pi)\beta(\lambda_A + \lambda_0) \tag{14}$$

$$\pi(1-\pi)\beta\left[\frac{dC_B^*(V_B')}{dV_B'} - \mu\right] = \pi(1-\pi)\beta(\lambda_A + \lambda_0) - \beta\lambda_B\beta\pi^2 + \gamma_1$$

$$\pi(1-\pi)\beta\left[\frac{dC_A^*(V_A')}{dV_A'} - \mu\right] = \pi(1-\pi)\beta(\lambda_B + \lambda_0) - \beta\lambda_A\beta\pi^2 + \gamma_2$$

$$(1-\pi)\left[\frac{dC_B^*(V_B')}{dV_B'} - \frac{dC_A^*(V_A')}{dV_A'}\right] = (\lambda_A - \lambda_B) + \frac{(\gamma_1 - \gamma_2)}{\pi} \tag{15}$$

and the Envelope Condition

$$\frac{dC_{ab}^*(V)}{dV} = \mu > 0$$

that with (14) gives

$$V' \leqslant V \tag{16}$$

The cheapest simultaneous contract provides a non-increasing sequence of promised utilities for successive attempts until one or both projects are completed.

We find that the agent's participation constraint can not bind, $V = 0$ cannot be provided by an incentive compatible and promise keeping contract. We define in the following Lemma the boundaries of the set of utilities that the principal may provide in an incentive compatible and promise keeping simultaneous contract.

Lemma 5 (Feasible utilities) *The set of feasible utilities that may be provided in the incentive compatible and promise keeping contract is in the interval of* $V \epsilon \left[\underline{V}^{sim}, W_{AB}\right)$, *where* $\underline{V}^{sim}$ *is the lifetime utility provided at the cheapest simultaneous contract.*

Proof. For $V > W_{AB}$ the agent is provided more utility than what he would get if both projects were completed, what contradicts incentive compatibility constraint.

For $V < \underline{V}^{sim}$, we have $V' \leqslant V < \underline{V}^{sim}$ by (16), and sequence of provided utilities converges to $\bar{V}$, $\underline{V}^{sim} > V = \bar{V} \geqslant 0$. If $\bar{V} = 0$, there is no incentive compatible and promise keeping contract that can provide this utility. If $\bar{V} > 0$, there exists an alternative stationary contract, $\underline{V}^{sim}$, that is cheaper. We reach a contradiction. ■

Proposition 4 (Cheapest simultaneous contract) *The cheapest simultaneous contract is a stationary contract that provides* $\underline{V}^{sim}$ *up to one of the projects is completed. The agent receives a constant transfer for all attempts until one (or both) projects are completed.*

When only one of the projects is completed, if that project gets a project bonus, the agent receives a decreasing sequence of transfers that converges to a stationary contract.

Proof. The cost function is an increasing function. The minimum level of utility that may be provided in an incentive compatible and promise keeping contract coincides with the cheapest one, and it is given by $\underline{V}^{sim}$, the cheapest simultaneous stationary contract. In the stationary contract, the same transfers are provided for all attempts until one (or both) projects are completed.

When a project receives a project bonus, the sequence of transfers is decreasing and converges to the stationary contract when this given project has been completed. Proof is provided in Proposition 1. ■

From the project characteristics we know that the probability of success depends only on a discrete investment choice. Intuitively, the cheapest contract is the one that gives the agent the cheapest combination of incentives that induces him to invest in both projects. And this minimum coincides with the cheapest stationary contract.

The agent gets a **Participation Bonus** when he signs the contract, that is given by $\underline{V}^{sim}$. Like in the simultaneous case, since the agent values the completed projects, even if he gets the minimum transfer feasible to invest, he gets a positive utility from the expected value of the projects in which he is simultaneously investing.

3.3.1 Comparative statics: Agents and Projects characteristics

We proceed now to study how project characteristics and agents' preferences affect **Project Bonuses** in the cheapest simultaneous contract, i.e. when do we have promised utilities over the minimum feasible level when only one of the projects is successful. As agent's characteristics, the agent's risk aversion and time preferences play a key role. As project characteristics, we look at the agent's valuation of the projects, investment costs and probabilities of success for both projects.

Proposition 5 (Project bonus and agent's preferences) *At the cheapest simultaneous contract for technically symmetric projects, a project bonus is provided to agents with Arrow-Pratt absolute risk aversion*

$$r(\tau) > \frac{\beta\pi(1-\pi)}{[1-\beta(1-\pi)]\,(\Psi_{AB}-\Psi)} \tag{17}$$

Proof is provided in Appendix D. The more risk averse agents are the more likely to receive a project bonus, since they are the agents that need greater compensation for the risk they undertake with investment. The set of candidate agents to receive project bonuses decreases as the time discount increases. When agents discount less the future, smaller bonuses need to be provided to make them undertake the risk of investing with delayed returns.

We now look at how the optimal contract changes when the projects involved are not technically symmetric and the agent values them differently. We start with the case where projects only differ in their value for the agent, and we find that project bonuses go to the less valued project. We continue with how different projects' technical characteristics may give project bonuses to the more valued project when it has greater investment cost or a smaller probability of success.

Proposition 6 (Project bonus for different project valuations) *In the simultaneous cheapest contract with technically symmetric projects, either no project bonus is promised, or if there is a project bonus it is greater for the less valued project.*

Proof is provided in Appendix D. The aim of project bonuses is to compensate the agent for the effort he makes in the investment. For technically symmetric projects, the less valued project may require a greater extra compensation to induce the agent to invest in it.

Proposition 7 (Asymmetric investment costs) *At the cheapest simultaneous contract, when the two projects have different investment costs, we can have project bonuses for the more valued project when this project is the one with greater investment cost.*

Proof is provided in Appendix D. The intuition is as follows: project bonuses are given to compensate the agent for the investment he undertakes. The agent compares costs and benefits of the investment in each of the projects. When a project has a greater investment cost, even if it is the more valued project, he may need an extra bonus to compensate for the extra effort it requires.

Proposition 8 (Asymmetric probabilities of success) *At the cheapest simultaneous contract, when the projects have different probabilities of success, we can have that the more valued project gets project bonus if this project is the one with smaller probability of success.*

Proof is provided in Appendix D. Despite of valuation differences, projects that are less likely to succeed are less appealing for the agent, and the principal may need to compensate for this fact.

3.4 Cost comparison of the sequential and simultaneous contracts

We compare the cost for the principal of the two alternative timing structures: simultaneous versus sequential completion of the projects. We do this comparison under two possible information scenarios: observable and non-observable agent's investment.

Together with cost, an important variable to consider is length of the contract. **Expected length** of the simultaneous contract is smaller than expected length of the sequential contract. Expected length for each timing alternative is given by

$$\begin{aligned} Expected\ Length(simultaneous) &= \frac{\pi^2 - \pi + 2}{\pi} \\ Expected\ Length\ (\text{seq}uential) &= \frac{3 - 2\pi}{\pi(2 - \pi)} \end{aligned}$$

and the difference is

$$EL(sim) - EL(\text{seq}) = -\pi^3 + \pi^2 + 6\pi + 1 > 0 \text{ for all } \pi \epsilon\, [0, 1]$$

3.4.1 First best: Observable investment

Let us start comparing the cost of the two timing alternatives when investment is observable and so transfers equal investment cost are the cheapest feasible alternative. In our model, even with incomplete information, transfers equal to investment cost may be chosen since for some sets of parameters they may be incentive compatible. Let C_{seq}^{FI} and C_{sim}^{FI} be the expected costs of the full information contract for the sequential and simultaneous timing respectively. The difference in cost for the two alternatives is given by

$$\begin{aligned} [C_{\text{seq}}^{FI} - C_{sim}^{FI}] = \frac{1}{1 - \beta(1 - \pi_B)}(\Psi_B + \frac{\beta \pi_B \Psi_A}{1 - \beta(1 - \pi_{A|B})}) \\ - \frac{1}{[1 - \beta(1 - \pi_B)(1 - \pi_A)]}(\Psi_{AB} + \frac{\beta \pi_A (1 - \pi_B)\Psi_B}{1 - \beta(1 - \pi_{B|A})} + \frac{\beta \pi_B (1 - \pi_A)\Psi_A}{1 - \beta(1 - \pi_{A|B})}) \end{aligned}$$

We find that when projects are totally symmetric and independent, the cheapest alternative is the sequential timing. When $\pi_{i|j} > \pi_i$ for any project i (probability of success of project i increases when project j is already completed), sequential contract that starts with the project that generates the externality is the cheapest timing. When $\Psi_{AB} < 2\Psi$, we have that whenever

$$2\Psi - \Psi_{AB} \geqslant \Psi \frac{\left[(1-\beta)(1+\beta(1-\pi)^2) + (1-\pi)2\beta\pi\right]}{[1-\beta(1-\pi)]}$$
$$\text{where } \frac{\left[(1-\beta)(1+\beta(1-\pi)^2) + (1-\pi)2\beta\pi\right]}{[1-\beta(1-\pi)]} < 1$$

simultaneous contract is cheaper than the sequential one. Whenever cost to invest in both projects simultaneously is significantly smaller than the sum of costs to invest on both projects by themselves, a simultaneous contract is the cheapest contract under complete information.

3.4.2 Second best: Unobservable investment

When investment is not observable, it is necessary to provide the agent the appropriate incentives to invest. We find that the simultaneous contract is cheaper than the sequential one whenever the sequential contract provides a transfer big enough to cover investment in both projects. When the incentives (in form of transfer over the investment cost) are such that with sequential transfers simultaneous investment is feasible, the principal can always design an alternative simultaneous contract including Project Bonuses that is cheaper. As we showed in the sequential contract comparative statics, sequential contract transfers are likely to fall in this category when technical externalities among projects are not important (Claim 1), when valuation of the last project in the sequence has little effect on the joint valuation of the projects (Claim 3), and when Ψ_{AB} is significantly smaller than the sum of investment costs in each of the projects independently (Claim 2). For totally independent projects, we are likely to fall in this category when costs of investment are relatively high with respect to the agent's valuation of the projects.

The intuition is as follows: when investment is not observable, the principal needs to provide the agent incentives to undertake the risk associated to investment. If the transfer that needs to be provided is large enough, the principal can provide an alternative simultaneous contract where the agent "diversifies" the risk of the investment he is undertaking that is cheaper.

Proposition 9 *For technically symmetric projects (i.e.$\pi_A = \pi_B, \Psi_A = \Psi_B$), when transfers of the cheapest sequential contract are such that simultaneous investment is feasible, i.e. $\tau^{\text{seq}} \geqslant \Psi_{AB}$, simultaneous contract is cheaper.*

Proof is provided in Appendix E. The intuition is as follows: when cost of investment is big enough, for any sequential contract we can construct an alternative simultaneous contract with the same sequence of transfers that is cheaper (given that expected length of the contract decreases), what contradicts the sequential being the cheapest feasible contract.

We show in Appendix A that the fact that when investment is not observable simultaneous contract is the cheapest alternative under some circumstances does not depend on the commitment to a sequence of transfers assumption. At the stationary contract that does not allow for project bonuses, we also find that moral hazard makes simultaneous investment the cheapest alternative under some circumstances.

4 Principal's Expected Benefits

The principal compares expected cost and expected benefits of each timing alternative to choose the strategy that maximizes his utility. We derive and compare here the principal's expected benefits for the two alternative timing structures to complete both projects.

It is realistic to imagine that principals may prefer some projects over the rest. In a Foreign Aid setting, political, religious, and moral factors of the donor society may affect the donor's valuation of the different projects. In that case, the result of the expected benefit comparison of the two timing alternatives may not coincide with the timing that maximizes the recipient's expected well-being from the completed projects. This bias in donor's preferences may generate inefficiencies in the contract: there may exist a cheaper alternative that increases the agent's expected wellbeing that is not chosen due to the principal's biased preferences.

In a corporate finance setting, some projects may maximize the value of the firm but may not be the ones preferred by the managers. In that case, the possibility of the shareholders to impose their preferred timing acts as a protection mechanism against the manager's prioritizing their preferences over the value of the firm.

The function $Z(W)$ measures how projects (and agent's valuation of the projects) enter the principal's utility function. We allow this function to vary among projects to reflect the principal's preferences over the projects. We assume that the principal's valuation of the projects completed by the agent has the form

$$Z_i(W_i) = \alpha_i W_i \text{ for } i = A, B$$

and

$$Z_{AB|i}(W_{AB}) = \alpha_j W_j + \max(\alpha_A, \alpha_B)\left[W_{AB} - W_A - W_B\right]$$

denotes principal's extra-utility when project j is completed whenever i had already been accomplished.

The expected benefit for the principal of the sequential contract that starts with project A is given by

$$\begin{aligned}
E(Z(W))_{Seq} &= \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^t\Big[\left[\pi + (1-\pi)\sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^t\right] Z_A(W_A) \\
&\quad + \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^t\pi Z_{AB|A}(W_{AB})\Big] = \\
&= \frac{\beta\pi}{1-\beta(1-\pi)}\left[Z_A(W_A) + \beta\frac{(\pi Z_{AB|A}(W_{AB}) + (1-\pi)Z_A(W_A))}{1-\beta(1-\pi)}\right] = \\
&= \frac{\beta\pi}{1-\beta(1-\pi)}\left[Z_A(W_A) + \frac{\beta\pi Z_{AB|A}(W_{AB})}{1-\beta(1-\pi)}\right]
\end{aligned}$$

and we see that to start with the more valued project maximizes the expected benefits of the sequential contract. This ordering coincides with the cost minimizing sequence for technically symmetric projects.

When investment in both projects is simultaneous, the expected benefit for the principal is given by

$$\begin{aligned}
E(Z(W))_{\mathrm{Si}\,m} &= \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^{2t}\left[\pi^2 Z(W_{AB}) + \pi(1-\pi)\left(Z_A(W_A) + Z_B(W_B)\right)\right] \\
&+ \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^{2t+1}\pi\left[\begin{array}{c} \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^t\pi(Z_{AB|A}(W_{AB}) + Z_{AB|B}(W_{AB})) + \\ + \sum_{t=0}^{\infty}\beta^{t+1}(1-\pi)^t(1-\pi)Z_A(W_A) + Z_B(W_B) \end{array}\right]
\end{aligned}$$

$$\begin{aligned}
E(Z(W))_{\mathrm{Si}\,m} &= \frac{\beta\pi}{1-\beta(1-\pi)^2}[\pi Z(W_{AB}) \\
&+ (1-\pi)\left[\begin{array}{c} Z_A(W_A) + Z_B(W_B) + \\ + \frac{\beta\pi Z_{AB|A}(W_{AB}) + \beta\pi Z_{AB|A}(W_{AB}) + (1-\pi)\beta(Z_A(W_A) + Z_B(W_B))}{1-\beta(1-\pi)} \end{array}\right]] =
\end{aligned}$$

$$\begin{aligned}
E(Z(W))_{\mathrm{Si}\,m} &= \frac{\beta\pi^2 Z(W_{AB})}{1-\beta(1-\pi)^2} \\
&+ \frac{\beta\pi(1-\pi)}{1-\beta(1-\pi)^2}\left(\frac{Z_A(W_A) + Z_B(W_B) + \beta\pi Z_{AB|A}(W_{AB}) + \beta\pi Z_{AB|B}(W_{AB})}{1-\beta(1-\pi)}\right)
\end{aligned}$$

To account for externalities in probabilities of success, let us simplify notation and define $\pi_{i/j}$ as the probability of success of project i when project j is already completed. The expected benefits of each timing structure become:

$$E(Z(W))_{Seq} = \frac{\beta\pi_A}{1-\beta(1-\pi_A)}\left[Z_A(W_A) + \frac{\beta\pi_{B/A}Z_{AB\backslash A}(W_{AB})}{1-\beta(1-\pi_{B/A})}\right]$$

$$E(Z(W))_{\mathrm{Si}\,m} = \frac{\beta}{1-\beta(1-\pi_A)(1-\pi_B)}[\pi_A Z_A(W_A) + \pi_B Z_B(W_B)$$
$$+\frac{\beta\pi_A\pi_{B|A}Z_{AB|A}(W_{AB})}{1-\beta(1-\pi_{B|A})} + \frac{\beta\pi_B\pi_{A|B}Z_{AB|B}(W_{AB})}{1-\beta(1-\pi_{A|B})} + \pi_A\pi_B Z(W_{AB})]$$

The simultaneous contract provides greater expected benefit to the agent when W_{AB} is significantly greater than $W_A + W_B$, i.e., when the projects have important value interactions. When the utility the projects provide increases significantly with their joint completion, a contract where both projects receive simultaneous investment maximizes expected benefits. When there are important externalities in the probabilities of success of the projects, sequential contract is more likely to provide greater expected benefits than the simultaneous timing. When preferences of the principal are biased towards a project (i.e. $\alpha_A \neq \alpha_B$) with respect to the agent's, the sequential contract that starts with the principal's preferred project is the timing alternative that maximizes principal's expected benefits.

5 Optimal contract

Once we determine the costs and benefits of each timing alternative, we can compare the two to choose the principal's utility maximizing contract.

From the **cost of the contract**, we find that the simultaneous contract is cheaper than the sequential one when the latter provides a transfer greater than the simultaneous investment cost. From the cheapest sequential contract, we find that transfers are more likely to be greater than the simultaneous investment cost when there are no technical externalities among the projects, and when there is small interaction among project valuations. When investment costs are such that simultaneous cost is significantly smaller than the sum of independent investment costs in each of the projects, it is likely that the sequential transfer is greater than the simultaneous investment cost.

From the **expected benefits** for the principal, we see that a simultaneous contract provides greater expected benefits when there are important value interactions among the projects. When there are technical externalities among the projects, sequential timing is more likely to be the expected benefits maximizing alternative.

	Projects with small valuation interaction	Projects with big valuation interaction ($W_A + W_B << W_{AB}$)
Big technical externalities among projects	*Sequential* is the timing that maximizes expected benefits when the externality is big enough.	Expected benefits maximizing alternative depends on the size of the value interaction with respect to the size of the technical externality.
Small technical externalities among projects	Independent Projects: *Simultaneous* is the timing that maximizes expected benefits.	*Simultaneous* is the expected utility maximizing timing

Figure 2: Optimal Contract

The table shows the optimal timing for different combinations of technical externalities and valuation interactions among the projects. We find that, for large technical externalities, the optimal contract is a sequential one that starts with the project that produces the technical externality. When value interaction among the projects is critical, a simultaneous contract is the optimal strategy so long as the interactions are sufficiently high.

For independent projects, i.e. projects with negligible value interactions and technical externalities, the optimal contract depends on the relation between investment costs and probabilities of success, as well as the agent's valuation of the projects. When projects are relatively devalued by the agent, or require high or very risky investments, the optimal timing is simultaneous.

When both technical externalities and value interactions are important, optimal timing depends on the relative value of each force. When value interaction is important enough, simultaneous is the optimal timing. And when technical externalities are more important, the optimal timing is sequential.

6 Conclusions

We present the optimal contract for use when a principal provides funds to an agent to build two projects. In a multitask dynamic model wherein the principal cannot commit to cut the flow of funds to the agent prior to both projects' completion, and whereby both principal and agent value the projects, we derive optimal incentive payments. The agent makes a (discrete) decision as to whether to invest in one, both or neither of the projects. The contract has an indeterminate length; it does not end until both projects are completed.

It is our purpose to use this model as a "cookbook" of sorts: we show how the optimal contract adapts to different agent and project characteristics. We derive the optimal timing and sequence of transfers for different projects that accounts for donor and recipient preferences

All agents receive a participation bonus when they sign the contract. This bonus exists because the agent values the completed projects. The agent's project valuations are also involved in inducing the agent to invest in multiple tasks in a dynamic setting: the agent values the projects by himself, and the contract can compensate for differences in valuations and technical characteristics of the projects to provide high-powered incentives to the agent to invest in both projects. We find that, for some projects and recipients, the valuation of the projects by the agent allows for an incentive compatible contract where transfers equal investment costs.

Candidate agents to receive project bonuses are those agents with more concave utility functions; the more risk averse. The principal must provide incentives to the agent for him to invest from the transfer received. For the agent, to invest is to take a risk: forgoing consumption today so that tomorrow there will be a positive probability that the project is completed. Agents with higher discount factors are more likely to receive project bonuses: the more the agent discounts the future, the higher the incentives must be to make him invest. Aid recipients and managers in more unstable environments require even better rewards to undertake investment. The projects less valued by the agent, and those projects with high investment costs and/or small probabilities of success, are more likely to receive project bonuses.

We find that the inability to observe the investment on the part of the principal makes simultaneous timing the optimal alternative in some circumstances, when sequential timing would be chosen under observable investment.

Comparing costs and benefits for the two timing alternatives has important implications in the design of multiple project contracts. We find that when technical externalities among the projects are important, the optimal timing is sequential. And when there are important value interactions among the projects, simultaneous timing is the principal's utility maximizing alternative. When projects are unrelated, we find that the optimal contract depends on the investment costs and valuation considerations of the agent: the optimal contract is simultaneous for more costly and less valued projects, and sequential for less costly and more valued projects.

In the foreign aid example, when the principal has special preferences for any of the projects, the contract may be inefficient: there may exist an alternative cheaper contract that provides the agent greater expected value than the alternative that maximizes the principal's utility. Foreign aid literature studies the negative effects on the efficiency of aid from donor-driven development. From a theoretical point of view, this is a puzzling situation: even if the agent is committed to the project and transfers equal to investment funds are incentive compatible, the principal's preferences may lead to inefficiency. How to provide appropriate incentives to the principal, perhaps by extending the proposed framework to account for a richer set of participants in the aid network (governments, multilateral organizations, NGO's, etc.) remains open for future research.

In the corporate finance example, we see that the principal's ability to set the timing provides him with additional assurance against the agent's discretionary use of funds: shareholders may set their utility maximizing timing in the contract to avoid the manager maximizing his own utility in the timing decision.

Further, in the corporate finance example, the ability of the shareholders to decide the timing of projects works as a protection mechanism: they ensure that the manager follows the timing that maximizes their benefit instead of the manager's preferred timing.

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A Appendix: Stationary contract

A.1 One project already completed

At the stationary contract when project X has already been completed, for incentive compatibility and promise keeping constraints to be satisfied transfer needs to satisfy

$$u(\tau)(1-\beta(1-\pi)) - u(\tau-\Psi_y)(1-\beta) \leqslant \beta\pi_y\left(W_{XY}(1-\beta) - w_x\right) \tag{18}$$

Let $\hat{\tau}$ be the smallest feasible transfer when project X has already been completed. The cheapest stationary contract when project X is already completed provides the agent a utility

$$\underline{S}_x = \frac{u(\tilde{\tau}-\Psi_y) + w_x + \beta\pi_y W_{XY}}{1-\beta(1-\pi_y)} > W_x$$

Let $g(\tau) = u(\tau)(1-\beta(1-\pi)) - u(\tau-\Psi_y)(1-\beta)$. For $g'(\tau) > 0$, transfer $\tilde{\tau} = \Psi_y$ is the cheapest feasible transfer, since for all $\tau > \Psi_y$ we can propose an alternative contract with smaller transfer that satisfies (18) and is cheaper. For $g'(\tau) < 0$, either $\tilde{\tau} = \Psi_y$ is feasible or (18) binds.

The cost of the cheapest stationary contract when X is completed is given by

$$C_x = \frac{\hat{\tau}}{[1-\beta(1-\pi_y)]}$$

Claim 4 *Cost function is non-increasing with the valuation of the project already completed*

Proof. Case 1: $\gamma = 0$, $V' \geqslant \underline{V}_X$. From First Order Conditions and convexity of the cost function we have that

$$\begin{gathered}\beta(1-\pi_y)\left[\frac{dC_x^*(V')}{dV'} - \frac{dC_x^*(V)}{dV}\right] + \lambda = 0 \\ W_x < \underline{V}_X \leqslant V' \leqslant V \\ \frac{dC_x^*(V)}{dW_x} = -\frac{dC_x^*(V')}{dV'}(1-\beta) - \frac{\lambda\beta(1-\beta)}{(1-\pi_y)} < 0\end{gathered} \tag{19}$$

The cost function is decreasing with the valuation of the completed project. The optimal contract provides a non-increasing sequence of promised utilities for the successive unsuccessful attempts until the remaining project is completed.

Case 2: $\gamma > 0$ In that case, $V' = \underline{V}_X \leqslant V$. We have an stationary contract from the second trial at $V = \underline{V}_X = V'$. In this situation,

$$\frac{dC_x^*(\underline{V}_X)}{dW_x} \leqslant 0 \tag{20}$$

Suppose not, suppose transfers at the stationary contract were increasing with the valuation of the completed project. Let $\tau(w_x)$ be the cheapest stationary transfer for valuation of the project already completed w_x. For $\hat{w}_x > w_x$, $\tau(w_x)$ does satisfy incentive compatibility constraint for the agent, but we can propose a contract with smaller transfer such that

$$u(\tau(w_x) - \varepsilon) - u(\tau(w_x) - \varepsilon - \Psi_y) = \beta\pi_y \left[V'_B - V\right]$$

where

$$V = \frac{u(\tau(w_x) - \varepsilon - \Psi_y) + \hat{w}_x + \beta\pi_y W_{XY}}{1 - \beta(1-\pi)} = \underline{V}_X$$

that satisfies promise keeping and incentive compatibility constraint for $\hat{w}_x$ and is cheaper, what contradicts transfers increasing with the valuation of the project already completed. ∎

Claim 5 *The cost function is increasing in the investment cost and decreasing in the probability of success.*

Proof. To get the effect of changes in investment costs on the cost function, we check how it affects the constraints of the problem. From the promise keeping constraint, we see that greater transfers and promises are required when the cost of investment increases, in order to provide same level of utility.

From the envelope we get that

$$\frac{dC_x^*(V)}{d\Psi_y} = (\lambda + \mu) u(\tau - \Psi_y) > 0$$

so the cost function is increasing in investment cost.

How the probability of success affects the cost function is given by the Envelope condition

$$\frac{dC_x^*(V)}{d\pi_y} = -\beta C_x^*(V') - [\mu\beta + \lambda](W_{AB} - V') < 0$$

∎

Claim 6 *Transfers are non-increasing with the probability of success of the project still to be completed.*

Proof. Let $\hat{\pi} > \pi$. When probability of success increases, from (18) smaller transfers are feasible. For τ to be incentive compatible we need

$$u(\tau) - (W_{XY}(1-\beta) - w_x) < 0$$

and change in utility provided to the agent is given by

$$\begin{aligned}&\left[u'(\tau)(1-\beta(1-\pi))-u'(\tau-\Psi_y)(1-\beta)\right]\Delta\tau\\&=\left[\beta\left(W_{XY}(1-\beta)-w_x\right)-\beta u(\tau)\right]\Delta\pi\end{aligned}$$

and either $g'(\tau) > 0$ and $\Delta\tau = 0$, so transfer is constant at the investment cost and $\underline{S}_x(\hat{\pi}) > \underline{S}_x(\pi)$. Or $g'(\tau) < 0$, $\underline{S}_x(\hat{\pi})$ versus $\underline{S}_x(\pi)$ depends on the agent's utility function. ■

Claim 7 *Transfers are non-increasing with the project's value interactions (W_{AB})*

Proof. Suppose not, suppose $\hat{\tau}(W_{AB}) < \hat{\tau}(\hat{W}_{AB})$ for $W_{AB} < \hat{W}_{AB}$. There exist an alternative contract with $\tau(\hat{W}_{AB}) \leqslant \hat{\tau}(W_{AB})$ that is feasible and cheaper when value of both projects completed is $\hat{W}_{AB}$, what contradicts increasing transfers with the value interaction among the projects.

With respect to the promised utility when a project is already completed, either $g'(\tau) > 0$ and $\Delta\tau = 0$, so transfer is constant at the investment cost and $\underline{S}_x(\hat{W}_{AB}) > \underline{S}_x(W_{AB})$. Or $g'(\tau) < 0$, $\underline{S}_x(\hat{W}_{AB})$ versus $\underline{S}_x(W_{AB})$ depends on the agent's utility function. ■

A.2 Stationary sequential contract

At the stationary sequential contract, the principal chooses the transfer that minimizes

$$C_{A\_B} = \min_{\tau} \frac{[\tau + \beta\pi_B C_B^*(\underline{S}_B)]}{[1-\beta(1-\pi_B)]}$$

$$s.t. \quad u(\tau)(1-\beta(1-\pi_B)) - u(\tau-\Psi_B)(1-\beta) \leqslant \beta\pi_B(1-\beta)\underline{S}_B \tag{21a}$$

$$\tau \geqslant \Psi_y \tag{21b}$$

that provides the agent a utility

$$\underline{S}^{\text{seq}} = \frac{u(\tau^* - \Psi_B) + \beta\pi_B\underline{S}_B}{1-\beta(1-\pi_B)}$$

and transfer τ^* is either $\tau^* = \Psi_y$ or $(21a)$ binds.

With a parallel reasoning to the preceding section, we find that cheapest sequential transfer, τ^*, is non-increasing with the probability of success and non-decreasing with investment cost Ψ_B. Cheapest simultaneous transfer is non-increasing with $\underline{S}_B$.

A.3 Stationary simultaneous contract

At the stationary simultaneous contract, the principal chooses the transfer that minimizes

$$C_{ab} = \min_{\tau} \frac{[\tau + \beta\pi(1-\pi)\,[C_B^*(\underline{S}_B) + C_A^*(\underline{S}_A)]]}{[1-\beta(1-\pi)^2]}$$

$$s.t. \quad u(\tau)(1-\beta(1-\pi)^2) - u(\tau-\Psi_{AB})(1-\beta) \leqslant$$

$$\leqslant \beta\pi(1-\beta)\,[\pi W_{AB} + (1-\pi)\,[\underline{S}_A + \underline{S}_B]] \tag{22a}$$

$$u(\tau-\Psi_i)(1-\beta(1-\pi)^2) - u(\tau-\Psi_{AB})(1-\beta) \leqslant$$

$$\beta\pi\,[\pi(1-\beta(1-\pi)\pi]\,(W_{AB} - \underline{S}_j) + (1-\pi)[1-\beta(1-\pi)^2]\underline{S}_i \text{ for i,j=A,B} \tag{22b}$$

$$\tau \geqslant \Psi_{AB} \tag{22c}$$

and utility this contract provides to the agent is

$$\underline{S}^{sim} = \frac{u(\tilde{\tau}-\Psi_{AB}) + \beta\pi(1-\pi)(\underline{S}_A + \underline{S}_B)}{1-\beta(1-\pi)^2}$$

Transfer $\tilde{\tau}$ is either $\tilde{\tau} = \Psi_{AB}$ or $(22a)$ binds.

A.4 Cost comparison

A.4.1 Feasible transfers

To compare the set of incentive compatible transfers for each timing alternative, we compare

$$g_{\text{seq}}(\tau) = u(\tau)(1-\beta(1-\pi)) - u(\tau-\Psi_y)(1-\beta)$$

and

$$g_{sim}(\tau) = u(\tau)(1-\beta(1-\pi)^2) - u(\tau-\Psi_{AB})(1-\beta)$$

We find that, for a given transfer,

$$g'_{\text{seq}}(\tau) - g'_{sim}(\tau) = \left[u'(\tau-\Psi_{AB}) - u'(\tau-\Psi_y)\right](1-\beta) + u'(\tau)\beta\pi^2 > 0$$

what implies that whenever $g'_{sim}(\tau) > 0$, $g'_{\text{seq}}(\tau) > 0$, transfers equal to investment costs are feasible in both timings. Whenever $g'_{\text{seq}}(\tau) < 0$, we have that $g'_{sim}(\tau) < 0$ and is feasible to have sequential transfer greater than the simultaneous.

Whenever $(22b)$ binds and $\tau^* > \Psi_{AB}$, we find that $\tilde{\tau} < \tau^*$ since at $\tilde{\tau}$ sequential contract is not feasible (the agent prefers to invest on both projects than in the first project in the sequence).

We need to consider, for $\tau^* > \Psi_{AB}$, four cases for the transfer comparison with respect to $(22a)$:

1. $(22a)$ does not bind and $g'_{sim}(\tau) > 0$. Transfers equal to investment costs are feasible in both timings, so sequential contract is the cheapest alternative.

2. $(22a)$ does bind and $g'_{sim}(\tau) > 0$. Transfers equal to investment costs are feasible in both timings, so sequential contract is the cheapest alternative

3. $(22a)$ does bind and $g'_{sim}(\tau) < 0$. Let

$$\begin{gathered} T\,\mathrm{seq}(\tau) = u(\tau)(1-\beta(1-\pi_B)) - u(\tau-\Psi_B)(1-\beta) \\ -\beta\pi_B(1-\beta)\underline{S}_B \leqslant 0 \\ Tsim(\tau) = u(\tau)(1-\beta(1-\pi)^2) - u(\tau-\Psi_{AB})(1-\beta) \\ -\beta\pi(1-\beta)\left[\pi W_{AB} + (1-\pi)\left[\underline{S}_A + \underline{S}_B\right]\right] \leqslant 0 \end{gathered}$$

   When $Tsim(\tilde{\tau}) = 0$ and so $(22a)$ binds, we have that

$$\begin{gathered} T\,\mathrm{seq}(\tilde{\tau}) = u(\tau)\beta\pi(1-\pi) - \left[u'(\tau-\Psi_B)(\Psi_{AB}-\Psi_B)\right](1-\beta) \\ -\beta\pi(1-\beta)\left[\pi(\underline{S}_B - W_{AB}) - (1-\pi)\underline{S}_A\right] \end{gathered}$$

   for small $(\Psi_{AB}-\Psi_B)$ we have that $T\,\mathrm{seq}(\tilde{\tau}) > 0$. If $g'_{\mathrm{seq}}(\tau) < 0$, we have that $\tilde{\tau} < \tau^*$. If $g'_{\mathrm{seq}}(\tau) > 0$, we would have that transfer in the sequential case would equal investment cost, and so contracts would not be comparable.

4. $(22a)$ does not bind, and $g'_{sim}(\tau) < 0$ and $(21a)$ does bind with a transfer $\tau^* \geqslant \Psi_{AB}$. Then by the symmetric argument we have that $Tsim(\tau^*) < 0$, and since $g'_{sim}(\tau) < 0$ we have that $\tilde{\tau} < \tau^*$.

   Whenever neither $(22a)$ nor $(21a)$ bind, transfers are equal to investment cost in both cases and we are not in the set of contracts we can compare.

A.4.2 Cost functions

The difference of the costs for the simultaneous and sequential timing is given by

$$\begin{aligned} C_{ab} - C_{A\_B} &= \left[\tau^* - \tilde{\tau}\right](1-\beta(1-\pi)) + \tau^*\beta\pi(1-\pi) \\ &\quad + (1-\beta(1-\pi))\beta\pi C_x \end{aligned}$$

Simultaneous contract is cheaper when

$$[\tilde{\tau} - \tau^*] < C_x\beta\pi(2-\pi)$$

Under full information, we have that sequential contract is the cost minimizing alternative. With moral hazard, the optimal timing depends on the need to provide incentives to the agent: when agent's valuation of the projects is small, or when they are relatively costly and probability of success is small, transfers need to be over the investment cost. The fact that in a simultaneous contract expected returns to investment are higher, and that partly compensates the agent for the risk he is undertaking, can make the simultaneous contract the cost minimizing alternative.

B Appendix: Cost function properties

Lemma 6 *The cost function when project x has already been completed,* C_x^* *,is increasing, convex and differentiable.*

Proof. (a) Increasing. Let (τ, V') be the optimal contract that provides V level of utility. We want to show that $C^*(V) < C^*(\tilde{V})$ for all $V < \tilde{V}$.

To provide $V = \hat{V} - \varepsilon$, the optimal contract needs to satisfy the promise keeping constraint for the new utility to be provided.

$$(\hat{V} - V) = \varepsilon = u'(\tau - \Psi)\Delta\tau + \beta(1 - \pi)\Delta V'$$

and for the incentive compatibility constraint to be satisfied we need that

$$\left[u'(\tau) - u'(\tau - \Psi)\right]\Delta\tau \leqslant -\beta\pi\Delta V'$$

Let's set $\Delta V' = 0$. To satisfy the two constraints, change in transfer has to satisfy

$$\left[u'(\tau) - u'(\tau - \Psi)\right]\Delta\tau \leqslant 0$$
$$\varepsilon = u'(\tau - \Psi)\Delta\tau$$

The contract $(\tau - \Delta\tau, V')$ is feasible and incentive compatible when the level of utility to be provided is V, but it may not be optimal. This implies

$$C_x^*(V) \leqslant C_x^*(\hat{V}) - \frac{\varepsilon}{u'(\tau - \Psi)} < C_x^*(\hat{V})$$

(b) Convexity. Let (τ_1, V_1') and (τ_2, V_2') be the optimal contracts for V_1 and V_2 respectively.

From concavity of the utility function, we know that $\tau < \lambda\tau(V_1) + (1-\lambda)\tau(V_2)$ is feasible when the level of utility to be provided is $(\lambda V_1 + (1-\lambda)V_2)$ keeping constant promised utilities in case of failure. This transfer is also Incentive Compatible. Comparing the cost functions we get

$$\begin{aligned} C_x^*(\lambda V_1 + (1-\lambda)V_2) &\leqslant \lambda C_x^*(V_1) + (1-\lambda)C_x^*(V_2) - \\ &- [\lambda\tau(V_1) + (1-\lambda)\tau(V_2) - \tau] < \\ &< \lambda C_x^*(V_1) + (1-\lambda)C_x^*(V_2) \end{aligned}$$

(c) To show that C^* is differentiable, we apply Benveniste-Scheinkman (1978). Define

$$\begin{aligned} C_x^*(V_0) &= \tau(V'(V_0), V_0) + \beta(1-\pi)C_x(V'(V_0)) \\ W(V) &= \tau(V'(V_0), V) + \beta(1-\pi)C(V'(V_0)) \end{aligned}$$

where $\tau(V'(V_0), V)$ is the transfer that satisfies promise keeping and IC for a V level of utility to be provided when promise in case of failure is fixed at $V'(V_0)$. We have that

$$\begin{aligned} C_x^*(V_0) &= W(V_0) \\ C_x^*(V) &\leqslant W(V) \text{ for } V \text{ in a neighborhood of } V_0 \end{aligned}$$

C^* is a convex function, and W is a differentiable function. From Benveniste-Scheinkman (1978), $C^*(V)$ is differentiable at V_0 with derivative

$$\frac{dC_x^*(V_0)}{dV_0} = \frac{dW(V_0)}{dV_0} = \frac{d\tau(V'(V_0), V)}{dV}$$

■

Lemma 7 *The cost function of the sequential contract that starts with project B, $C_{B\_A}^*$,is increasing, convex and differentiable.*

Proof. The T operator is a contraction on the complete metric space of continuous, increasing and convex functions, so it has a fixed point. This fixed point is an increasing and convex function by the symmetric argument used in Lemma 1.

To show that C^* is differentiable, we apply Benveniste-Scheinkman (1978). Define

$$\begin{aligned} C_{B\_A}^*(V_0) &= \tau(V'(V_0), V_B'(V_0), V_0) + \beta(1-\pi)C_{B\_A}^*(V'(V_0)) + \beta\pi C_B^*(V_B'(V_0)) \\ W(V) &= \tau(V'(V_0), V_B'(V_0), V) + \beta(1-\pi)C_{B\_A}^*(V'(V_0)) + \beta\pi C_B^*(V_B'(V_0)) \end{aligned}$$

where $\tau(V'(V_0), V_B'(V_0), V)$ is the transfer that satisfies promise keeping and IC for a V level of utility to be provided when promise in case of failure is fixed at $V'(V_0)$ and promise in case of success is fixed at $V_B'(V_0)$. We have

$$\begin{aligned} C_{B\_A}^*(V_0) &= W(V_0) \\ C_{B\_A}^*(V) &\leqslant W(V) \text{ for } V \text{ in a neighborhood of } V_0 \end{aligned}$$

W is a differentiable function. $C^*_{B\_A}(V)$ is differentiable at V_0 with derivative

$$\frac{dC^*_{B\_A}(V_0)}{dV_0} = \frac{dW(V_0)}{dV_0} = \frac{d\tau(V'(V_0), V_B'(V_0), V)}{dV}$$

■

Lemma 8 *The cost function of the simultaneous contract. C^*_{ab} ,is increasing, convex and differentiable.*

Proof. The T operator is a contraction on the complete metric space of continuous and increasing functions, so it has a fixed point. The fixed point is an increasing and concave function.

To show that C^*_{ab} is differentiable, we apply Benveniste-Scheinkman (1978). Let

$$\begin{aligned} C^*_{ab}(V_0) &= \tau(V'(V_0), V_A'(V_0), V_B'(V_0), V) + \beta\pi(1-\pi)C^*_A(V_A'(V_0)) + \\ &\quad + \beta\pi(1-\pi)C^*_B(V_B'(V_0)) + \beta(1-\pi)^2 C^*_{ab}(V'(V_0)) \\ W(V) &= \tau(V'(V_0), V_A'(V_0), V_B'(V_0), V) + \beta\pi(1-\pi)C^*_A(V_A'(V_0)) + \\ &\quad + \beta\pi(1-\pi)C^*_B(V_B'(V_0)) + \beta(1-\pi)^2 C^*_{ab}(V'(V_0)) \end{aligned}$$

where $\tau(V'(V_0), V)$ is the transfer that satisfies promise keeping and IC for a V level of utility to be provided when promise in case of failure is fixed at $V'(V_0)$ and promise in case of success is fixed at $V_B'(V_0), V_A'(V_0)$. We have

$$\begin{aligned} C^*_{ab}(V_0) &= W(V_0) \\ C^*_{ab}(V) &\leqslant W(V) \text{ for } V \text{ in a neighborhood of } V_0 \end{aligned}$$

W is a differentiable function. $C^*(V)$ is differentiable at V_0 with derivative

$$\frac{dC^*_{ab}(V_0)}{dV_0} = \frac{dW(V_0)}{dV_0} = \frac{d\tau(V'(V_0), V_A'(V_0), V_B'(V_0), V)}{dV}$$

■

C Appendix: Sequential contract characteristics

Proposition 10 *At the cheapest stationary contract, a project bonus upon completion of the first project may be provided to agents with Arrow-Pratt absolute risk aversion*

$$r(\tau) > \frac{\beta\pi_B}{(1-\beta)\Psi_B} \tag{23}$$

Proof. From the First Order Conditions of the stationary sequential contract, we know that constraint (8a) must bind whenever the optimal contract differs from $(\Psi_B, \underline{V}_B)$. The First Order Condition with respect to τ when (8a) is plugged into the objective function is given by

$$1+\beta\pi\left[\frac{dC_B^*(V_B')}{dV_B'}-\left(\frac{1-\beta(1-\pi_B)}{\beta\pi_B}\right)\gamma_B\right]\frac{dV_B'}{d\tau}=0$$

and from (8a) we have that

$$\frac{dV_B'}{d\tau}=-\frac{[1-\beta(1-\pi_B)]\,u'(\tau)-(1-\beta)u'(\tau-\Psi_B)}{-\beta\pi_B(1-\beta)}$$

We need to check how the optimal contract varies with the sign of this derivative. When

$$\frac{dV_B'}{d\tau}>0$$

we need

$$0<\frac{dC_B^*(V_B')}{dV_B'}<\left(\frac{1-\beta(1-\pi_B)}{\beta\pi_B}\right)\gamma_B$$

for the First Order Conditions to be satisfied, so $\gamma_B>0$ and $V_B'=\underline{V}_B$ and no project bonus is provided.

A necessary condition for $\gamma_B=0$, $V_B'\geqslant\underline{V}_B$ and project bonus is feasible, is that

$$\frac{dV_B'}{d\tau}<0$$

which implies

$$\begin{aligned}[1-\beta(1-\pi_B)]\,u'(\tau)-(1-\beta)u'(\tau-\Psi_B)&<0\\ -\frac{u''(\tau)}{u'(\tau)}=r(\tau)&>\frac{\beta\pi_B}{(1-\beta)\Psi_B}\end{aligned}\qquad(24)$$

The candidate agents to get project bonuses are the ones with more concave utility functions. ■

Claim 8 *As the value interaction among projects increases, the optimal contract provides non-increasing transfers.*

Proof. We proceed in two steps: we first derive the effect of the value interaction, W_{AB}, on the cost of promised utility when B is completed, and second we derive how this affects the transfer and promises at the optimal (stationary) sequential contract.

(1) Since effect of value interactions comes from the cost of the promise when the first project is completed, we start with the effect of the value interactions on the cost of promised

utility when project B is completed. The transfer when B has already been completed is non-increasing in W_{AB}. Suppose not. Let τ be the optimal transfer to be provided when the value of both projects completed is W_{AB}. For $\hat{W}_{AB} > W_{AB}$, τ is feasible (incentive compatibility and promise keeping are satisfied with this transfer) and there may be a cheaper transfer $\hat{\tau} \leqslant \tau$. So transfer when B is already completed is non-increasing in W_{AB}. Cost of promised utility becomes smaller as value interaction increases.

Let (τ, V_B') be the optimal contract for valuation of both projects completed W_{AB}.

(2a) When $\hat{W}_{AB} > W_{AB}$ implies $\underline{V}_B(\hat{W}_{AB}) > \underline{V}_B(W_{AB})$. We want to show that optimal sequential transfer is non-increasing in W_{AB}. Suppose not.

If $V_B' = \underline{V}_B(W_{AB})$, for $\hat{W}_{AB}$ the new contract must provide $\hat{V}_B' \geqslant \underline{V}_B(\hat{W}_{AB}) > \underline{V}_B(W_{AB})$. The contract $(\tau, \underline{V}_B(\hat{W}_{AB}))$ satisfies the incentive compatibility and promise keeping constraints when the value interaction is given by $\hat{W}_{AB}$, but we can propose an alternative contract with smaller transfers that is feasible, which contradicts increasing transfers in the valuation of both projects completed.

If $V_B' > \underline{V}_B(W_{AB})$, we can either have $V_B' > \underline{V}_B(\hat{W}_{AB})$, where contract for W_{AB} is feasible for $\hat{W}_{AB}$ but we can find a cheaper alternative contract with no greater transfers. Or we can have $\underline{V}_B(\hat{W}_{AB}) > V_B' > \underline{V}_B(W_{AB})$, and the argument would be symmetric to the $V_B' = \underline{V}_B(W_{AB})$ case.

(2b) When $\hat{W}_{AB} > W_{AB}$ implies $\underline{V}_B(\hat{W}_{AB}) < \underline{V}_B(W_{AB})$. All contracts feasible for W_{AB} are also feasible for $\hat{W}_{AB}$, and we can propose alternative contracts with smaller transfers and greater promises incentive compatible and promise keeping and cheaper, since cost of promise in case of success became smaller, which contradicts increasing transfers with size of the value interaction. ∎

D Appendix: Simultaneous contract characteristics

Proposition 11 *At the cheapest simultaneous contract for technically symmetric projects, a project bonus is provided to agents with absolute risk aversion*

$$r(\tau) > \frac{\beta\pi(1-\pi)}{[1-\beta(1-\pi)]\,(\Psi_{AB}-\Psi)} \tag{25}$$

Proof. In the cheapest simultaneous contract, which we showed in Proposition 4 to be stationary, two incentive compatibility constraints need to be taken into account. When the promise keeping $(12a)$ constraint is plugged into the incentive compatibility constraints

$(12c)$ and $(12d)$ we get

$$u(\tau-\Psi)(1-\beta(1-\pi)^2)-u(\tau-\Psi_{AB})(1-\beta(1-\pi))\geqslant$$
$$\geqslant \beta\pi(1-\beta(1-\pi))\left[\pi(W_{AB}-V_B')+(1-\pi)V_A'\right]$$
$$u(\tau-\Psi)(1-\beta(1-\pi)^2)-u(\tau-\Psi_{AB})(1-\beta(1-\pi))\geqslant$$
$$\geqslant \beta\pi(1-\beta(1-\pi))\left[\pi(W_{AB}-V_A')+(1-\pi)V_B'\right]$$

and the problem the principal solves to get the cheaper contract is:

$$C(V)=\min_{\tau,V_A',V_B'}\frac{\tau+\beta(1-\pi)\pi\left[C_B^*(V_B')+C_A^*(V_A')\right]}{[1-\beta(1-\pi)^2]}$$
$$s.t.\quad IC_A(\mu_1),IC_B(\mu_2) \tag{26}$$
$$V_B'\geqslant \underline{V}_B\ (\gamma_B),V_A'\geqslant \underline{V}_A\ (\gamma_A) \tag{27}$$

where (27) comes from the optimal contract when only project A or B are completed respectively, as do the cost functions $C_A^*(.)$ and $C_B^*(.)$. The First Order Conditions give:

$$\beta(1-\pi)\pi\frac{dC_A^*(V_A')}{dV_A'}= \tag{28}$$
$$=\gamma_A+\mu_1\left[\beta\pi(1-\beta(1-\pi))\right]-(\mu_2+\mu_1)\left[\beta\pi^2(1-\beta(1-\pi))\right]$$
$$\beta(1-\pi)\pi\frac{dC_B^*(V_B')}{dV_B'}= \tag{29}$$
$$=\gamma_B+\mu_2\left[\beta\pi(1-\beta(1-\pi))\right]-(\mu_2+\mu_1)\left[\beta\pi^2(1-\beta(1-\pi))\right]$$

We check for all possible combinations of constraints binding.

(i) Suppose $\mu_1>0$ and $\mu_2>0$. Both incentive compatibility constraints bind and we have $V_A'=V_B'$. The principal's problem becomes:

$$C(V)=\min_{\tau,V'}\frac{\tau+\beta(1-\pi)\pi\left[C_A^*(V_c')+C_B^*(V_c')\right]}{[1-\beta(1-\pi)^2]}$$
$$s.t.\quad u(\tau-\Psi)(1-\beta(1-\pi)^2)-u(\tau-\Psi_{AB})(1-\beta(1-\pi))= \tag{30a}$$
$$=\beta\pi(1-\beta(1-\pi))\left[\pi W_{AB}+(1-2\pi)V_c'\right]$$
$$V_c'\geqslant\max(\underline{V}_A\ ,\underline{V}_B)\ \ (\gamma) \tag{30b}$$

where V_c' denotes promised utility when one of the projects has been completed. When we plug in $(30a)$ the First Order Conditions is

$$1+\beta(1-\pi)\pi\left[\frac{dC_A^*(V_c')}{dV_c'}+\frac{dC_B^*(V_c')}{dV_c'}-\left(\frac{1-\beta(1-\pi)^2}{\beta(1-\pi)\pi}\right)\gamma\right]\frac{dV_c'}{d\tau}=0$$

From the Incentive Compatible-promise keeping constraint $(30a)$ we have

$$\frac{dV_c'}{d\tau} = -\frac{u'(\tau-\Psi)(1-\beta(1-\pi)^2) - u'(\tau-\Psi_{AB})(1-\beta(1-\pi))}{-\beta\pi(1-\beta(1-\pi))(1-2\pi)}$$

From these two equations we get that $\gamma > 0$ is only feasible if $\frac{dV'}{d\tau} > 0$. This means that $V' = \max(\underline{V}_A\ ,\underline{V}_B)$ and since projects are symmetric no project receives a project bonus.

We can have $\gamma = 0$, so bonus in both projects is feasible, when $\frac{dV'}{d\tau} < 0$. This is the case when

$$r(\tau) > \frac{\beta\pi(1-\pi)}{[1-\beta(1-\pi)]\,(\Psi_{AB}-\Psi)} \text{ and } (1-2\pi) < 0$$

Since when

$$r(\tau) < \frac{\beta\pi(1-\pi)}{[1-\beta(1-\pi)]\,(\Psi_{AB}-\Psi)} \text{ and } (1-2\pi) > 0$$

we can propose an alternative contract with smaller promises for more valued project and greater promises for less valued projects that is cheaper, and where one of the projects does not get a bonus that is cheaper, which contradicts optimally of bonus for both projects.

(ii) Take the case where $\mu_1 = 0$ and $\mu_2 > 0$ From (28) we get that $\gamma_A > 0$ which implies $V_A' = \underline{V}_A$. Then, from (29) we get

$$1 + \beta\pi\left[\frac{dC_B^*(V_B')}{dV_B'} - \gamma_B\right]\frac{dV_B'}{d\tau} = 0$$

and we have that $\gamma_B = 0$ is only feasible when

$$\frac{dV_B'}{d\tau} < 0$$

that implies

$$\left[1-\beta(1-\pi)^2\right]u'(\tau-\Psi) - [1-\beta(1-\pi)]\,u'(\tau-\Psi_{AB}) < 0$$
$$-\frac{u''(\tau)}{u'(\tau)} = r(\tau) > \frac{\beta\pi(1-\pi)}{[1-\beta(1-\pi)]\,(\Psi_{AB}-\Psi)}$$

and a symmetric argument applies for $\mu_1 > 0$ and $\mu_2 = 0$.

(iii) Suppose $\mu_1 = \mu_2 = 0$. Then for First Order Conditions to be satisfied we need $\gamma_A > 0, \gamma_B > 0$, and no project gets a project bonus. ■

Proposition 12 *In the simultaneous cheapest contract with technically symmetric projects, either no project bonus is promised, or if there is a project bonus it is greater for the less valued project.*

Proof. We need to consider four cases, for the four possible combinations of project bonus in each of the two projects. These cases are given by the binding and not binding of the feasibility constraints $V_A' \geqslant \underline{V}_A$ and $V_B' \geqslant \underline{V}_B$. From the cost minimization problem, λ_0, λ_A and λ_B are the Lagrange multipliers for the incentive compatibility constraints $(12b)$, $(12c)$ and $(12d)$, and μ is the multiplier for the promise keeping constraint $(12a)$.

1. Both projects get project bonus, i.e. none of the constraints binds, $V_A' > \underline{V}_A$ and $V_B' > \underline{V}_B$. We showed that in the stationary contract this can only be the case when both IC constraints are binding, so $V_A' = V_B'$. In this case, $V_A' = V_B' > \max(\underline{V}_B, \underline{V}_A)$. So greater bonuses are given for the less valued project.

2. $\gamma_1 = 0, \gamma_2 > 0$. $V_A' = \underline{V}_A$, project A does not get a project bonus.

   We consider four cases:

   (i) incentive compatibility constraint for project B binds, so $\lambda_A = 0, \lambda_B > 0$. This implies $V_A' = \underline{V}_A \leqslant V_B'$.From the First Order Conditions we get

$$\frac{dC_B^*(V_B')}{dV_B'} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

   If $W_A > W_B$, we have $W_B < W_A < \underline{V}_A < V_B'$, so a positive bonus is given when the less valued project is completed.

   If $W_A < W_B$, $\underline{V}_A < \underline{V}_B$. From the First Order Conditions and convexity of the cost function, we have that

$$\frac{dC_B^*(V_B')}{dV_B'} < \frac{dC_{ab}^*(V')}{dV'} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A} < \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B}$$

   what leads to a contradiction since it implies $V_B' < \underline{V}_B$.

   (ii) incentive compatibility for project A binds, $\lambda_A > 0, \lambda_B = 0$. This implies $V_A' = \underline{V}_A \geqslant V_B'$.

   From First Order Conditions we have that either

$$\frac{dC_B^*(V_B')}{dV_B'} > \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A} \quad \text{or} \quad \frac{dC_B^*(V_B')}{dV_B'} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

   depending on the relation of the values of the multipliers $\left[\lambda_A - \frac{\gamma_2}{\pi}\right]$

   If $W_A > W_B$, we have a positive bonus for the less valued project.

   If $W_A < W_B$, we have $V_B' < \underline{V}_A < \underline{V}_B$, which contradicts feasibility.

(iii) $\lambda_A = 0, \lambda_B = 0$. incentive compatibility A and incentive compatibility B may or may not be binding. We will go to points 1 or 2 depending on the situation.

(iv) Both incentive compatibility constraints bind, $\lambda_A > 0, \lambda_B > 0$ If both constraints are binding, we need $V_B^{'} = V_A^{'}$. The relative magnitude of the multipliers brings us to the previous cases.

3. $\gamma_1 > 0, \gamma_2 = 0$, no project bonus for project B. Symmetric to (2).

   In that case we find $\lambda_A > 0, \lambda_B = 0, V_A' > \underline{V}_B$ when $W_A < W_B$. Or $\lambda_A = 0, \lambda_B > 0, V_A' < \underline{V}_B$ when $W_A < W_B$

4. Neither project gets a bonus, $\gamma_1 > 0, \gamma_2 > 0$

   Both constraints are binding, so no bonus promised utility is provided in any situation.

■

Proposition 13 *In the simultaneous contract, when the two projects have different investment costs, we can have project bonus for the more valued project when this project is the one with greater investment cost.*

Proof. We consider the case $\Psi_A > \Psi_B$. We need to check the incentive compatibility constraints in this new situation:

$$\begin{aligned} &if \quad \beta\pi V_A^{'} - u(\tau - \Psi_B) = \beta\pi V_B^{'} - u(\tau - \Psi_A) \text{ both constraints bind simultaneously} \\ &if \quad V_A^{'} > V_B^{'} - \left[\frac{u(\tau - \Psi_B) - u(\tau - \Psi_A)}{\beta\pi}\right] \text{ IC}_A \text{ binds and IC}_B \text{ does not bind} \\ &if \quad V_A^{'} < V_B^{'} - \left[\frac{u(\tau - \Psi_B) - u(\tau - \Psi_A)}{\beta\pi}\right] \text{ IC}_B \text{ binds and IC}_A \text{ does not bind} \end{aligned}$$

Let

$$\gamma = -\left[\frac{u(\tau - \Psi_B) - u(\tau - \Psi_A)}{\beta\pi}\right]$$

We need again to consider the four possible cases:

1. Both projects get project bonus, neither $V_A^{'} \geqslant \underline{V}_A$ and $V_B^{'} \geqslant \underline{V}_B$ bind. We can mimic the argument of the symmetric costs case and find that one of them needs to bind at $\underline{V}^{sim}$.

2. $\gamma_1 = 0, \gamma_2 > 0, V_A^{'} = \underline{V}_A$, project A does not get project bonus.

   We need to consider four cases:

(i) $\lambda_A = 0, \lambda_B > 0$. Incentive compatibility for project B binds. This implies

$$(\underline{V}_A - V_B^{'}) < \gamma$$

and from the First Order Conditions we get

$$\frac{dC_B^*(V_B^{'})}{dV_B^{\prime}} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

If $W_A > W_B$, we have $W_B < W_A < \underline{V}_A < V_B^{'}$ to satisfy both incentive compatibility and promise keeping constraints. Positive bonus when less valued project is completed.

If $W_A < W_B, \underline{V}_A < \underline{V}_B$. From convexity of the cost function with respect to the value of the project already completed and First Order Conditions, we have that

$$\frac{dC_B^*(V_B^{'})}{dV_B^{\prime}} < \frac{dC_{ab}^*(V^{\prime})}{dV^{\prime}} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A} < \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B}$$

what leads to a contradiction since it implies $V_B^{'} < \underline{V}_B$ and that is not feasible.

(ii) $\lambda_A > 0, \lambda_B = 0$.Incentive compatibility for project A binds. This implies

$$(\underline{V}_A - V_B^{'}) > \gamma$$

From First Order Conditions, either

$$\frac{dC_B^*(V_B^{'})}{dV_B^{\prime}} > \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A} \quad \text{or} \quad \frac{dC_B^*(V_B^{'})}{dV_B^{\prime}} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

If $W_A < W_B$, we have $V_B^{'} < \underline{V}_A < \underline{V}_B$ what is not feasible.

If $W_A > W_B$, we have bonus when less valued project is completed.

(iii) $\lambda_A = 0, \lambda_B = 0$. incentive compatibility A and incentive compatibility B may or may not be binding. We go to points 1 and 2 depending on the situation.

(iv) $\lambda_A > 0, \lambda_B > 0$ If both incentive compatibility constraints are binding, we need $(\underline{V}_A - \underline{V}_B) = \gamma$.

3. $\gamma_1 > 0, \gamma_2 = 0$, $V_B^{'} = \underline{V}_B$. Project B does not get project bonus.

   We need to consider four cases:

   (i) $\lambda_A = 0, \lambda_B > 0$. Incentive compatibility constraint for project B binds. This implies

$$(V_A^{\prime} - \underline{V}_B) < \gamma$$

From First Order Conditions, either

$$\frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} < \frac{dC_A^*(V_A')}{dV_A'} \quad \text{or} \quad \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} > \frac{dC_A^*(V_A')}{dV_A'}$$

If $W_A > W_B$, we have or $V_A' < \underline{V}_B < \underline{V}_A$, that is a contradiction. Or we have positive bonus when the project more valued that requires a greater cost of investment is completed.

If $W_A < W_B$, we have positive bonus when less valued project is completed.

(ii) $\lambda_A > 0, \lambda_B = 0$. Incentive compatibility constraint for project A binds. This implies

$$(V_A' - \underline{V}_B) > \gamma$$

We have from First Order Conditions

$$\frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} > \frac{dC_A^*(V_A')}{dV_A'}$$

If $W_A > W_B$ we have

$$\frac{dC_A^*(V_A')}{dV_A'} < \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} < \frac{dC_B^*(\underline{V}_A)}{d\underline{V}_A}$$

what leads to a contradiction.

If $W_A < W_B$ we have positive bonus when less valued project is completed.

(iii) $\lambda_A = 0, \lambda_B = 0$. incentive compatibility A and incentive compatibility B may or may not be binding. We will go to points 1 and 2 depending on the situation.

(iv) $\lambda_A > 0, \lambda_B > 0$. Both incentive compatibility constraints are binding. Depending on the relative magnitude of the multipliers, we are in situation (1) or (2).

4. $\gamma_1 > 0, \gamma_2 > 0$, no bonus for any project.

   Both constraints are binding, so no bonus promised utility is provided in any situation.

■

Proposition 14 *In the simultaneous contract, when the projects have different probabilities of success, we can have that the more valued project gets project bonus if this project is the one with smaller probability of success.*

Proof. The First Order Conditions of the simultaneous principal's problem when $\pi_A > \pi_B$ are

$$\pi_B(1-\pi_A)\beta\left[\frac{dC_B^*(V_B')}{dV_B'} - \frac{dC_{ab}^*(V')}{dV'}\right] = (1-\beta)\lambda_A - \beta\lambda_B + \gamma_1$$
$$\pi_A(1-\pi_B)\beta\left[\frac{dC_A^*(V_A')}{dV_A'} - \frac{dC_{ab}^*(V')}{dV'}\right] = (1-\beta)\lambda_B - \beta\lambda_A + \gamma_2$$

And from the incentive compatibility constraints we get

$$\begin{array}{llll} if & \pi_A(V_B' - V_A') & = & -(\pi_A - \pi_B)(V_A' - V') \text{ both constraints bind simultaneously} \\ if & \pi_A(V_B' - V_A') & < & -(\pi_A - \pi_B)(V_A' - V') \text{ IC}_A \text{ binds and IC}_B \text{ does not bind} \\ if & \pi_A(V_B' - V_A') & > & -(\pi_A - \pi_B)(V_A' - V') \quad \text{IC}_B \text{ binds and IC}_A \text{ does not bind} \end{array}$$

We need to consider four cases:

1. Both projects get project bonus, neither $V_A' \geqslant \underline{V}_A$ and $V_B' \geqslant \underline{V}_B$ bind. We mimic the argument of the symmetric probabilities case and find that one of them needs to bind at $\underline{V}^{sim}$.

2. No bonus for project A. $\gamma_1 = 0, \gamma_2 > 0, V_A' = \underline{V}_A$

   We need to consider four cases:

   (i) Incentive compatibility constrain for project B binds, $\lambda_A = 0, \lambda_B > 0$. This implies

$$\pi_A(V_B' - \underline{V}_A) > -(\pi_A - \pi_B)(\underline{V}_A - V')$$

   and from First Order Conditions

$$\frac{dC_B^*(V_B')}{dV_B'} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

   If $W_A > W_B$, we have positive bonus when less valued project is completed.

   If $W_A < W_B, \underline{V}_A < \underline{V}_B$. To be feasible we need $\underline{V}_A < V_B'$, and we give bonus to the more valued project when it is the one with smaller success probability.

   (ii) Incentive compatibility for project A binds, $\lambda_A > 0, \lambda_B = 0$. This implies

$$\begin{aligned} \pi_A(V_B' - \underline{V}_A) &< -(\pi_A - \pi_B)(\underline{V}_A - V') \\ V_B' &< \underline{V}_A \end{aligned}$$

From First Order Conditions, either

$$\frac{dC_B^*(V_B')}{dV_B'} > \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A} \quad \text{or} \quad \frac{dC_B^*(V_B')}{dV_B'} < \frac{dC_A^*(\underline{V}_A)}{d\underline{V}_A}$$

If $W_A < W_B$, we have $V_B' < \underline{V}_A < \underline{V}_B$ what is a contradiction.

If $W_A > W_B$, we have bonus when less valued project is completed.

(iii) $\lambda_A = 0, \lambda_B = 0$. incentive compatibility A and incentive compatibility B may or may not be binding. We go to points 1 and 2 depending on the situation.

(iv) $\lambda_A > 0, \lambda_B > 0$ If both constraints are binding, we need $(\underline{V}_A - \underline{V}_B) = \gamma$.

3. No bonus for project B, $\gamma_1 > 0, \gamma_2 = 0$, $V_B' = \underline{V}_B$

   We need to consider four cases:

   (i) Incentive compatibility for project B binds, $\lambda_A = 0, \lambda_B > 0$. This implies

$$\pi_A(\underline{V}_B - V_A') > -(\pi_A - \pi_B)(V_A' - V')$$

   From First Order Conditions, either

$$\frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} < \frac{dC_A^*(V_A')}{dV_A'} \text{ or } \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} > \frac{dC_A^*(V_A')}{dV_A'}$$

   If $W_A > W_B$, we have or $V_A' < \underline{V}_B < \underline{V}_A$, that is a contradiction. Or we have project bonus for the more valued project that has a smaller probability of success.

   If $W_A < W_B$, we have positive bonus when less valued project is completed.

   (ii) Incentive compatibility constraint for project A binds, $\lambda_A > 0, \lambda_B = 0$. This implies

$$\begin{aligned} \pi_A(\underline{V}_B - V_A') &< -(\pi_A - \pi_B)(V_A' - V') \\ \underline{V}_B &< V_A' \end{aligned}$$

   We have from First Order Conditions

$$\frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} > \frac{dC_A^*(V_A')}{dV_A'}$$

   If $W_A > W_B$ we have

$$\frac{dC_A^*(V_A')}{dV_A'} < \frac{dC_B^*(\underline{V}_B)}{d\underline{V}_B} < \frac{dC_B^*(\underline{V}_A)}{d\underline{V}_A}$$

what leads to a contradiction.

If $W_A < W_B$ we have positive bonus when less valued project is completed.

(iii) $\lambda_A = 0, \lambda_B = 0$. incentive compatibility A and incentive compatibility B may or may not be binding. We will go to points 1 and 2 depending on the situation.

(iv) $\lambda_A > 0, \lambda_B > 0$. Both constraints are binding. Depending on the relative magnitude of the multipliers, we are in situation (1) or (2).

4. $\gamma_1 > 0, \gamma_2 > 0$

   Both constraints are binding, so no bonus promised utility is provided in any situation.

■

E Appendix: Cost comparison

Proposition 15 *For technically symmetric projects (i.e.$\pi_A = \pi_B, \Psi_A = \Psi_B$), when transfers of the cheapest sequential contract are such that simultaneous investment is feasible, i.e. $\tau^{\text{seq}} \geqslant \Psi_{AB}$, simultaneous contract is cheaper.*

Proof. Without loss of generality suppose $W_A \leqslant W_B$. By Lemma 3 optimal sequential contract starts with project B. Let $(\tau, V', V_B^{'})$ be the cheapest (and by Proposition 2 stationary) sequential contract that provides utility $\underline{V}^{\text{seq}}$ to the agent and is such that $\tau \geqslant \Psi_{AB}$. We want to compare it with the cheapest simultaneous contract for same promised utility level $\underline{V}^{\text{seq}}$.

We define a simultaneous contract $(\tilde{\tau}, \tilde{V}', \tilde{V}_B', \tilde{V}_A')$ with $\tilde{\tau} = \tau, \tilde{V}' = V'$ and $\tilde{V}_B = V_B'$ that provides the agent a utility $\bar{V} \geqslant \underline{V}^{\text{seq}}$. We set $\tilde{V}_A'$ so that the contract $(\tau, V', V_B', \tilde{V}_A')$ satisfies promise keeping and incentive compatibility constraints of the simultaneous contract. Since $\tau \geqslant \Psi_{AB}$, this transfer is feasible in a simultaneous contract. From the promise keeping constraint, we get that $\tilde{V}_A'$ should satisfy

$$\begin{aligned}
u(\tau - \Psi_{AB}) + \beta \left[\pi^2 W_{AB} + (1-\pi)^2 V' + \pi(1-\pi)\tilde{V}_A' \; + \pi(1-\pi)V_B'\right] \\
= u(\tau - \Psi) + \beta \left[\pi \tilde{V}_B' + (1-\pi)\tilde{V}'^{t}\right] \\
[u(\tau - \Psi) - u(\tau - \Psi_{AB})] = \beta \left[\pi^2 (W_{AB} - V_B') + \pi(1-\pi)(\tilde{V}_A' - \tilde{V}')\right] \\
\tilde{V}_A' = \frac{u(\tau - \Psi) - u(\tau - \Psi_{AB})}{\beta\pi(1-\pi)} - \frac{\pi^2(W_{AB} - V_B') - \pi(1-\pi)V'}{\pi(1-\pi)}
\end{aligned} \tag{31}$$

From the incentive compatibility constraint of the sequential contract we have that

$$u(\tau-\Psi_{AB})+\beta\left[\pi^2 W_{AB}+(1-\pi)^2 V'+\pi(1-\pi)\underline{V}_A\ +\pi(1-\pi)V'_B\right] \qquad (32)$$
$$\leqslant u(\tau-\Psi)+\beta\left[\pi V'_B+(1-\pi)V'\right]$$

since sequential transfer is greater than cost of investment in both projects simultaneously and the sequential contract aims to provide incentives to the agent to invest only on project B, the first project in the sequence. Together (31) and (32) imply $\tilde{V}'_A \geqslant \underline{V}_A$.

The incentive compatibility constraints to satisfy are given by:

$$u(\tau-\Psi)+\beta\left[\pi\tilde{V}'_A+(1-\pi)V'\right]\leqslant$$
$$\leqslant u(\tau-\Psi_{AB})+\beta\left[\pi^2 W_{AB}+(1-\pi)^2 V'+\pi(1-\pi)\tilde{V}'_A+\pi(1-\pi)V'_B\right] \qquad (33)$$
$$u(\tau-\Psi)+\beta\left[\pi V'_B+(1-\pi)V'\right]\leqslant$$
$$\leqslant u(\tau-\Psi_{AB})+\beta\left[\pi^2 W_{AB}+(1-\pi)^2 V'+\pi(1-\pi)\tilde{V}'_A+\pi(1-\pi)V'_B\right] \qquad (34)$$
$$u(\tau)+\beta V'\leqslant$$
$$\leqslant u(\tau-\Psi_{AB})+\beta\left[\pi^2 W_{AB}+(1-\pi)^2 V'+\pi(1-\pi)\tilde{V}'_A+\pi(1-\pi)V'_B\right] \qquad (35)$$

Incentive compatibility (35) is satisfied by construction of the contract $(\tau, V', V'_B, \tilde{V}'_A)$. Incentive compatibility (34) is binding by construction of $\tilde{V}'_A$. For (33) to be satisfied, we need to ensure that $\tilde{V}'_A \leqslant V'_B$. For this to hold we need

$$\tilde{V}'_A - V_B = \frac{[u(\tau-\Psi)-u(\tau-\Psi_{AB})]}{\beta\pi(1-\pi)} - \frac{\pi W_{AB}}{(1-\pi)} + V' - \frac{V'_B}{(1-\pi)}$$
$$= \frac{[u(\tau-\Psi)-u(\tau-\Psi_{AB})]-\beta\pi^2\left(W_{AB}-V'_B\right)-\beta\pi(1-\pi)\left[V'_B-V'\right]}{\beta\pi(1-\pi)}$$
$$= \frac{[u(\tau-\Psi)-u(\tau-\Psi_{AB})]-\beta\pi\left[\pi\left(W_{AB}-V'_B\right)+(1-\pi)\left[V'_A-V'\right]-(1-\pi)\left[\tilde{V}'_A-V_B\right]\right]}{\beta\pi(1-\pi)}$$

and by (34)we have that

$$\left[\tilde{V}'_A-V_B\right](1+\beta\pi(1-\pi))$$
$$= \frac{[u(\tau-\Psi)-u(\tau-\Psi_{AB})]-\beta\pi\left[\pi\left(W_{AB}-V'_B\right)+(1-\pi)\left[V'_A-V'\right]\right]}{\beta\pi(1-\pi)} = 0$$

what implies $\tilde{V}'_A = V'_B$ and (33) also binds.

Let $C^*_{B\_A}(\underline{V}^{\mathrm{seq}})$ and $C^*_{ab}(\underline{V}^{\mathrm{seq}})$ be the minimum cost to provide the agent utility $\underline{V}^{\mathrm{seq}}$ in a sequential and simultaneous contract respectively, and let $C_{ab}(\bar{V})$ be the cost of the simultaneous contract $(\tau, V', V'_B, \tilde{V}'_A)$ that provides the agent a utility $\bar{V} \geqslant \underline{V}^{\text{seq}}$. By Proposition 2 the optimal sequential contract is stationary, so $V' = \underline{V}^{\text{seq}}$. The difference in cost of each timing structure is given by

$$\begin{aligned}
C^*_{B_A}(\underline{V}^{\text{seq}}) - C^*_{ab}(\underline{V}^{\text{seq}}) &\geqslant C^*_{B_A}(\underline{V}^{\text{seq}}) - C_{ab}(\underline{V}^{\text{seq}}) \\
\geqslant C^*_{B_A}(\underline{V}^{\text{seq}}) - C_{ab}(\bar{V}) &= \beta\pi C^*_B(V'_B) + \beta(1-\pi)C^*_{B_A}(V') \qquad (36)\\
-\beta\Big[\pi(1-\pi)C^*_B(V'_B) &+ \pi(1-\pi)C^*_A(\tilde{V}'_A) + (1-\pi)^2 C_{ab}(V')\Big]
\end{aligned}$$

$$= \beta(1-\pi)\Big[C^*_{B\_A}(V') - C_{ab}(V')\Big] + \beta\pi^2 C^*_B(V'_B) + \beta\pi(1-\pi)\Big[C^*_{ab}(V') - C^*_A(\tilde{V}'_A)\Big] \quad (37)$$

Rearranging terms, we obtain

$$(1-\beta(1-\pi))\Big[C^*_{B\_A}(V') - C^*_{ab}(V')\Big] \geqslant \beta\Big[\pi^2 C^*_B(\tilde{V}'^t_B) + \pi(1-\pi)\Big[C^*_{ab}(V'^s) - C^*_A(\tilde{V}'_A)\Big]\Big] \quad (38)$$

Since $W_A \leqslant W_B$ and $\tilde{V}'_A = V'_B$, by (**??**) and (**??**) we have that $C^*_B(\tilde{V}'^t_B) \leqslant C^*_A(V'^t_A)$. Using this, (38) becomes

$$\begin{aligned}
&\pi C^*_B(V'_B) + (1-\pi)\Big[C_{ab}(V') - C^*_A(\tilde{V}'_A)\Big] \\
&\geqslant \frac{\tau + \pi(1-\pi)\Big[C^*_A(\tilde{V}'_A) + C^*_B(V'_B)\Big]}{1-\beta(1-\pi)} + (2\pi-1)C^*_B(V'_B) \\
&= \frac{\Phi(\pi)C^*_B(V'_B) + \tau}{1-\beta(1-\pi)} > 0
\end{aligned}$$

since

$$\Phi(\pi) = -1 + 2\pi(2-\pi) - (2\pi\beta - 1)(1-\pi)^2 > 0 \text{ for all } \pi \in [0,1]$$

what implies

$$C^*_{B\_A}(\underline{V}^{\text{seq}}) > C^*_{ab}(\underline{V}^{\text{seq}})$$

hence a sequential contract is more expensive than the simultaneous one. ■