Trust, Ability-to-Pay, and Charitable Giving

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Abstract

In the literature on privately provided public goods, altruism has been motivated by what contributions can accomplish (public goods philanthropy), by the pleasure of giving (warm-glow philanthropy), or by the desire to personally make a difference (impact philanthropy). We revisit these motives but allow for income heterogeneity and distrust in the institutional structures involved. We also model socially motivated philanthropy when income-heterogeneous donors take trust and ability-to-pay into account. We show key differences across the four models in terms of crowding out and in the effects of income distribution. In the socially motivated model, low-income donors may contribute more than high-income donors, giving theoretical foundation to the frequently observed "U-shaped" pattern of giving.

Keywords: Philanthropy, Social Motivation, Trust, Ability to Pay, Crowding Out.

JEL Classifications: H41, H31, H50.
1 Introduction

Charitable giving comes from different sources: individuals, foundations, corporations, and bequests. According to the World Giving Index (WGI) of the Charities Aid Foundation (CAF), 28 percent of the global population (or 1.4 billion people) is estimated to have donated money in 2013, 21 percent (or 1.0 billion people) to have volunteered time, and 48 percent (or 2.3 billion people) to have helped a stranger. At the country level, the U.S.A. has the highest (overall) score (64 percent) in 2013, together with Myanmar, and is the only country to rank in the top 10 for all three kinds of giving covered by the WGI (CAF, 2014). Other countries with high scores are Canada and Ireland (3rd and 4th ranked with a common score of 60 percent), New Zealand (5th ranked with a score of 58 percent), and Australia (6th ranked with a score of 56 percent). The proportion of the population donating money in these cases ranges from 62 percent in New Zealand to 91 percent in Myanmar. More generally, of the top 20 countries in the 2014 WGI, more than 50 percent of the population is estimated to have donated money in all but three cases.

While the high percentage of the population donating money is suggestive of the importance of individual giving, we can further look at individual giving levels to get a clearer picture and, at least for the U.S.A., compare its contribution to charitable giving to the other sources of giving. According to Giving USA 2014, donations by individuals represent 72 percent of the $335.17 billion collected in charitable giving in 2013, consistent with the 72% of the $316.23 collected in 2012 and the 73% of the $298.42 collected in 2011. When we examine individual giving as a percentage of individual income in the U.S.A. (Figure 1a) and as a percentage of household income in Canada (Figure 1b), we find a “U-shaped” pattern of giving, with the share of income donated initially decreasing in income but eventually increasing, consistent with findings in Auten et al. (2000). To the best of our knowledge, there are no theoretical models explaining this U-shaped pattern of giving but some anecdotal evidence suggests that the distribution of contributions across different causes may offer a possible explanation in that religious organizations tend to receive the greatest support through individual contributions and low-income people tend to give more to religious causes (Turcotte, 2012). As noted in Andreoni (2006), it is also plausible that low-income people may be young and thus expect to see their wages increase so that they may be willing to give more.

Although individuals continue to donate their time and money, a number of high-profile scandals involving embezzlement, misuse of donations, slow disbursements of disaster relief, and generally inefficient operations, such as the United Way in the 1990s, the American Red Cross Liberty Fund after 9/11, and, more recently, the Breast Cancer Society (FTC, 2015), have generated distrust among potential philanthropists. It is not unusual to come across examples of charities spending only a small fraction of money raised on the activities they promise. In Light (2008), survey data from 2008 indicate that only 25 percent of Americans believed charitable organizations to be “very good” at helping people and

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1Myanmar’s high ranking results from a strong culture of giving that has religious roots. In fact, 90 percent of the country’s population follows the Theravada school of Buddhism under which the lives of ordained monks and nuns are supported by the religion’s followers through charitable giving.

2For the U.S.A., we compute the shares of individual income devoted to contributions as the ratio between average contribution (total contributions of individuals within each adjusted gross income bracket divided by the number of returns) and average total income (aggregated income of individuals within each adjusted gross income bracket divided by the number of returns). For Canada, we compute the shares of household income contributed to charitable giving as the ratio between the average annual donation and the mid-point of the household income range. For the lowest income range, we use the upper bound of the range; for the highest income range, we use the lower bound of the range.
70 percent of Americans reported charitable organizations to waste “a great deal” or “fair amount” of money. Accordingly, as public confidence in charities falls, we can expect donations to decline, but the extent of this decline should ultimately depend on the reasons why individuals donate in the first place. Experimental evidence that charity quality can influence contributions is available in Landry et al. (2010).

So why do people give away their earnings in a seemingly unselfish manner? How can we reconcile unselfish giving with economists’ view of human nature as reflected in the hypothesis of the pursuit of self-interest? These questions have puzzled economists since the late 1960s (Hochman and Rogers, 1969; Kolm, 1969; Becker, 1974), but the bulk of the academic work attempting to answer the questions has flourished since the late 1980s, identifying and modelling a number of motivational channels giving rise to philanthropic behaviour. Aside from its significance in pursuing new theoretical facets of individual decision making, this work has coincided with a strong public policy interest in private philanthropy as a substitute for public sector provision of goods and services. If private contributions and public provision are two separate but equally viable means of providing for certain goods and services, be they private or public, there is clearly an important policy dimension in the study of what drives private charity as the nature of philanthropic motives likely impacts how private charity interacts with public provision in supporting the common cause and thus conditions the appropriate configuration of the two strategies based on efficiency concerns. And, of course, understanding the drivers of private philanthropy is a fundamental step in the design of effective policy tools to promote or restrict its level as need be. With this in mind, we set out to build on the theoretical philanthropy literature along two lines: (1) by adding income heterogeneity and institutional trust in existing philanthropy models and (2) by proposing and modelling social pressures based on ability-to-pay considerations as an additional motivator of philanthropy.

In the literature on the provision of charitable goods, altruism is motivated by what contributions can accomplish (public goods philanthropy model), by the pleasure of giving (private consumption or warm-glow philanthropy model), or by the desire to personally make a difference (impact philanthropy model). To illustrate the differences across the three models, we can express the problem of the representative donor $i$ as choosing the level of voluntary contribution ($g_i$) that maximizes utility $V_i(c_i, m_i)$, which depends positively on private consumption ($c_i$) and philanthropic motives ($m_i$), subject to the constraint that $w_i - t_i = c_i + g_i$, where $w_i$ and $t_i$ represent income and taxes. If we let $G$ and $T$ represent aggregate voluntary contribution and tax levels, $G_{-i}$ is $G$ minus the representative donor’s voluntary contribution, and $z(\cdot)$ represents the production of the charitable good, we can capture the essence of each of the three models through $m_i$, with $m_i = z(G + T)$ in the public goods model, $m_i = g_i$ in the warm-glow

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3For example, charitable giving to the Catholic Church declined after the U.S. Catholic clergy scandals (Bottan and Perez-Truglia, 2015).

4In this paper, we relate giving to trust in institutions (i.e., charity, government). For trust among individuals in a society, see Glaeser et al. (2000) and Alesina and La Ferrara (2002).

5Hochman and Rogers (1969) and Kolm (1969) are the first to recognize the public good nature of charities even when they are intended to support the provision of private goods (e.g., day care, housing). Inasmuch as individuals contributing to the charities experience altruistic feelings towards those who consume the charitable goods, their private consumption becomes a public good.

6For a review of this work, see Andreoni (2006).

7For the public goods model, see Warr (1982), Roberts (1984), Bergstrom et al. (1986), Bernheim (1986), Andreoni (1988) and Auten et al. (2002). For the warm-glow model, see Andreoni (1990) and Ribar and Wilhelm (2002). For the impact model, see Duncan (2004). A separate model of status signalling of contributions (prestige) is explored in Glazer and Konrad (1996).
model, and $m_i = z(G + T) - z(G_{-i} + T)$ in the impact model. As such, in the last model, the impact of one’s contribution is equal to the difference between the total charitable activity level, $z(G + T)$, and the charitable activity level in the absence of the individual contribution, $z(G_{-i} + T)$. Under the assumption that $z(\cdot)$ is linear, the impact philanthropy model collapses to the warm-glow model; under the assumption that $V_i(\cdot)$ is linear in $m_i$, the impact philanthropy model collapses to the public goods model. These results, however, do not necessarily apply if potential donors do not fully trust the charitable organization and/or the government to utilize their entire donation toward the cause. It is rather straightforward to see that, if there is distrust, in the sense that agents believe that only a fraction ($\lambda$) of each unit of money they contribute is actually spent on the charitable good, the equivalence between the warm-glow model and the impact philanthropy model when $z''(\cdot) = 0$ no longer holds; in fact, while the utility function in the former remains unchanged, the utility function in the latter becomes $V_i(w_i - g_i - t_i, \lambda g_i)$, and the utility-maximizing contribution choices thus differ between the two models. In the first part of the paper, we thus explore the implications of incomplete institutional trust for the contribution level and crowding out effect of taxation within and between the public goods model and the impact philanthropy model under the assumption that there are two types of donors who differ in income. We include income heterogeneity here for consistency and comparison with a fourth model we propose in this paper to motivate philanthropy which is based on the desire to be seen as socially responsible citizens.

The notion that social considerations matter in individual decision making, particularly when voluntary time and money contributions are concerned, is not novel, and, in fact, there is empirical evidence suggesting that social interactions matter in charitable giving (e.g., List and Price, 2009; Meer, 2011), but we are not aware of any formal theoretical analysis of the implications of social motives for charitable contribution levels and, more importantly, crowding in/out effects of taxation. Social considerations are typically modelled as a function of (a) the difference between one’s contribution and the average contribution and (b) some measure of the cost of not adhering to the social norm that relates to the benefit of the activity financed via voluntary contributions or to the benefit of living in some ideal state in which everyone acts according to some socially desirable norm (e.g., the cost of the social disapproval from not acting socially responsibly may be reflected in the utility difference between a society in which everyone acts responsibly and a society in which no one does). In this paper, in addition to formally modelling socially motivated philanthropy, we extend the discussion of social motivation on several fronts: (i) we allow for income heterogeneity, (ii) we incorporate ability-to-pay in determining the impact of social pressures, and (iii) we introduce distrust in the institution responsible for the production of the charitable good.

2 Model

We consider a society with a population of size one in which individuals are faced with the decision of how much to contribute to a charitable good and how much to set aside for private consumption. There are two types of agents differing in income: the high-income agents earn $w$ while the low-income agents earn $\delta w$, where $\delta \in (0, 1)$. Individuals derive utility from the consumption of the private good ($c$) and philanthropy

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8Rege (2004) examines the decision to contribute or not contribute (rather than contribution levels) in a model of social interaction where the stable equilibria involve everyone contributing or everyone not contributing to the public good.
Philanthropy can result from one of four sources: (1) from what it can accomplish (i.e., the charitable good itself), (2) from the pleasure of giving, (3) from the difference it can make, or (4) from social pressure. While one main contribution in this paper is in the modelling of socially motivated philanthropy, we re-visit the other three sources of philanthropic motivation within a framework that allows for income heterogeneity and distrust in the bodies directly involved in collecting the funds intended for the charitable good. Before we can define \( f_i \) in each of the four cases, we need to be more specific about distrust, income distribution, and the production of the charitable good. To this end, we introduce \( \lambda_i \) to denote what donors believe to be actually spent on the charitable good for each unit of money contributed whether through voluntary contributions \( (j = g) \) or taxes \( (j = t) \). With both the number of potential donors and the high income level normalized to one, we let \( \sigma \) represent the fraction of the population with an income level less than one (i.e., \( w_l = \delta < 1 \)), so that the rest of the population \( (1 - \sigma) \) has an income level equal to one (i.e., \( w_h = 1 \)). Finally, we capture the production of the charitable good in \( z(\cdot) \equiv z(\lambda_g \overline{g} + \lambda_t t) \), with \( z'(\cdot) > 0 \) and \( z''(\cdot) \leq 0 \), where \( \overline{g} \) is societal average contribution, that is, \( \overline{g} = \sigma g_l + (1 - \sigma) g_h \); the impact of agent \( i \)'s voluntary contribution is then equal to \( z(\lambda_g \overline{g} + \lambda_t t) - z_i (\lambda_g \overline{g} + \lambda_t t - \lambda_g g_i) \), for \( i = l, h \).

Hence, we have that \( f_l = f(g_i) \) in the warm-glow model, \( f_i = f(z(\cdot)) \) in the public goods philanthropy model, and \( f_i = f(z(\cdot) - z_i) \) in the impact philanthropy model, respectively. For the socially motivated philanthropy model, some additional qualifications are in order. In line with previous modelings of social motivation, there are three elements we account for when defining social approval: (i) the individual’s contribution, (ii) the difference between the individual’s contribution and the average contribution, and (iii) the benefit of contributing. We combine the first two elements into a weighted average with \( (1 - \beta) \) as the weight assigned to the individual’s contribution and, correspondingly, \( \beta \) as the weight assigned to the difference between the individual’s contribution and the average contribution; a larger \( \beta \) thus implies greater emphasis on the frame of reference (that is, society’s average contribution). The third element captures the idea that, the more costly actions are, the greater the social disapproval they yield if they do not conform to the social norm. This element is modelled as a fixed component of social approval equal to the benefit of living (or cost of not living) in an ideal state (e.g., Nyborg and Rege, 2003) or to the difference between living in a society in which everyone behaves responsibly and a society in which no one behaves responsibly (e.g., Rege, 2004). More formally, this element is intended to reflect the approval rate which, in Holländer (1990), is defined as the hypothetical benefit, measured in terms of the private good, that an individual would enjoy if everyone else in society increased his/her contribution marginally. If \( z(\cdot) \) is a linear function, we can express the approval rate merely as a function of \( \lambda_g \) in that \( \lambda_g \) reflects the
benefit from a unit increase in society’s average contribution.

Unlike previous analyses of social motivation, we allow for individuals to consider their ability-to-pay in their decisions. Specifically, we define social approval in terms of share of income spent on the public good as opposed to in terms of income spent on the public good. In our socially motivated philanthropy model, we can then write $f_i$ as

$$f_i \equiv s_i = \lambda \left( \frac{g_i}{w_i} - \beta \frac{\tilde{g}}{\tilde{w}} \right) w_i,$$

where $\lambda = \lambda_g$ (as the distinction between $\lambda_g$ and $\lambda_t$ is not relevant in this setup, we drop the subscript $g$ for convenience) and $\tilde{w} = \sigma \tilde{\vartheta} + (1 - \sigma)$, with $\vartheta = \delta$ if ability-to-pay matters in social considerations. We replace $\delta$ with $\vartheta$ in the social approval function to be able to isolate the effect of adding income in the social approval function to account for ability-to-pay by simply looking at the effect of $\vartheta$ or by comparing the results unde $\vartheta = \delta$ with those under $\vartheta = 1$. In the above specification, and unlike typical specifications of social approval, the frame of reference differs between the two income groups in that society’s average contribution is scaled according to the decision-making agent’s income relative to society’s average income; put differently, a donor’s frame of reference or social norm is the contribution level such that the share of the donor’s income invested on the charitable good is the same across donors and equal to the ratio between society’s average contribution and society’s average income.

We divide the rest of this section into two parts: in the first part, we formally review and expand upon the existing philanthropy models by considering (i) a population of donors who differ in income and (ii) lack of complete trust in the charitable organization and/or government; in the second part, we present and analyze a model in which voluntary contributions are motivated by individuals’ desire to be seen as socially responsible citizens. For simplicity, throughout the rest of the paper, we may refer to voluntary contributions simply as contributions.

2.1 Warm-Glow, Public Goods, and Impact Philanthropy Models

In the warm-glow model, donors choose their private consumption and contribution to the charitable organization in order to maximize

$$V_i = u(c_i) + f(g_i)$$

subject to the constraint that

$$c_i = w_i - g_i - t.$$  

Whether donors have complete trust in the charitable organization and/or the government does not enter in their decision making as it is the act of giving that provides them with utility and not what their act yields; in other words, whether their contributions are utilized fully or only partially in support of the charitable good is irrelevant.

The utility-maximizing contribution of donor $i$ is thus such that

$$u'(c_i) = f'(g_i),$$

which gives that

$$\frac{dg_i}{dt} = -\frac{u''}{f'' + u''} = -\frac{1}{1 + \frac{f''}{u''}} \Rightarrow -1 \leq \frac{dg_i}{dt} < 0$$

(7)
and
\[
\frac{dg_i}{dw_i} = \frac{u''}{f'' + u''} = \frac{1}{1 + \frac{f''}{u''}} \implies 0 < \frac{dg_i}{dt} = \frac{dg_i}{dt} \leq 1. \tag{8}
\]

The effect of a tax increase is equivalent to the effect of an income decrease of identical magnitude. The type-\(h\) donor contributes more than the type-\(l\) donor (i.e., \(g_h > g_l\)), which implies that \(f'(g_l) \geq f'(g_h)\), so that \(u'(c_l) \geq u'(c_h)\); for \(u'(c_l) \geq u'(c_h)\), we must have that \(c_l \leq c_h\) or \(g_h - g_l \leq 1 - \delta\). The high-income donors always allocate at least a portion of their extra income to charitable giving, and crowding out is incomplete; in the extreme case in which the marginal utility of giving is constant, they spend the extra income solely on charitable giving, and crowding out is complete. Although income changes are not explicitly considered in the warm-glow model of Andreoni (1990), our results are consistent with that framework, yet serve as a comparison point with the other models.

The crowding out effect of taxation is income-dependent, unless the marginal utility of giving is constant, and can be decreasing or increasing in income as
\[
\frac{d^2g_i}{dt dw_i} = -\frac{d^2g_i}{dt^2} = -\frac{d^2g_i}{dw_i^2} = -\frac{(f'' u'')^2}{(f'' + u'')^3} \left[ \frac{u''}{(u'')^2} - \frac{f''}{(f'')^2} \right]_{>0}
\]
is positive (negative) if the additional contribution needed for a marginal increase in utility grows with the level of contribution at a faster (slower) rate than the additional consumption needed for the same utility increase grows with the level of consumption.\(^9\) As the above derivative implies, the crowding out effect of taxation also varies with the tax rate and in exactly the opposite direction from how it varies with income. For illustrative purposes, if \(V_i = c_i^\alpha + g_i^{1-\alpha}\), we have that the crowding out effect of taxation is smaller (larger) among high-income donors and increasing in the tax rate when \(\alpha > 0.5\) (\(\alpha < 0.5\)) and independent of income and of the tax rate when \(\alpha = 0.5\); when \(\alpha > 0.5\) (\(\alpha < 0.5\)), we also have that the marginal propensity to contribute, that is, \(\frac{dg_i}{dwi}\), is decreasing (increasing) in income.

In sum, although trust is not relevant in the warm-glow model (individuals care about their giving but not about the results of their giving), high-income donors contribute more than low-income donors, and government taxation crowds out private contributions at different rates for different income levels.

### 2.1.1 Public Goods Philanthropy Model

In the public goods philanthropy model, donor \(i\)'s utility is given by
\[
V_i = u(c_i) + f(z (\lambda_g g_i + \lambda_g G_{-i} + \lambda t)), \tag{10}
\]
where again \(G_{-i}\) is society’s total contribution minus donor \(i\)’s contribution and is thus impacted by other donors’ contributions; however, in maximizing this utility subject to (5), the donor takes the contribution of every other donor as given, and the Nash equilibrium then requires that the first-order conditions to the maximization problems of all donors be simultaneously satisfied. As we have only two types of donors,

\(^9\)Noting that \(\frac{dg_i}{dvi} = \frac{1}{f'’}\) and \(\frac{dg_i}{dwi} = \frac{1}{f'’}\), we can express the condition for an income-increasing crowding out effect as
\[
\frac{dg_i}{dvi} - \frac{d^2g_i}{dwi^2} < 0, \text{ that is, } \frac{dg_i}{dvi}, \text{ which gives the consumption needed for a unit increase in utility, must increase with } c_i \text{ at a greater rate than } \frac{dg_i}{dvi}, \text{ which gives the contribution needed for a unit increase in utility, increases with } g_i.
we know that, in equilibrium, a fraction $\sigma$ of the population chooses $g_l$ and the remaining $(1 - \sigma)$ chooses $g_h$ so that $\lambda_y g_l + \lambda_y G_{-i} = \sigma \lambda_y g_l + (1 - \sigma) \lambda_y g_h$. To characterize the Nash equilibrium, it thus suffices to consider the utility-maximizing conditions of the representative donors in the two income groups, namely,

$$u'_t = \lambda_y f'(\cdot) z'(\cdot) \quad \text{and} \quad u'_h = \lambda_y f'(\cdot) z'(\cdot),$$

which, combined, give that $u'_t = u'_h$ so that $g_h - g_l = 1 - \delta$. The private consumption level is the same between the two income groups, and the high-income donors thus spend their extra income entirely on the charitable good, that is, the difference in contribution level between the two income groups is always equal to the income difference. The implication of this result is that all the parameters of the model other than $\delta$ affect $g_h$ and $g_l$ in exactly the same manner.

More specifically, we have

**Proposition 1** In the public goods philanthropy model, donors’ income affects the choice of how much to contribute, with richer donors contributing more, but does not affect how this choice responds to changes in $t$, $\lambda_y$, $\lambda_t$, and $\sigma$, that is, the effects of the changes on $g_l$ and $g_h$ are exactly the same in direction and magnitude, with a larger $t$, a smaller $\lambda_y$, a larger $\lambda_t$, and a larger $\sigma$ amounting to lower contribution levels. However, as $\delta$ increases, $g_h$ decreases while $g_l$ increases, but the latter effect exceeds the former effect so that, overall, the difference between the two contribution levels falls.

Using the results from the total differentiation of the two equilibrium conditions as given in the Appendix, we can easily show that

$$\frac{dg_l}{dt} = \frac{dg_h}{dt} = \frac{dg}{dt} = -1 + \frac{\lambda_y (\lambda_y - \lambda_t) A}{u'' + \lambda_y^2 A} < 0,$$

$$\frac{dg_l}{d\lambda_y} = \frac{dg_h}{d\lambda_y} = \frac{dg}{d\lambda_y} = -f'(\cdot) z'(\cdot) + \lambda_y A \lambda_y \frac{A}{u'' + \lambda_y^2 A} > 0,$$

$$\frac{dg_l}{d\lambda_t} = \frac{dg_h}{d\lambda_t} = \frac{dg}{d\lambda_t} = -\frac{\lambda_y A}{u'' + \lambda_y^2 A} \leq 0,$$

and

$$\frac{dg_l}{d\sigma} = \frac{dg_h}{d\sigma} = \frac{dg}{d\sigma} = \frac{\lambda_y^2 A (1 - \delta)}{u'' + \lambda_y^2 A} \geq 0,$$

where $A = z'(\cdot) f''(\cdot) z'(\cdot) + f'(\cdot) z''(\cdot) \leq 0$, which implies that the marginal utility benefit from the public good is non-increasing, and $u'' = u'_l = u'_h$; furthermore, we have that

$$\frac{dg_l}{d\delta} = 1 + \frac{dg_h}{d\delta} = 1 + \left( -\frac{\sigma \lambda_y^2 A}{u'' + \lambda_y^2 A} \right) = \frac{u'' + (1 - \sigma) \lambda_y^2 A}{u'' + \lambda_y^2 A} > 0.$$

Based on the above results, it is clear that, while taxation always crowds out (i.e., reduces) voluntary contributions, the extent of crowding out depends on whether donors perceive the charitable institution to be less or more trustworthy than the government. If donors trust the charitable organization more than they trust the government (that is, $\lambda_y > \lambda_t$), there is less than 100 percent crowding out; conversely, if they trust the government more than they trust the charitable organization (that is, $\lambda_y < \lambda_t$), there is more
than 100 percent crowding out. The extent to which donors trust the two bodies does not matter for the crowding out effect of taxation provided that \(0 < \lambda_g = \lambda_t \leq 1\); in such a case, there is always 100 percent crowding out. The trustworthiness of the bodies is also irrelevant for the extent of crowding out, in that it remains at 100 percent, when \(f'' = z'' = 0\); in such a case, the extent of trust in the government and the proportion of the population in the low-income categories have no effect on contributions, and an increase in the income level of the low-income group does not affect the contribution level of the high-income group but increases that of the low-income group by a dollar for each dollar increase in income.

The effects of \(\lambda_g\) and \(\lambda_t\) on contributions are rather straightforward. In the public goods philanthropy model, donors value the charitable good and are thus indifferent between contributing to the provision of the good via taxes, which the government collects, or via voluntary contributions, which the charitable organization collects, provided that the two collection bodies are equally trustworthy: the two contribution channels are perfectly substitutable in such a case and this explains why, for each dollar increase in taxes, voluntary contributions fall by a dollar. However, if donors have reasons to believe that one body is less reliable than the other, they adjust their contributions as to favour the more trustworthy channel: if the charitable organization is more reliable, they increase contributions; if the government is more reliable, they decrease contributions. The effect of \(\sigma\) is also quite natural: as low-income donors contribute less than high-income donors, an increase in the proportion of the population of donors in the low-income bracket reduces the provision level of the charitable good which, in turn, increases the marginal benefit of contributing; donors thus respond to the increase in \(\sigma\) by contributing more. Finally, an increase in \(\delta\) allows low-income donors to spend more on private consumption as well as to contribute more to the charitable good (the marginal effect of \(\delta\) on \(g_t\) is less than unity unless the marginal benefit of contributing is constant); the provision of the charitable good thus increases, and the high-income donors respond to the resulting increase in their marginal benefit of contributing by decreasing their contributions to a lesser extent, though, than the low-income donors increase their contributions so that, overall, the provision of the charitable good increases in response to an increase in \(\delta\).

From (13) and (14), we also obtain

**Corollary 1** Societies with more trustworthy governments have lower contribution levels provided that \(f(\cdot)\) and \(z(\cdot)\) are not both linear, while societies with more trustworthy charitable organizations enjoy higher contribution levels independently of the curvatures of \(f(\cdot)\) and \(z(\cdot)\), but disparity in charitable organizations’ trustworthiness can explain variation in philanthropic behaviour across societies to a greater extent than disparity in governments’ trustworthiness.

The last point follows from a comparison of how an increase in \(\lambda_g\) affects voluntary contributions relative to an increase in \(\lambda_t\). In absolute terms, the marginal effects of \(\lambda_t\) falls short of the marginal effects of \(\lambda_g\) so that societies with more trustworthy charitable organizations but also more trustworthy governments enjoy higher contributions levels; to put it in another way, from a position of equal trustworthiness of both charitable organizations and governments, an increase in \(\lambda_g\) yields a larger increase in contributions than a decrease in \(\lambda_t\) of equal magnitude so that differences in philanthropic behaviour across societies are more likely to emerge if there exist differences in charitable organizations’ trustworthiness. In the extreme case in which \(f'' = z'' = 0\), differences in charitable organizations’ trustworthiness across societies result in differences in philanthropic behaviour while differences in governments’ trustworthiness do not.
Before moving on to analyzing the impact philanthropy model, it is useful to derive and comment on the reaction functions of the two types of donors in the public goods philanthropy model. Using the total differentiation results in the Appendix, we can express the slopes of the two reaction functions as

\[
\frac{dg_l}{d\lambda_l} \bigg|_{\lambda_l = \lambda_l f'(\cdot) z(\cdot)} = -\frac{u'' + \sigma \lambda_g^2 A}{(1 - \sigma) \lambda_g^2 A} \quad \text{and} \quad \frac{dg_h}{d\lambda_l} \bigg|_{\lambda_l = \lambda_l f'(\cdot) z(\cdot)} = -\frac{\sigma \lambda_g^2 A}{u'' + (1 - \sigma) \lambda_g^2 A},
\]

which are negative for \( f'' < 0 \) and/or \( z'' < 0 \) and thus imply that \( g_l \) and \( g_h \) are strategic substitutes: type-\( h \) donor reacts to an increase in type-\( l \) donor’s contribution by decreasing his/her contribution, and vice versa, but whether one type is more responsive than the other type depends on \( \sigma \); in particular, if \( \sigma \leq \frac{1}{2} \), the low-income donors are more responsive to changes in contribution by the high-income donors than the other way around.\(^{10}\)

While this strategic relationship holds in the impact philanthropy model, we show shortly that the two types of contributions are strategic complements in the socially motivated philanthropy model.

Upon inspection of (12), we also obtain

**Proposition 2** *In the public goods philanthropy model, when \( \lambda_g \neq \lambda_l \) and \( A < 0 \), incomplete trust affects the extent of crowding out, but, while greater trust in the government increases crowding out, it is not always the case that greater trust in the charitable organization results in less crowding out. The tax rate also affects the extent of crowding out, with a higher tax rate potentially resulting in less crowding out.*

We capture the impact of trust on crowding out in the derivatives of (12) with respect to \( \lambda_g \) and \( \lambda_l \), which we can express as

\[
\frac{d^2 g}{dt d\lambda_g} = \lambda_g (\lambda_g - \lambda_l) \left[ A' u'' \left( \frac{u''}{u'' + \lambda_g^2 A} \right) + A u'' \frac{dg}{d\lambda_g} \right] + (2 \lambda_g - \lambda_l) A u'' + \lambda_g^2 A^2 \lambda_l \left( u'' + \lambda_g^2 A \right)^2
\]

and

\[
\frac{d^2 g}{dt d\lambda_l} = \lambda_g (\lambda_g - \lambda_l) \left[ A' u'' \left( \frac{u''}{u'' + \lambda_g^2 A} \right) t + A u'' \frac{dg}{d\lambda_l} \right] - \lambda_g A \left( u'' + \lambda_g^2 A \right)^2
\]

\[
= \lambda_g (\lambda_g - \lambda_l) t \left[ (u'')^2 A' - \lambda_g (A)^2 u'' \right] - \lambda_g A \left( u'' + \lambda_g^2 A \right)^2 \left( u'' + \lambda_g^2 A \right)^3
\]

where \( A' = f''(z)^3 + 3z' f'' z'' + f' z'' > 0 \).\(^{11}\) As long as \( \lambda_g < \lambda_l < 2 \lambda_g \), \( \frac{d^2 g}{dt d\lambda_g} > 0 \), which implies that, the more donors trust the charitable organization, the smaller the crowding out effect is. However, if \( \lambda_g > \lambda_l \), greater trust in the charitable organization may result in an increase in the crowding out effect; in other words, it is possible for \( \frac{d^2 g}{dt d\lambda_g} \) to be negative. As reflected in (19), the crowding out effect is, on the other

\(^{10}\)The condition that \( \sigma \leq \frac{1}{2} \) for low-income donors to be strategically more responsive is only sufficient. In fact,

\[
\left| \frac{dg_l}{dg_h} \right|_{\lambda_l = \lambda_l f'(\cdot) z'(\cdot)} - \left| \frac{dg_h}{dg_l} \right|_{\lambda_l = \lambda_l f'(\cdot) z'(\cdot)} = \frac{A \left[ (1 - 2\sigma) u'' - (\sigma^2 - 1) \lambda_g^2 A \right]}{\left( u'' + \sigma \lambda_g^2 A \right) \left[ u'' + (1 - \sigma) \lambda_g^2 A \right]}
\]

\(^{11}\)The third-order derivatives of \( u(\cdot), f(\cdot), \) and \( z(\cdot) \) are non-negative.
hand, always increasing in the extent to which donors trust the government, that is, \( \frac{d^2 g}{dt^2} < 0 \). The effect of \( t \) on the crowding out effect is instead equal to

\[
\frac{d^2 g}{dt^2} = \frac{[\lambda_g (\lambda_g - \lambda_t) A u''(\cdot)]^2}{(u'' + \lambda_g^2 A)^3} \left[ u'' - \frac{\lambda_g A'}{\lambda_g A} \right];
\]

where \( \lambda_g, \lambda_t \) are parameters, \( A, u''(\cdot), A' \) are constants, and \( \lambda_g A \) is the marginal benefit of investing on private consumption relative to the convexity of the marginal benefit of investing on the public good.

For illustrative purposes, to underscore the possibility for greater trust in the charitable organization or a higher tax rate to amount to both a stronger and a weaker crowding out effect, we compute the effect under the assumptions that \( \lambda_g = 0.7 \), and \( \lambda_t = 0 \). For \( \alpha = \frac{1}{4} \), \( \rho = 1 \), and \( t = 0.10 \), we can show that an improvement in the trustworthiness of the charitable organization generates an increase in the crowding out effect of taxation if \( 0 < \lambda_t < 0.36 \) and a decrease if \( 0.36 \leq \lambda_t \leq 1 \). For \( \alpha = \frac{1}{4} \), we can also visualize the different effects of \( \lambda_g \) on crowding out by plotting the absolute value of the marginal effect of taxation on voluntary contributions when both \( \lambda_t = 0.3 \) and \( \lambda_t = 0.5 \) and allowing for \( \lambda_g \) to increase from 0.7 to 0.9 in each case. As we depict in Figure 2a over the range of feasible \( t \) values, the increase in \( \lambda_g \) shifts the profile of the marginal effect of taxation up when \( \lambda_t = 0.3 \) (that is, crowding out increases) and down when \( \lambda_t = 0.5 \) (that is, crowding out decreases). Finally, to emphasize the ambiguous effect of the tax rate on crowding out, we plot the profile of the marginal effect of taxation in Figure 2b for \( \lambda_g = 1 \), \( \lambda_t = 0.8 \), and \( \rho = 10 \) when both \( \alpha = \frac{1}{4} \) and \( \alpha = \frac{3}{4} \); in the former case, we obtain that \( \frac{d^2 g}{dt^2} > 0 \) over the range of feasible \( t \) values (that is, crowding out decreases); in the latter case, \( \frac{d^2 g}{dt^2} < 0 \) (that is, crowding out increases). Hence, societies with stronger philanthropic motives (i.e., lower \( \alpha \)) can afford higher tax rates not only to generate additional resources in support of charitable organizations but also to limit the amount of erosion of voluntary contributions ensuing from taxation.

### 2.1.2 Impact Philanthropy Model

In the impact philanthropy model, donors maximize

\[
V_i = u(c_i) + f(z (\lambda_g g_i + \lambda_g G_{-i} + \lambda_t t)) - z (\lambda_g G_{-i} + \lambda_t t))
\]

subject to (5) and thus choose \( g_i \) such that

\[
u_i' = \lambda_g f_i' z(\cdot).
\]

We can then characterize the Nash equilibrium as satisfying

\[
u_i' = \frac{f_i'}{f_h'},
\]

which yields the following:

\[12\text{The results also hold for } \rho = 1 \text{ but the range of feasible } t \text{ values (i.e., } t \text{ values such that } g_h > g_i > 0 \text{) is narrower.} \]
Proposition 3 In the impact philanthropy model, \( g_h > g_l \) but \( g_h - g_l \leq 1 - \delta; \) furthermore, contributions are at least as large as in the public goods philanthropy model.

If \( g_l > g_h, z_l < z_h \) so that \( z(\cdot) - z_l > z(\cdot) - z_h \) and \( f'_l \leq f'_h \); however, if \( \frac{d_f}{d_h} \leq 1, \frac{u_i}{u_h} \leq 1 \) which would require that \( a_l \geq c_h \) or \( g_h - g_l \geq 1 - \delta > 0 \), clearly a contradiction. We must then have that \( g_h > g_l \), in which case \( \frac{d_f}{d_h} = \frac{u_i}{u_h} \geq 1 \) and \( c_l \leq c_h \) or \( 0 < g_h - g_l \leq 1 - \delta \). The high-income donors thus spend their extra income by consuming more of the private good and contributing more to the charitable good; however, if the marginal benefit of contributing is constant, they spend their extra income exclusively on the charitable good.

That contributions are at least as large in the impact philanthropy model as in the public goods philanthropy model follows from a comparison of donor \( i \)'s utility-maximizing conditions in the two models. Letting \( g^*_l \) and \( g^*_h \) denote the Nash equilibrium contributions of donor \( i \) in the public goods and impact philanthropy models, we have that \( u'_i(g^*_l) = \lambda_g f'(z(g^*_l, g^*_h)) z'(g^*_l, g^*_h) \leq \lambda_g f'(z(g^*_l, g^*_h) - z_i(g^*_l, g^*_h)) z'(g^*_l, g^*_h) \) so that \( B = u'_i(g^*_l) - \lambda_g f'(z(g^*_l, g^*_h) - z_i(g^*_l, g^*_h)) z'(g^*_l, g^*_h) \leq 0 \); given that \( \frac{d_B}{d_g} > 0 \), as we show in the Appendix, we obtain that \( g^*_l > g_l \) when \( f'_l < 0 \) and \( g^*_h = g^*_l \) when \( f'_l = 0 \), where \( g^*_l \) is such that \( u'_i(g^*_l) - \lambda_g f'(z(g^*_l, g^*_h) - z_i(g^*_l, g^*_h)) z'(g^*_l, g^*_h) = 0 \). Combining that \( g^*_l > g_l, g^*_h - g^*_l = 1 - \delta \), and \( g^*_h - g^*_l < 1 - \delta \) when \( f''_l < 0 \) for \( i = l, h \), we then obtain that \( g^*_h - g^*_l > g^*_h - g^*_l > 0 \), that is, impact philanthropy results in higher contributions for both types of donors but its incremental effect is decreasing in income or larger among the low-income donors.

As in the public goods philanthropy model, the contribution levels of the two types of donors in the impact philanthropy model are strategic substitutes: the slopes of the two reaction functions are equal to

\[
\frac{dg_h}{dg_l} \bigg|_{u'_i = \lambda_g f'_l z'(\cdot)} = \frac{-u'_i + \sigma \lambda^2_g (z'' f''_l z' + f''_l z'') + (1 - \sigma) \lambda^2_g z'' f''_l z''}{(1 - \sigma) \lambda^2_g [f'_l z'' + z'' f''_l z'']} \tag{24}
\]

and

\[
\frac{dg_h}{dg_l} \bigg|_{u'_i = \lambda_g f'_l z'(\cdot)} = \frac{-u'_i}{u'_h} + \frac{\sigma \lambda^2_h [f'_l z'' + z'' f''_l z'']}{(1 - \sigma) \lambda^2_h [f'_l z'' + z'' f''_l z'']} \tag{25}
\]

which are both negative. As the effects of \( \lambda_g, \lambda_t, \sigma \), and \( \delta \) in the impact philanthropy model are qualitatively identical to those in the public goods philanthropy model (although they differ quantitatively between the two income groups), we only summarize them here and refer the reader to the Appendix for their expressions. Specifically, assuming that \( z'' < 0 \), we have that both types of donors contribute more at high \( \lambda_g \) and high \( \sigma \) but less at high \( \lambda_t \), the low-income donors contribute more at high \( \delta \), and the high-income donors contribute more at low \( \delta \); if \( z'' = 0 \), both types of donors become unresponsive to changes in \( \lambda_t \) and \( \sigma \) and the high-income donors become unresponsive to changes in \( \delta \) (this unresponsiveness also arises in the public good philanthropy model but under the assumption that both \( z'' \) and \( f'' \) are equal to zero). Contrary to what happens in the public goods philanthropy model, however, income does affect the extent to which donors respond to changes in \( \lambda_g, \lambda_t, \) and \( \sigma \).

When we analyze the impact of taxation for \( f'' < 0 \), we confirm the result in Duncan (2004) that, when trust is not in question (i.e., \( \lambda_g = \lambda_t = 1 \)), impact philanthropists reduce their contributions by less than a dollar for each dollar increase in taxes (there is less than perfect crowding out) but also establish that the less than 100 percent crowding out result holds even in instances in which donors distrust the charitable
organization as much as they distrust the government (i.e., \( \lambda_g = \lambda_t < 1 \)) and/or the marginal physical product of contributions is constant (i.e., \( z'' = 0 \) or \( z' = z'_i = z'_h \)). The crowding out effect of taxation in the impact philanthropy model is given by

\[
\frac{dg_i}{dt} = -\frac{(u''_i + \lambda^2_g z' f''_{-i} z'_i) \{ u''_i + \lambda_g \lambda_t (z'_i z_i' + f'_{-i} z'_{-i}) \} - \lambda^2_g (d_i - \sigma) \Phi_t}{D^{IP}} = -\frac{N^{IP}_i}{D^{IP}} < 0, \tag{26}
\]

for \( i, -i = l, h \) and \( i \neq -i \), where \( D^{IP} > 0 \) as shown in the Appendix, \( d_l = 1, d_h = 0 \), and

\[
\Phi_t = u''_h [z' f''_h (z' - z'_h) + f'_{-i} z'_{-i}] - u''_i [z' f''_i (z' - z'_i) + f'_{-i} z'_{-i}]. \tag{27}
\]

We can then show that \( D^{IP} - N^{IP}_i > 0 \), and thus conclude that \(-1 < \frac{dg_i}{dt} < 0 \) for \( i = l, h \), when \( \lambda_g = \lambda_t = \lambda \) and/or \( z'' = 0 \). Writing out the difference of interest as

\[
D^{IP} - N^{IP}_i = \lambda_g u''_i (\lambda_g - \lambda_t) (z'_i f''_{-i} z'_i + f'_{-i} z'_{-i}) + \lambda^2_g (\sigma_i \lambda_g - \lambda_t) z'_i f''_{-i} z'_i z''_{-i} f''_i z'_i + \lambda_g \lambda_t u''_{-i} z'_i f''_i z'_i
\]

\[
+ \lambda^4 (1 - \sigma_i) (z'_i f''_{-i} z'_i + f'_{-i} z'_{-i}) z'_i f''_{-i} z'_i z''_{-i} f''_i z'_i [\lambda_t (z'_i - z'_h) + \sigma_i \lambda_g z'_i], \tag{28}
\]

for \( i, -i = l, h \) and \( i \neq -i \), where \( \sigma_i = \sigma \) and \( \sigma_h = 1 - \sigma \), we note that every item on the right-end side of (28) has a positive effect on \( D^{IP} - N^{IP}_i \) but the first two items which can increase or decrease the difference depending on the relationship between the two trust parameters. In particular, \emph{ceteris paribus}, the largest possible negative (positive) effect on \( D^{IP} - N^{IP}_i \) arises when \( \lambda_t = 1 \) (\( \lambda_g = 1 \)), a conclusion that resonates with the result in the public goods philanthropy model that there is more than 100 percent crowding out when \( \lambda_t > \lambda_g \) and that, the larger the difference between \( \lambda_t \) and \( \lambda_g \) is, the greater the extent to which crowding out exceeds (falls short of) the 100 percent level. If \( z'' = 0 \), which, unlike in Duncan (2004), does not collapse the impact philanthropy model to the warm-glow model unless \( \lambda_g = 1 \), the difference measuring the extent of crowding out reduces to

\[
D^{IP} - N^{IP}_i = \lambda^2_g z'_i f''_i z'_i (u''_i + \lambda^2_g z'_i f''_i z'_i) > 0, \tag{29}
\]

which is independent of \( \lambda_t \); when \( \lambda_g = \lambda_t = \lambda \), the difference becomes

\[
D^{IP} - N^{IP}_i = \lambda^4 (1 - \sigma_i) \{ z'_i f''_i z'_i (z'_i f''_{-i} z'_i + f'_{-i} z'_{-i}) - z'_i f''_{-i} z'_i [z'_i f''_i (z' - z'_i) + f'_{-i} z'_{-i}] \} +
\]

\[
+ \lambda^2 u''_{-i} z'_i f''_{-i} z'_i + \lambda^4 \sigma_i z'_i f''_i z'_i f''_{-i} z'_i > 0. \tag{30}
\]

By (28), for \( z'' < 0 \) and \( \lambda_t > \lambda_g \), we can then envision situations in which impact philanthropists respond to a tax increase by reducing their voluntary contributions by more than the tax increase, that is, there is more than 100 percent crowding out. Although analytical conditions under which more than 100 percent crowding out also arises in the impact philanthropic model are difficult to obtain even if we were to specify the functional form of the benefits from private consumption and philanthropy, (28) clarifies that the relationship among \( \lambda_g, \lambda_t, \) and \( \sigma_i \) is key to determining the magnitude of the crowding out effect. With this in mind, we set out below to illustrate the possibility numerically and graphically. In addition to highlighting that more than 100 percent crowding out is plausible in the impact philanthropy model, we also show that trust plays a greater role in this model than in the public goods model in that, under equality between the two trust parameters, the crowding out effect becomes invariant with respect to trust in the latter model but not in the former model.
representing donor $i$’s preferences over private consumption and philanthropy, where $e$ denotes the charitable good’s endowment, we illustrate crowding out effects under parametric assumptions: under (a) we allow $t$ to vary from 0 to 0.35 and demonstrate that $\frac{d\gamma}{dt} > 1$ for $i = l, h$; under (b), we allow $t$ to vary from 0 to 0.4 for $\lambda = 0.6$ and $\lambda = 1$ and demonstrate that $\frac{d^2\gamma}{d\alpha^2} \neq 0$ for $i = l, h$.\footnote{Specifically, for (a) $\alpha = \frac{1}{2}$, $\rho = 2$, $e = 1$, $d = 0.7$, $\sigma = 0.6$, $\lambda_g = 0.3$, and $\lambda_l = 1$, and $t$ varies from 0 to 0.35; for (b) $\alpha = \frac{1}{2}$, $\rho = 1$, $e = 1$, $d = 0.7$, $\sigma = 0.7$, and $\lambda_g = \lambda_l = \lambda$, and $t$ varies from 0 to 0.4 at both $\lambda = 0.6$ and $\lambda = 1$.} We sum up the results in Figures 3a and 3b, where we plot the absolute values of the average crowding out effect over incremental changes in $t$ equal to 0.05 from $t = 0$ to the highest $t$ value that ensures an interior solution, that is, $\delta - g_l - t > 0, 1 - g_h - t > 0, g_l > 0$, and $g_h > 0$. Hence, more than 100 percent crowding out can also come about in the impact philanthropy model when there is complete trust in the government but little trust in the charitable organization, the crowding out effect differs between the two income-differentiated groups of donors, and the crowding out effect remains dependent on trust when governments’ trustworthiness coincides with charitable organizations’ trustworthiness.

2.2 A Socially Motivated Philanthropy Model

To this point, we have considered the roles of incomplete institutional trust and ability-to-pay in existing charitable giving models which emphasize the warm-glow feeling of giving, the benefit of the public good giving contributes to, or the benefit of the impact of giving. Another possible explanation for philanthropy is that individuals may voluntarily give to gain social approval from their peers (or avoid social disapproval). In defining social approval, we account for: (i) the individual’s contribution, (ii) the difference between the individual’s contribution and the average contribution, and (iii) the benefit of contributing. Differently from previous studies of social motivation, however, we specify (i) and (ii) in terms of the share of income spent on the public good as opposed to the level of income spent on the public good. With $\beta$ denoting the weight assigned to the frame of reference (i.e., the ratio of average contribution to average income), we can then write $f_i$ as

$$ f_i \equiv s_i = s \left( \lambda \left( \frac{g_i}{w_i} - \frac{\overline{g}}{\overline{w}} \right) w_i \right), $$

where $\lambda$ reflects the benefit from a unit increase in society’s average contribution under the assumption that the production of the charitable good is linear ($\lambda = \lambda_g$ but we can drop the subscript as the distinction between $\lambda_g$ and $\lambda_l$ is not relevant here), $\overline{g} = \sigma g_l + (1 - \sigma) g_h$, and $\overline{w} = \sigma \overline{\vartheta} + (1 - \sigma)$. If ability-to-pay matters in social considerations (i.e., $\overline{\vartheta} = \delta$), the frame of reference differs between the two income groups in that society’s average contribution is scaled according to the decision-making agent’s income relative to society’s average income.

Socially motivated philanthropists thus maximize

$$ V_i = u \left( c_i \right) + s \left( \lambda \left( \frac{g_i}{w_i} - \frac{\overline{g}}{\overline{w}} \right) w_i \right) $$

(33)
subject to (5). Using the qualifications about \( w_i \) noted above, and assuming that \( \beta + \sigma(1 - \vartheta) < 1 \), we can express the Nash equilibrium as satisfying

\[
u'_l = s'_l \lambda \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \vartheta \quad \text{and} \quad \nu'_h = s'_h \lambda \left( 1 - \frac{\beta}{w} \right),
\]

which, combined, give that

\[
u'_l = \frac{s'_l}{s'_h} \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \vartheta = \frac{s'_l}{s'_h} \left( 1 - \frac{\beta}{w} \right).
\]  

(35)

Upon inspection of (35), we obtain

**Proposition 4** In the socially motivated philanthropy model, the contribution levels of the two income groups are strategic complements and, provided that ability-to-pay matters in social considerations, low-income donors may end up contributing more than high-income donors.

We capture the strategic relationship between the choices of the two types of philanthropists in the slopes of their reaction functions which, using the total differentiation results in the Appendix, we can write as

\[rac{dg_h}{dg_l} \bigg|_{u'_l = s'_l \lambda \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \vartheta} = \frac{-a_{SP}^{g_h}}{a_{SP}^{g_l}} \frac{\nu''_l + \lambda^2 s''_l \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \left( \frac{1}{\vartheta} - \frac{\sigma \beta}{w} \right) \vartheta^2}{\lambda^2 s''_l \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \frac{(1-\sigma\beta)\beta}{w} \vartheta^2} > 0
\]

and

\[rac{dg_h}{dg_l} \bigg|_{u'_h = s'_h \lambda \left( 1 - \frac{\beta}{w} \right)} = \frac{-a_{SP}^{g_h}}{a_{SP}^{g_l}} \frac{\lambda^2 s''_h \left( 1 - \frac{\beta}{w} \right) \frac{\sigma \beta}{w}}{\nu''_h + \lambda^2 s''_h \left( 1 - \frac{\beta}{w} \right) \frac{1 - (1-\sigma\beta)\beta}{w}} > 0.
\]

(36)

(37)

To prove that low-income donors may contribute more, it suffices to show that the Nash equilibrium condition in (35) does not necessitate that \( g_l < g_h \). Later in this subsection, and for illustrative purposes to re-enforce key results, we solve the model with explicit functional forms and show that, depending on values of key parameters (e.g., \( \sigma \) and \( \beta \)), both a positive gap and a negative gap between \( g_h \) and \( g_l \), as well as equal contributions, can support the Nash equilibrium. Letting \( b = \left( \frac{1}{\vartheta} - \frac{\beta}{w} \right) \frac{a}{1 - (1-\sigma\beta)\beta} \geq 1 \), we know that, in equilibrium, \( \frac{u'_l}{u'_h} \geq \frac{s'_l}{s'_h} \). If donors do not take their ability-to-pay into account in their social considerations (this is equivalent to setting \( \vartheta = 1 \)), \( b = 1 \) and, in equilibrium, \( \frac{u'_l}{u'_h} = \frac{s'_l}{s'_h} \). If \( g_l > g_h \), \( s'_l < s'_h \) so that \( u'_l < u'_h \) for which we need that \( g_h - g_l > 1 - \delta > 0 \); we thus have a contradiction and can conclude that \( g_l > g_h \) does not represent a feasible solution. If \( g_l < g_h \), on the other hand, \( s'_l > s'_h \) so that \( u'_l > u'_h \) for which we need that \( g_h - g_l < 1 - \delta \), a solution that is both feasible and in line with the one arising in the impact philanthropy model. If, however, the frame of reference in the social approval function varies by income (that is, ability-to-pay matters), \( b > 1 \) and both \( g_l - g_h \geq 0 \) and \( g_h - g_l > 0 \) are possible solutions. In equilibrium, \( \frac{u'_l}{u'_h} > \frac{s'_l}{s'_h} \) so that a number of possibilities can arise: (i) \( \frac{u'_l}{u'_h} = \frac{s'_l}{s'_h} > 1 \); \( \frac{s'_l}{s'_h} \); (ii) \( \frac{s'_l}{s'_h} < b \frac{s'_l}{s'_h} = \frac{u'_l}{u'_h} < 1 \); (iii) \( \frac{s'_l}{s'_h} < b \frac{s'_l}{s'_h} = \frac{u'_l}{u'_h} = 1 \); (iv) \( \frac{u'_l}{u'_h} = b \frac{s'_l}{s'_h} > \frac{s'_l}{s'_h} > 1 \); (v) \( \frac{u'_l}{u'_h} = b \frac{s'_l}{s'_h} > \frac{s'_l}{s'_h} = 1 \). For \( g_l - g_h \geq 0 \) to obtain in equilibrium, \( \frac{s'_l}{s'_h} < 1 \) and \( \frac{u'_l}{u'_h} > 1 \) so that the relationship in (i) holds; for

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\(^{14}\)This condition is automatically satisfied if ability-to-pay does not matter in social considerations (i.e., \( \vartheta = 1 \)).
\[ g_l - g_h < 0 \] to obtain in equilibrium, the relationship in (i), (iv), or (v) may apply, which would require that \( 0 < g_h - g_l < (1 - \delta)\beta \frac{\varphi}{\varphi} < 1 - \delta, \) \( (1 - \delta)\beta \frac{\varphi}{\varphi} < g_h - g_l < 1 - \delta, \) or \( g_h - g_l = (1 - \delta)\beta \frac{\varphi}{\varphi} < 1 - \delta, \) respectively.\(^{15}\)

The possibility for \( g_l > g_h \) is important in that data typically show (e.g., Figure 1b) that low-income households tend to donate a larger share of their income than high-income households, an observation which existing philanthropy models cannot easily support. In the socially motivated philanthropy model, low-income individuals can give more in absolute terms than high-income individuals, and the U-shaped pattern of giving can thus follow more easily and naturally. In attempting to provide an intuitive explanation of the \( g_l > g_h \) result, we consider how parametric changes affect the likelihood of the result by closely examining the above condition (i). We then arrive at\

**Proposition 5** In a world of socially motivated philanthropists who account for ability-to-pay in their social considerations, an equilibrium involving a \( g_l \) level in excess of the \( g_h \) level is more likely to arise at high \( \sigma \), high \( \beta \), and/or low \( \lambda \). The effect of \( \delta \) on the likelihood that \( g_l > g_h \) is, on the other hand, ambiguous.

Under condition (i), \( s'_l(R_l) < s'_h(R_h) \), where \( R_i = \lambda \left( \frac{g_i}{w_i} - \beta \frac{\varphi}{\varphi} \right) w_i \) for \( i = l, h \), which requires that \( R_l - R_h = \frac{\lambda}{\varphi} \left\{ [1 - s(1 - \beta)(1 - \delta)](g_l - g_h) + \beta (1 - \delta) g_h \right\} > 0 \). If our starting point is a situation in which \( g_l = g_h \), we know that \( R_l > R_h \) and \( s'_l < s'_h \); as \( R_l - R_h \) is increasing in \( g_l \) and decreasing in \( g_h \), a decrease in \( s'_l \) via an increase in \( R_l \) necessitates either an increase in \( g_l \) or a decrease in \( g_h \) or a combination of the two changes, and thus a \( g_l \) level in excess of the \( g_h \) level. When \( g_l = g_h \), \( R_l - R_h = b \) and \( b \frac{s'_l(R_l)}{s'_h(R_h)} \) is an increasing function of \( b \); we can then deduce the effects of parametric changes on the likelihood of the \( g_l - g_h > 0 \) result by studying how such changes affect \( b \): a parametric change that increases \( b \) but does not affect \( \frac{w_l}{w_h} \) triggers an increase in \( g_l \) relative to \( g_h \) which, in turn, decreases \( \frac{s'_l}{s'_h} \) to restore equilibrium under condition (i). It is straightforward to show that \( b \) is increasing in \( \sigma \) and \( \beta \) and decreasing in \( \varphi = \delta \); a change in \( \delta \), however, also affects \( \frac{w_l}{w_h} \) and in exactly the same direction as it affects \( b \), so that, while we can conclude that \( g_l \) is more likely to exceed \( g_h \) at high \( \sigma \) and/or \( \beta \), we cannot make a conclusive statement about the effect of \( \delta \). On the other hand, the higher the \( \lambda \), the less likely it is for \( g_h \) to fall short of \( g_l \): as \( R_l - R_h \) is increasing, and \( \frac{s'_l}{s'_h} \) is thus decreasing, in \( \lambda \), but \( b \) and \( \frac{w_l}{w_h} \) are directly independent of \( \lambda \), an increase in \( \lambda \) calls for a decrease in \( R_l - R_h \), or an increase in \( g_h \) relative to \( g_l \), to restore equilibrium under condition (i).

Intuitively, when the social norm, or the weight assigned to it, increases and/or the cost of social nonconformity decreases, social approval becomes less relevant and the benefit of acting socially responsibly by contributing to the charitable good thus decreases. In a world in which ability-to-pay matters in social considerations, the social norm is not uniform between the two income groups but is lower for the low-income donors as it reflects the contribution level such that the share of income allotted to the charitable good is equal to the ratio between society’s average contribution and society’s average income. Specifically, the social norm consists of two parts: society’s average contribution as a share of society’s average income, which is common between the two income groups, and the donor’s income level; hence, the lower the donor’s

\[^{15}\text{For relationships (ii) and (iii), } \frac{w_l}{w_h} \leq 1 \text{ requires that } g_h - g_l \geq 1 - \delta \text{ while } \frac{s'_l}{s'_h} < 1 \text{ requires that } g_h - g_l < (1 - \delta)\beta \frac{\varphi}{\varphi} < 1 - \delta; \text{ as it is not possible to satisfy both requirements, these relationships cannot hold in equilibrium.}\]
income level, the less relevant the social norm is. An increase in $\beta$ increases the relevance of the frame of reference or social norm, an increase in $\sigma$ increases the ratio of society’s average contribution to society’s average income provided that $g_t > \delta g_h$, and a decrease in $\lambda$ decreases the cost of not adhering to the social norm; in each of these cases, social approval declines but, as the effect depends on the donor-specific component of the social norm (i.e., the donor’s income level), the low-income donors experience a smaller reduction than the high-income donors. Finally, an increase in $\delta$ decreases the high-income donors’ social norm via an increase in society’s average income which, in turn, reduces the common component of the social norm, while it increases the low-income donors’ social norm via an increase in the donor-specific component of the social norm which more than offsets the decrease in the common component; at the same time, however, the increase in $\delta$ has a direct positive effect on both the private consumption and the voluntary contribution of the low-income donors, and the overall effect on the likelihood that $g_t > g_h$ is thus unclear.

Through a comparative statics analysis with focus on the effect of taxation, we obtain

**Proposition 6** When donors are motivated to contribute to a charitable organization by social pressure, there is less than 100 percent crowding out if $s_i'' < 0$ and complete crowding out if $s_i'' = 0$.

Using the details in the Appendix, we can write the effects of a tax increase on the contribution levels of the two types of donors as

$$\frac{dg_t}{dt} = \frac{a_{1t}^{SP} a_{2g_t}^{SP} - a_{1i}^{SP} a_{2g_t}^{SP}}{D^{SP}} = -1 + \frac{N_t^{SP}}{D^{SP}} < 0$$

and

$$\frac{dg_h}{dt} = \frac{a_{1t}^{SP} a_{2g_h}^{SP} - a_{1i}^{SP} a_{2g_h}^{SP}}{D^{SP}} = -1 + \frac{N_h^{SP}}{D^{SP}} < 0,$$

where

$$N_t^{SP} = \lambda^4 s_i'' s_h'' (1 - \beta) \left( \frac{1}{\theta} - \frac{\beta}{\theta} \right) \left( 1 - \frac{\beta}{\theta} \right) \theta + \lambda^2 u_i'' s_i'' \left( \frac{1}{\theta} - \frac{\beta}{\theta} \right)^2 \varphi^2 > 0$$

and

$$N_h^{SP} = \lambda^4 s_i'' s_h'' (1 - \beta) \left( \frac{1}{\theta} - \frac{\beta}{\theta} \right) \left( 1 - \frac{\beta}{\theta} \right) \theta + \lambda^2 u_i'' s_h'' \left( 1 - \frac{\beta}{\theta} \right)^2 > 0,$$

with

$$D^{SP} - N_t^{SP} = u_i'' u_h'' + \lambda^2 u_i'' s_h'' \left( 1 - \frac{\beta}{\theta} \right) \left[ 1 - \frac{(1 - \sigma) \beta}{\theta} \right] + \lambda^2 u_i'' s_h'' \left( \frac{1}{\theta} - \frac{\beta}{\theta} \right) \sigma^2 \frac{(1 - \sigma) \beta}{\theta} > 0$$

and

$$D^{SP} - N_h^{SP} = u_i'' u_h'' + \lambda^2 u_i'' s_h'' \left( 1 - \frac{\beta}{\theta} \right) \left[ 1 - \frac{(1 - \sigma) \beta}{\theta} \right] + \lambda^2 u_i'' s_h'' \left( 1 - \frac{\beta}{\theta} \right) \sigma^2 > 0,$$

so that $\frac{N_t^{SP}}{D^{SP}} < 1$ and $\frac{N_h^{SP}}{D^{SP}} < 1$. If the utility effect of social approval is increasing but at a constant rate, that is, $s_i'' = 0$, a tax increase results in a 100 percent crowding out of voluntary contributions.

We sum up the effects of changes in the other parameters of the model on contribution levels in

**Proposition 7** In the socially motivated philanthropy model, voluntary contributions are decreasing in $\beta$ and $\sigma$ and increasing in $\lambda$ and $\delta$. When ability-to-pay does not matter, low-income donors are more responsive to a change in $\delta$ than high-income donors; however, when ability-to-pay matters, the high-income donors are more responsive to a change in $\delta$.

---

16In the previous paragraph, we argue that an increase in $\sigma$ positively affects the likelihood that $g_t > g_h$ starting from a position of equality between the two contribution levels; under equality, we have that $g_t > \delta g_h$. 
Borrowing again from the Appendix, we can summarize the various effects in

\[
\begin{align*}
\frac{dg_l}{d\lambda} &= \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0 \quad \& \quad \frac{dg_h}{d\lambda} = \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0, \\
\frac{dg_l}{d\sigma} &= \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} < 0 \quad \& \quad \frac{dg_h}{d\sigma} = \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} < 0, \\
\frac{dg_l}{d\beta} &= \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} < 0 \quad \& \quad \frac{dg_h}{d\beta} = \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} < 0, \\
\frac{dg_l}{d\delta} \bigg|_{\vartheta=1} &= \frac{a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0 \quad \& \quad \frac{dg_h}{d\delta} \bigg|_{\vartheta=1} = -\frac{a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0, \\
\text{and} \quad \frac{dg_l}{d\delta} &= \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} < 0 \quad \& \quad \frac{dg_h}{d\delta} = \frac{a_{1\lambda}a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0.
\end{align*}
\]

When ability-to-pay does not matter (i.e., \(\vartheta = 1\)),

\[
\frac{dg_h}{d\delta} - \frac{dg_l}{d\delta} = -\frac{u''_h \left[u''_h + \lambda^2 s''_h (1-\beta)^2\right]}{D^{SP}} < 0; \tag{43}
\]

when ability-to-pay matters (i.e., \(\vartheta = \delta\)), we can show that

\[
\begin{align*}
\frac{dg_l}{d\delta} &= \frac{dg_l}{d\delta} \bigg|_{\vartheta=1} + \frac{dg_l}{d\delta} < \frac{dg_l}{d\delta} \bigg|_{\vartheta=1} \quad \& \quad \frac{dg_h}{d\delta} &= \frac{dg_h}{d\delta} \bigg|_{\vartheta=1} + \frac{dg_h}{d\delta} > \frac{dg_h}{d\delta} \bigg|_{\vartheta=1}, \tag{44}
\end{align*}
\]

that is, low-income (high-income) donors are less (more) responsive to a change in \(\delta\); although not as responsive, however, low-income donors remain positively affected by a change in \(\delta\) as

\[
\frac{dg_l}{d\delta} = \frac{(a_{1\lambda} + a_{1\delta}) a_{2\lambda} - a_{1\lambda}a_{2\lambda}}{D^{SP}} > 0. \tag{45}
\]

While the effect of \(\lambda\) is consistently positive across the public goods, impact, and socially motivated philanthropy models, and the effect of \(\beta\) only applies in the last model so that we drop it from consideration, the effects of \(\sigma\) and \(\delta\) differ; in particular, the effect of \(\sigma\) is positive in the public goods and impact models but negative in the socially motivated model and the effect of \(\delta\) on \(g_h\) is negative in the public goods and impact models but positive in the socially motivated model. Furthermore, while the effect of \(\lambda\) results even when \(f'' = z'' = 0\) in the public goods model and when \(s''_i = 0\) (for \(i = l, h\)) in the socially motivated model (that is, when the marginal utilility of philanthropy is constant and the public good is produced with a constant returns to scale technology), the effects of \(\sigma\) on \(g_l\) and \(g_h\) and the effect of \(\delta\) on \(g_h\) vanish in the former but not in the latter;\(^{17}\) the effect of \(t\), on the other hand, remains in both models, decreasing in the former and increasing in the latter to the perfectly crowding out level.\(^{18}\)

\(^{17}\)The effect of \(\delta\) on \(g_h\) would also disappear in the socially motivated philanthropy model if donors did not take into account their ability-to-pay in relation to society's ability-to-pay in their social considerations.

\(^{18}\)Although we model the benefit of contributing in the social approval function to reflect the approval rate which we can simply express in terms of \(\lambda_s\) under the assumption that \(z(z)\) is a linear function, we do not constrain ourselves to comparing the socially motivated philanthropy model with the public goods model only when \(z'' = 0\) given that other modelling options for the benefit of contributing exist as we note in the general discussion in the Model section (e.g., a fixed amount equal to the benefit of leaving in the ideal state which we could scale down by \(\lambda_s\)).
2.3 A Numerical Example

To emphasize key features of socially motivated philanthropy and analytically explore conditions under which low-income donors may contribute more than high-income donors, we solve the socially motivated philanthropy model assuming specific functional forms for the utility benefits from consumption and social approval. Specifically, we assume that

$$V_i = \ln (1 + c_i) + \ln (1 + s_i),$$  \hspace{1cm} (46)

where \(c_i = w_i - g_i - t\) and \(s_i = \lambda \left( \frac{g_i}{w_i} - \beta \frac{\bar{w}}{w_i} \right) w_i\). In equilibrium, we must then have that

$$g_h = \frac{\lambda (w - \beta) (1 + \delta - t)}{\lambda (1 - \sigma) \beta \bar{w}} g_l + \frac{2 \bar{w} - \beta (1 + \sigma)}{\beta (1 - \sigma) \bar{w}} g_l,$$  \hspace{1cm} (47)

and

$$g_h = \frac{\lambda (w - \beta) (2 - t) - \lambda \bar{w}}{-\lambda (1 - \sigma) \beta (2 - \sigma)} g_l + \frac{\sigma \beta}{\bar{w} - \beta (2 - \sigma)} g_l,$$  \hspace{1cm} (48)

which we can combine to obtain an expression for the contribution gap that reads as

$$g_h - g_l = \frac{\lambda (1 - \delta) (\bar{w}^2 + \beta^2 \bar{w}) - \lambda \beta (1 - \delta) (1 + \bar{w}) + \beta (1 - \bar{w})}{\lambda \beta (1 - \delta) (2 - \sigma) (\bar{w} - \beta) + \sigma \beta (1 - \delta) \bar{w}}.$$

Equations (47) and (48) represent type-\(l\) and type-\(h\) donors’ reaction functions which, in \((g_l, g_h)\) space, we can describe as positively sloped straight lines; given that the line depicting (47) is steeper than that depicting (48), we must have that the vertical intercept of the former is smaller while its horizontal intercept is larger than the corresponding intercepts of the latter for the two lines to cross in the positive quadrant so that the equilibrium may entail interior solutions for \(g_l\) and \(g_h\) (i.e., \(0 < g_l < \delta\) and \(0 < g_h < 1\)).

If \(\bar{w} = 1\), the contribution gap in (49) reduces to

$$0 < g_h - g_l = \frac{(1 - \delta) (1 - \beta)}{(2 - \beta)} < (1 - \delta),$$  \hspace{1cm} (50)

if \(\bar{w} = \delta\), the contribution gap is equal to

$$g_h - g_l = \frac{1 - \delta}{\lambda (1 - \sigma) \beta (2 - \sigma) (\bar{w} - \beta)} \left\{ \lambda (\bar{w}^2 + \beta^2 \bar{w}) - \beta [\lambda (1 + \bar{w}) + 1 + \bar{w}] \right\}.$$  \hspace{1cm} (51)

which needs not be positive, \(^{20}\) and, in fact,

$$g_h > g_l \quad \text{if} \quad \lambda \left( \frac{1 - \beta}{\bar{w}} \right) \left( 1 - \frac{\delta \beta}{\bar{w}} \right) \frac{\bar{w}}{\beta} > 1,$$

$$g_h = g_l \quad \text{if} \quad \lambda \left( \frac{1 - \beta}{\bar{w}} \right) \left( 1 - \frac{\delta \beta}{\bar{w}} \right) \frac{\bar{w}}{\beta} = 1,$$

$$g_h < g_l \quad \text{if} \quad \lambda \left( \frac{1 - \beta}{\bar{w}} \right) \left( 1 - \frac{\delta \beta}{\bar{w}} \right) \frac{\bar{w}}{\beta} < 1.$$  \hspace{1cm} (52)

\(^{19}\) That the two lines cross in the positive quadrant is a necessary but not sufficient condition for interior solutions; in fact, it only ensures that \(g_l > 0\) and \(g_h > 0\).

\(^{20}\) If we write \(g_h - g_l = (1 - \delta) \frac{N}{\bar{w}}\), we can easily show that, when \(N > 0\) (that is, \(g_h > g_l\)), \(g_h - g_l < 1 - \delta\) as \(N - D = -\lambda \bar{w} [\sigma \beta (1 - \delta) + (\bar{w} - \beta)] - \beta \bar{w} < 0\).
Whether the contribution gap is positive or negative or even zero does not depend on \( t \) but is increasing in \( \lambda \) and decreasing in \( \beta \) and \( \sigma \): \(^{21}\)

\[
\frac{d\Gamma}{d\lambda} = \frac{(\bar{w} - \beta)(\bar{w} - \delta \beta)}{\bar{w}^2} > 0, \quad \frac{d\Gamma}{d\beta} = -\lambda \frac{(\bar{w}^2 - \delta \beta^2)}{\bar{w}^2} < 0, \quad \text{and} \quad \frac{d\Gamma}{d\sigma} = -\lambda \frac{(1 - \delta)(\bar{w}^2 - \delta \beta^2)}{\bar{w} \beta} < 0, \tag{53}
\]

where \( \Gamma = \lambda (1 - \frac{\beta}{\bar{w}}) (1 - \frac{\delta \beta}{\bar{w}}) \bar{w} - 1 \), so that the lower the \( \lambda \), the higher the \( \beta \), and/or the higher the \( \sigma \), the more likely it is for the low-income donors to contribute more than the high-income donors. However, the effect of \( \delta \) on whether the gap is positive or negative is ambiguous as the sign of

\[
\frac{d\Gamma}{d\delta} = \lambda \left[ \frac{\bar{w}^2 (\sigma - \beta) + (1 - \sigma) \beta^2}{\bar{w}^2 \beta} \right] \tag{54}
\]

is unclear, but we know that \( \sigma \geq \beta \) is sufficient, although not necessary, for \( \frac{d\Gamma}{d\delta} > 0 \) and \( \sigma < \beta \) is necessary, although not sufficient, for \( \frac{d\Gamma}{d\delta} < 0 \). If it is reasonable to assume that \( \sigma \geq 0.5 \) and \( \beta \leq 0.5 \), that is, at least 50 percent of the population falls in the low-income category and the weight assigned to the frame of reference in the social approval function is at most 50 percent, \( \frac{d\Gamma}{d\delta} > 0 \) and, the larger the income gap between the two groups (i.e., the lower the \( \delta \)), the less likely it is for the low-income donors to contribute more than the high-income donors. A decrease in \( \delta \) affects the importance of the frame of reference differently between the two income groups: for both types of donors, the change increases the importance of the frame of reference through a decrease in society’s average income; however, for the low-income donors, the change has an additional effect which works in the opposite direction, and is larger, to decrease the importance of the frame of reference through a decrease in own income relative to society’s average income. As an increase (decrease) in the importance of the frame of reference reduces (increases) the marginal social approval benefit of contributing, a decrease in \( \delta \) reduces the incentive to contribute for the high-income donors while it increases it for the low-income donors. \(^{22}\)

Although under more restrictive parameter values, it is possible for low-income donors to end up contributing more than high-income donors in response to an increase in \( \delta \). Letting \( \beta = \gamma \sigma \), with \( \gamma > 1 \), and noting that \( \frac{d^2\Gamma}{d\delta^2} = -\frac{2\gamma^2(1-\sigma)\lambda}{\bar{w}^2} < 0 \), we have that \( \frac{d\Gamma}{d\delta} < 0 \) for \( \delta > \delta^* \), where \( \delta^* = \frac{1-\sigma}{\gamma+1} + \gamma \sqrt{\frac{(1-\sigma)}{\sigma(\gamma-1)}} \).

For \( \delta^* > 0 \), \( \sigma > \frac{1}{\gamma+1} \); for \( \delta^* < 1 \), \( \sigma < \frac{1}{\gamma} < \frac{1}{\gamma-1} \) if \( \gamma > 2 \) and \( \sigma < \frac{1}{\gamma-1} \leq \frac{1}{\gamma} \) if \( \gamma \leq 2 \). As we show below, a change in \( \delta \) affects \( g_l \) and \( g_h \) in the same way independently of the sign of \( \frac{d\Gamma}{d\delta} \); however, when \( \frac{d\Gamma}{d\delta} > 0 \), the effect on \( g_h \) is larger than the effect on \( g_l \), so that \( g_l \) is likely to exceed \( g_h \) at lower \( \delta \) values, while, when \( \frac{d\Gamma}{d\delta} < 0 \), the effect on \( g_h \) is smaller than the effect on \( g_l \), so that \( g_l \) is likely to exceed \( g_h \) at higher \( \delta \) values.

In a more technical manner, we can analyze the gap in contributions by solving for the \( \beta \) value (\( \beta^* \)), expressed in terms of the other parameters of the model, such that the gap is zero, namely,

\[
\beta^* = \frac{\sqrt{1 + \lambda + \lambda \delta - \sqrt{(1 + \lambda + \delta \lambda)^2 - 4\lambda^2 \delta}}}{2\lambda \delta \xi} < 0.
\]

As \( \frac{d\Gamma}{d\delta} < 0 \), we then have that \( g_h < g_l \) for \( \beta > \beta^* \) and \( g_h > g_l \) for \( \beta < \beta^* \), but \( \beta^* \) depends on \( \sigma \), \( \lambda \), and \( \delta \); in particular, \( \frac{d\beta^*}{d\delta} = \frac{(1-\delta^2)\beta^*}{\bar{w}} < 0 \), \( \frac{d\beta^*}{d\lambda} = \frac{\beta^*}{\xi \bar{w}} > 0 \), and \( \frac{d\beta^*}{d\delta} = \frac{\lambda(1-\delta^2)(1-\sigma)\beta^*}{\delta \xi} > 0 \) if \( \sigma > \frac{2\sqrt{5} - 4}{3\sqrt{5} - 5} \approx 0.28 \).

\(^{21}\) Independently of the sign of the contribution gap, high-income individuals enjoy a higher level of private consumption than low-income individuals, that is, \( c_h > c_l \): when \( g_h > g_l \), \( -\delta > g_h - g_l \) so that \( 1 - t - g_h = c_h > \delta - t - g_l = c_l \); when \( g_h < g_l \), \( 1 - t - g_h = c_h > 1 - t - g_l = 1 - \delta - g_l = c_l \).

\(^{22}\) Our emphasis here is only on the effect of a change in \( \delta \) via or on social approval; in other words, it is on the effect of \( \delta \).
where \((1 + \lambda) < \varphi = \sqrt{(1 + \lambda + \delta \lambda)^2 - 4\delta \lambda^2} < (1 + \lambda + \delta \lambda)\).\(^{23}\) so that, the larger the \(\lambda\) or the smaller the \(\sigma\) or the larger the \(\delta\), the larger \(\beta^*\) is and, thus, the less likely for a type-\(l\) donor to contribute more than a type-\(h\) donor.

To illustrate the above results, we graph reaction functions and corresponding Nash equilibria in Figure 4 through Figure 7 for different values of parameter \(\zeta\), where \(\zeta = \delta, \sigma, \beta, \lambda\), but under the same values for the other parameters. For one of the three values of \(\zeta\), we solve \(\lambda \left(1 - \frac{\beta}{\varphi}\right) \left(1 - \frac{\delta^2}{\varphi}\right) \frac{w}{\beta} = 1\) and call the solution \(\hat{\zeta}\); we then compute the Nash equilibrium for \(\zeta = \hat{\zeta} - \varepsilon, \zeta = \hat{\zeta}\), and \(\zeta = \hat{\zeta} + \varepsilon\), where \(\varepsilon = 0.10\) for \(\zeta = \delta, \sigma\) and \(\varepsilon = 0.05\) for \(\zeta = \beta, \lambda\). In Figure 8, we have reaction functions and corresponding Nash equilibria for three values of \(\delta\), namely, \(\delta = \hat{\delta} - 0.15, \delta = \hat{\delta}, \text{ and } \delta = \hat{\delta} + 0.15\), where \(\hat{\delta}\) satisfies \(\lambda \left(1 - \frac{\beta}{\varphi}\right) \left(1 - \frac{\delta^2}{\varphi}\right) \frac{w}{\beta} = 1\), but under the same values for other parameters which we choose, however, such that \(\frac{dT}{d\delta} < 0\). Hence, the contribution of type-\(l\) donors is more likely to exceed the contribution of type-\(h\) donors in response to a decrease in \(\delta\) in Figure 4, that is, when \(\frac{dT}{d\delta} > 0\), and in response to an increase in \(\delta\) in Figure 8, that is, when \(\frac{dT}{d\delta} < 0\).

For the effects of changes in the values of the model’s parameters, we provide the results of the comparative statics in the Appendix and simply note here that these results are consistent with those derived for the general case; specifically, we find that

\[
\frac{dg_i}{dt} = -\frac{1}{2}, \quad \frac{dg_i}{d\beta} < 0, \quad \frac{dg_i}{d\lambda} > 0, \quad \frac{dg_i}{d\sigma} < 0, \quad \text{and} \quad \frac{dg_i}{d\delta} > 0 \tag{56}
\]

for \(i = l, h\). In Figures 9 through 11, we illustrate the comparative statics’ results: in Figure 9, we allow for both \(\beta\) and \(\delta\) to change; in Figure 10, we consider changes in \(\lambda\) and \(\sigma\); in Figure 11, we look at the contribution profiles for different \(\delta\) values as well as different \(\sigma\) values. In all cases, we assume \(t = 0\) as its effect is constant. In addition to the comparative statics’ results, we include in the Figures the \(\beta^*\) condition which determines whether high-income individuals contribute more or less than low-income individuals. In particular, in Figure 9, we assume that \(\lambda = 1\) and \(\sigma = 0.7\) and show that \(g_i\) is increasing in \(\delta\) but decreasing in \(\beta\) and that, for a given \(\beta\), there is a feasible \(\delta\) value (i.e., \(0 < \delta < 1\)) below which \(g_l > g_h\); unless \(\sigma\) is sufficiently small (see Figure 11), the \(\delta\) value at which \(g_l = g_h\) increases as \(\beta\) increases. In Figure 10, we assume that \(\beta = 0.32\) and \(\delta = 0.7\) and show that \(g_i\) is increasing in \(\lambda\) and decreasing in \(\sigma\) and that the \(\beta\) value above which \(g_l > g_h\) is also increasing in \(\lambda\) and decreasing in \(\sigma\): when \(\sigma = 0.4\), \(g_h > g_l\) over the entire range of feasible \(\lambda\) values; when \(\sigma = 0.6\), \(g_l > g_h\) for \(\lambda \in [0.88, 1]\); when \(\sigma = 0.8\), \(g_l > g_h\) over the entire range of feasible \(\lambda\) values. Finally, in Figure 11, we assume that \(\beta = 0.3\) and \(\lambda = 0.9\) and show that \(g_i\) is decreasing in \(\sigma\) and increasing in \(\delta\), that the \(\beta\) value above which \(g_l > g_h\) is decreasing in \(\sigma\) but may decrease (at very low \(\sigma\) values) or increase (at high \(\sigma\) values) as \(\delta\) increases, and that, for a given \(\delta\), there is a feasible \(\sigma\) value above which \(g_l > g_h\): when \(\delta = 0.6\), \(g_l > g_h\) for \(\sigma \in [0.65, 1]\); when \(\delta = 0.7\), \(g_l > g_h\) for \(\sigma \in [0.88, 1]\).

As a way of summarizing the above results as concisely as possible, we include all possible scenarios

\(^{23}\)Letting \(\Lambda = \lambda \varphi (1 - \delta \zeta) - (1 - \sigma) \varphi \zeta\), we have that \(\frac{d\Lambda}{d\sigma} = \frac{\delta^2 \beta}{\varphi} > 0\) when \(\sigma = \sigma^* = \frac{\varphi \beta - \lambda (1 - \delta \zeta)}{\varphi \beta - \lambda (1 - \delta \zeta)}\), which is the \(\sigma\) value expressed in terms of \(\lambda\) and \(\delta\) that solves \(\Lambda = 0\); hence, \(\Lambda\) is increasing in \(\sigma\) around \(\sigma^*\) and is thus positive for \(\sigma > \sigma^*\). As \(\frac{d\Lambda}{d\lambda} > 0\) and \(\frac{d\Lambda}{d\sigma} > 0\), we know that the largest \(\sigma^*\) value ensues when \(\lambda = \delta = 1\), that is, \(\sigma^* = 0.28\), and, the lower the \(\lambda\) and/or the lower the \(\delta\), the smaller the \(\sigma\) value above which \(\frac{d\Lambda}{d\sigma} > 0\). We then have that \(\sigma > 0.28\) is a sufficient, although not necessary, condition for \(\frac{d\sigma^*}{d\sigma} > 0\).
involving interior solutions in Table 1 for both \( \vartheta = \delta \) and \( \vartheta = 1 \), along with corresponding results for the public goods model which we briefly review below. Independently of whether ability-to-pay matters in social considerations, voluntary contributions are decreasing in \( t, \sigma, \) and \( \beta \) and increasing in \( \delta \) and \( \lambda \); however, while the contribution gap \( (g_h - g_l) \) is decreasing in \( \sigma \) and increasing in \( \delta \) and \( \lambda \) when \( \vartheta = \delta \), it is unaffected by a change in \( \sigma \) and \( \lambda \) and is decreasing in \( \delta \) when \( \vartheta = 1 \).

For comparison purposes, we derive the Nash equilibrium under the same functional assumptions as above, but with the added simplification that the production of the charitable good exhibits constant returns to scale, when individuals are motivated to give by what their giving can accomplish in terms of the aggregate level of the charitable good (i.e., public good or \( z \)). To maximize

\[
V_i = \ln (1 + c_i) + \ln (1 + z),
\]

(57)

where \( c_i = w_i - g_i - t \) and \( z = \lambda_g g_i + \lambda_g G_{-i} + \lambda_t t \), donor \( i \) thus chooses to contribute according to

\[
g_i = \frac{2w_i + \sigma (1 - \delta)}{2} - \frac{1}{2\lambda_g} - \left( \frac{\lambda_g + \lambda_t}{2\lambda_g} \right) t,
\]

(58)

which immediately gives that \( g_i \) is increasing in \( \sigma \) and \( \lambda_g \), decreasing in \( \lambda_t \) and \( t \), and increasing in \( \delta \) for low-income donors (i.e., \( i = l \) and \( w_i = \delta \)) but decreasing in \( \delta \) for high-income donors (i.e., \( i = h \) and \( w_i = 1 \)); the expression in (58) also underscores that the effect of taxation on voluntary contributions is complete crowding out if \( \lambda_g = \lambda_t \), less than complete crowding out if \( \lambda_g > \lambda_t \), and more than complete crowding out if \( \lambda_t > \lambda_g \).

In contrast to philanthropy motivated by social considerations, public goods philanthropy responds positively to a change in \( \sigma \) and, among the high-income donors, negatively to a change in \( \delta \), with the negative effect on the contribution of the high-income donors falling short in magnitude of the positive effect on the contribution of the low-income donors so that, overall, the contribution gap \( (g_h - g_l) \) decreases as the income gap \( (1 - \delta) \) decreases. Although the effect of a change in \( \delta \) on \( g_h \) is positive among socially motivated philanthropists who ignore ability-to-pay in their social considerations, it is smaller than the positive effect on \( g_l \) so that, overall, the contribution gap remains decreasing in \( \delta \) in the socially motivated philanthropy model when \( \vartheta = 1 \) but to a lesser extent than in the public goods philanthropy model. Finally, when \( \vartheta = 1 \), an increase in \( \sigma \) strengthens the effect of \( \delta \) on the contributions of the low-income philanthropists who are motivated by social considerations but weakens the effect on the contributions of the low-income philanthropists who are motivated by what their giving can accomplish in terms of the aggregate level of the charitable/public good (the effect of \( \delta \) on the contributions of the high-income donors is stronger at high \( \sigma \) in both cases, although in opposite directions).

Using that \( z'' = 0 \), we can also obtain, in a quite straightforward manner, the equilibrium contribution for a donor who values the impact of his/her giving and thus maximizes

\[
V_i = \ln (1 + c_i) + \ln (1 + \lambda_g g_i),
\]

(59)

---

\footnote{Given their appealing simplicity, we provide the derivatives describing the effects of the various parameters on the two contribution levels when \( \vartheta = 1 \) directly in Table 1.}

\footnote{As the case for the socially motivated philanthropy model when \( \vartheta = 1 \), we provide the derivatives describing the effects of the various parameters on the two contribution levels in the context of the public goods model directly in Table 1.}
namely,
\[ g_i = \frac{1 + w_i}{2} - \frac{1}{2\lambda_g} - \frac{t}{2}, \] (60)
which corresponds to the warm-glow equilibrium when \( \lambda_g = 1 \). With the Nash equilibria of the various models, we can examine the income profile of the difference between the shares of income donated by the high- and low-income donors, that is, \( g_h - g_l \), to illustrate the potential for explaining the U-shaped pattern of donations. Under the same parametric assumptions, we show in Figure 12a that the warm-glow, public goods, and impact philanthropy models, as well as the socially motivated philanthropy model that does not account for ability-to-pay (i.e., \( \vartheta = 1 \)), all predict that high-income individuals contribute a larger share of their income and that this share is monotonically decreasing in the income gap. Hence, we cannot reconcile the observation that the lowest income groups contribute the highest share of income (as per Figures 1a and 1b) with these theories; furthermore, the non-negativity of the difference in shares leaves no room for a group of lower income individuals to contribute a larger share of their income than a higher income group. However, the socially motivated philanthropy model that accounts for ability-to-pay (i.e., \( \vartheta = \delta \)) yields a non-monotonic difference in the shares of giving, as we show in Figure 12b. In this case, socially motivated low-income individuals give a smaller share of their income when their income differs significantly from that of the high-income individuals but give a larger share of their income when the income gap is not extreme. For large income gaps (i.e., \( 1 - \delta = 0.8 \) in Figure 12b, when we compare the rich group with the poorest group), we see that the richest individuals would give a larger share; however, for small income gaps (i.e., \( 1 - \delta = 0.2 \) in Figure 12b, when we compare the lower-middle income group with the poorest group), we do find support for a situation in which the lower-income group gives a larger proportion of its income, a result that does not arise in the other models. As such, the U-shaped pattern becomes a theoretically viable result in the socially motivated philanthropy model when donors account for their ability-to-pay in their social considerations.26

3 Conclusions

In the literature on the provision of charitable goods, altruism has been motivated by what contributions can accomplish (public goods philanthropy), by the pleasure of giving (warm-glow philanthropy), or by the desire to personally make a difference (impact philanthropy). In the first part of the paper, we extend these motives but allow for income heterogeneity and distrust in the institutional structures involved in the provision/support of the charitable good (i.e., charitable organization and government). As warm-glow philanthropists benefit from what they contribute, their decisions are independent of the income distribution of donors, the income gap between high- and low-income donors, and how reliable the government and the charitable organization are; instead, warm-glow philanthropists only consider their own preferences and budget constraint, increasing their contributions in response to an income increase and decreasing them by the same amount in response to an equal tax increase. Provided that the marginal benefit of contributing is declining in the contribution level, the crowding out effect of taxation is less than complete and the

26 In this comparison, we are retaining the assumption of two income groups for comparability, while in the data there are clearly more than two groups. Nonetheless, the non-monotonicity in the share difference suggests that the socially motivated model that accounts for ability-to-pay can predict the U-shaped pattern in the data whereas the other models, including the socially motivated model that ignores ability-to-pay, cannot.
contribution gap between high- and low-income donors is less than the income gap, so that low-income donors enjoy a lower private consumption level than high-income donors; when the marginal benefit of contributing is constant, there is complete crowding out and the contribution gap coincides with the income gap which implies that the private consumption level is the same between the two types of donors.

Public goods and impact philanthropists are, on the other hand, responsive to changes in income distribution, income gap, and confidence in the government and/or the charitable organization; specifically, they contribute more when the proportion of low-income donors is higher, the income gap is lower, the charitable organization is more reliable, and/or the government is less reliable. While the effects of income distribution and trust are identical for the two types of donors in the public goods philanthropy model, they differ in the impact philanthropy model. As for the crowding out effect of taxation, several possibilities arise in the public goods philanthropy model depending on how reliable the government is relative to the charitable organization, with more (less) than 100 percent crowding out if the government is more (less) trustworthy. Hence, for public goods philanthropists, lack of confidence in the institutions involved in the provision and support of the charitable good does not necessarily condition the extent of crowding out which remains at 100 percent when donors perceive the two institutions to be equally reliable or unreliable. For impact philanthropists, on the other hand, when the level of trust is the same between the two institutions, the crowding out effect is less than complete and dependent on the trust level. Unlike what previous results suggest, it is however possible for impact philanthropists to experience more than complete crowding out if their trust in the government is high while their trust in the charitable organization is low and the marginal physical product of contributions is declining. Finally, in the public goods philanthropy model, the contribution gap is always equal to the income gap so that the two types of donors enjoy the same level of private consumption; in the impact philanthropy model, the contribution gap is always smaller than the income gap, unless the marginal benefit from the impact is constant (in which case the model reduces to the public goods philanthropy model), so that high-income donors consume more of the private good than low-income donors.

In the second part of the paper, we consider a distinct yet plausible motive for charitable contributions that is based on the desire to be seen as socially responsible citizens. To our knowledge, this is the first formal analysis of socially motivated philanthropy but, more importantly, it is the first analysis of social motives in which ability-to-pay and (less crucially) trust are taken into account. Socially motivated decision-making agents derive utility from social approval for engaging in a particular activity that depends on the activity’s societal benefit and on the weighted average of the individual level of engagement and the difference between this level and society’s average level; trust thus enters the social approval function through the activity’s societal benefit whereas ability-to-pay enters through the weighted average. In the context of voluntary contributions to a charitable organization, we introduce trust in a very straightforward manner as a scaler of the benefit of contributing and compute the weighted average in terms of contributions relative to incomes and society’s average contribution relative to society’s average income; for socially-motivated philanthropists who take ability-to-pay into account, the frame of reference is thus not the contribution level they expect to see on average but the proportion of society’s per capita income spent on the charitable good.

Irrespective of whether ability-to-pay enters the social approval function, socially-motivated philan-
thropists reduce their contributions by less than a dollar for each dollar increase in taxes (i.e., crowding out is less than complete), provided that the marginal benefit of social approval is declining in social approval, and respond to changes differently than public goods or impact philanthropists. In particular, a larger proportion of low-income donors amounts to smaller contributions for socially-motivated philanthropists but to larger contributions for public goods or impact philanthropists; a lower income gap results in larger contributions for both high- and low-income donors in the socially-motivated philanthropy model but in larger contributions for low-income donors and lower contributions for high-income donors in the public goods and impact philanthropy models. A key difference between the former model and the latter models is the nature of the strategic relationship between low- and high-income donors’ contributions: when low-income donors increases their contributions, high-income donors increase their contributions in one case while they decrease their contributions in the other case; the contributions of the two types of donors are thus strategic complements in the socially motivated philanthropy model and strategic substitutes in the other two models.

When ability-to-pay matters, an interesting result that arises in the socially motivated philanthropy model but not in the other models is that, under certain parametric restrictions, low-income donors end up contributing more than high-income donors; furthermore, low-income donors become less responsive to a decrease in the income gap while high-income donors become more responsive. In general, low-income donors are more likely to contribute more than high-income donors when the frame of reference is more important, the proportion of the population in the low-income category is higher, and/or the income gap is larger; in the numerical case we carry out, we also establish that the outcome is more likely to arise at lower levels of trust in the charitable organization and/or lower income gaps. As such, the socially motivated philanthropy model permits the possibility of explaining the U-shaped relationship between income and charitable giving observed in the data that previous models of giving could not explain.

4 Appendix

By total differentiation of the equilibrium conditions, we obtain the following:

\[
\begin{bmatrix}
  a_{1g_l}^j & a_{1g_h}^j \\
  a_{2g_l}^j & a_{2g_h}^j
\end{bmatrix}
\begin{bmatrix}
  dg_l \\
  dg_h
\end{bmatrix} = 
\begin{bmatrix}
  a_{1f_l}^j & a_{1\lambda g}^j & a_{1\lambda t}^j & a_{1\sigma}^j & a_{1\delta}^j & a_{1\theta}^j & a_{1\beta}^j \\
  a_{2f_l}^j & a_{2\lambda g}^j & a_{2\lambda t}^j & a_{2\sigma}^j & a_{2\delta}^j & a_{2\theta}^j & a_{2\beta}^j
\end{bmatrix}
\begin{bmatrix}
  dt \\
  d\lambda g \\
  d\lambda t \\
  d\sigma \\
  d\delta \\
  d\theta \\
  d\beta
\end{bmatrix},
\]

where we use the superscript \( j \) to differentiate between the public goods philanthropy model \((j = PG)\), the impact philanthropy model \((j = IP)\), and the socially motivated philanthropy model \((j = SP)\).

4.1 Public Goods Philanthropy Model

For \( j = PG \), we have that

\[
a_{1g_l}^{PG} = u_{l_i}^{''} + \sigma \lambda_g^2 \left[(z')^2 f'' + f'z''\right] < 0,
\]

\[
a_{1g_h}^{PG} = (1-\sigma) \lambda_g^2 \left[(z')^2 f'' + f'z''\right] \leq 0,
\]

\[
a_{2g_l}^{PG} = \sigma \lambda_g^2 \left[(z')^2 f'' + f'z''\right] \leq 0,
\]
\[ a_{2gh}^{PG} = u_{h}'' + (1 - \sigma) \lambda_{g}^{2} \left[ (z')^{2} f'' + f'z'' \right] < 0, \]
\[ a_{1i}^{PG} = -u_{i}'' - \lambda_{g} \lambda_{l} \left[ (z')^{2} f'' + f'z'' \right] > 0, \]
\[ a_{1i\lambda_{g}}^{PG} = -f' z' - \lambda_{g} \bar{g} \left[ (z')^{2} f'' + f'z'' \right] > 0, \]
\[ a_{1i\lambda_{l}}^{PG} = -\lambda_{g} t \left[ (z')^{2} f'' + f'z'' \right] \geq 0, \]
\[ a_{1i\sigma}^{PG} = -\lambda_{g}^{2} (g_{l} - g_{h}) \left[ (z')^{2} f'' + f'z'' \right] \leq 0, \]
\[ a_{1i}^{PG} = u_{i}'' < 0 \quad \& \quad a_{2gh}^{PG} = 0, \]

and

\[ a_{i\bar{g}}^{PG} = a_{i\bar{g}}^{PG} = 0, \]

for \( i = l, h \). Hence,

\[ D^{PG} = u_{i}'' u_{h}'' + u_{i}'' \lambda_{g}^{2} \left[ (z')^{2} f'' + f'z'' \right] = u_{i}'' \left\{ u_{i}'' + \lambda_{g}^{2} \left[ (z')^{2} f'' + f'z'' \right] \right\} > 0, \]

where \( u'' = u_i'' = u_h'' \).

### 4.2 Impact Philanthropy Model

For \( j = IP \), we have that

\[ a_{14j}^{IP} = u_{i}'' + \sigma \lambda_{g}^{2} \left( z' f_{i}'' z' + f_{i}'' z' \right) + (1 - \sigma) \lambda_{g}^{2} z' f_{i}'' z' < 0, \]
\[ a_{14h}^{IP} = (1 - \sigma) \lambda_{g}^{2} \left[ z' f_{h}'' (z' - z_l) + f_{h}'' z'' \right] \leq 0, \]
\[ a_{12g}^{IP} = \sigma \lambda_{g}^{2} \left[ z' f_{h}'' (z' - z_h) + f_{h}'' z'' \right] \leq 0, \]
\[ a_{2gh}^{IP} = u_{h}'' + (1 - \sigma) \lambda_{g}^{2} \left( z' f_{h}'' z' + f_{h}'' z'' \right) + \sigma \lambda_{g}^{2} z' f_{h}'' z'' < 0, \]
\[ a_{14t}^{IP} = -u_{i}'' - \lambda_{g} \lambda_{l} \left[ z' f_{i}'' (z' - z_l) + f_{i}'' z'' \right] > 0, \]
\[ a_{14\lambda_{g}}^{IP} = -\lambda_{g} \bar{g} \left[ z' f_{i}'' (z' - z_l) + f_{i}'' z'' \right] - z' \left( f_{i}'' + \lambda_{g} g_{l} z' f_{i}'' \right) < 0, \]
\[ a_{14\lambda_{l}}^{IP} = -\lambda_{g} t \left[ z' f_{i}'' (z' - z_l) + f_{i}'' z'' \right] \geq 0, \]
\[ a_{14\sigma}^{IP} = -\lambda_{g}^{2} (g_{l} - g_{h}) \left[ z' f_{i}'' (z' - z_l) + f_{i}'' z'' \right] \leq 0, \]
\[ a_{14g}^{IP} = u_{i}'' < 0 \quad \& \quad a_{2gh}^{IP} = 0, \]

and

\[ a_{i\bar{g}}^{IP} = a_{i\bar{g}}^{IP} = 0, \]

where \( z' f_{i}'' (z' - z_l) + f_{i}'' z'' \approx z'' \left( f_{i}'' + \lambda_{g} g_{l} z' f_{i}'' \right) \leq 0 < z' \left( f_{i}'' + \lambda_{g} g_{l} z' f_{i}'' \right) > -\lambda_{g} \bar{g} \left[ z' f_{i}'' (z' - z_l) + f_{i}'' z'' \right] \), for \( i = l, h \). Hence,

\[ D^{IP} = u_{i}'' u_{h}'' + u_{i}'' \lambda_{g}^{2} \left\{ z' f_{h}'' \left[ (1 - \sigma) z' + \sigma z_h' \right] + (1 - \sigma) f_{h}'' z'' \right\} + u_{h}'' \lambda_{g}^{2} \left\{ z' f_{i}'' \left[ \sigma z' + (1 - \sigma) z_l' \right] + \sigma f_{i}'' z'' \right\} + + \lambda_{g}^{2} z' f_{i}'' z' f_{h}'' \left[ \sigma z_h' + (1 - \sigma) z_l' \right] + (1 - \sigma) \lambda_{g} z' f_{h}'' z' f_{i}'' z'' + \sigma \lambda_{g}^{2} z' f_{i}'' f_{h}'' f_{i}'' z'' > 0. \]
We can then express the effects of $\lambda_g$, $\lambda_l$, $\sigma$, and $\delta$ as

$$
\frac{dg_i}{d\lambda_g} = - (u''_{i-i} + \lambda_g z' f''_{i-i} z'_{-i}) \{z' (f'_{i-i} + \lambda_g g_i z' f''_{i-i}) + \lambda_g g_i z' (f''_{i-i} (z' - z'_{-i}) + f'_i z'_{-i})\} + (d_i - \sigma) \lambda_g^2 z' \Phi_\lambda > 0,
$$

where $d_l = 1$, $d_h = 0$, and

$$
\Phi_\lambda = [z' f''_{i} (z' - z'_{i}) + f'_i z'_{-i}] (f'_{i} + \lambda_g g_i z' f''_{i}) - [z' f''_{h} (z' - z'_{h}) + f'_h z'_{h}] (f'_{i} + \lambda_g g_i z' f''_{i}),
$$

$$
\frac{dg_i}{d\lambda_l} = - \lambda_g t \{z' f''_{i} (z' - z'_{i}) + f'_i z'_{-i}\} (u''_{i-i} + \lambda_g^2 z' f''_{i-i} z'_{-i}) \leq 0,
$$

$$
\frac{dg_i}{d\sigma} = - \lambda_g^2 (g_i - g_h) [z' f''_{i} (z' - z'_{i}) + f'_i z'_{-i}] (u''_{i-i} + \lambda_g^2 z' f''_{i-i} z'_{-i}) \geq 0,
$$

for $i, -i = l, h$ and $i \neq -i$,

$$
\frac{dg_i}{d\delta} = u''_{i-i} \{u''_{h} + (1 - \sigma) \lambda_g^2 (z' f''_{h} z'_{-i} + f'_h z'_{-i}) + \sigma \lambda_g^2 z' f''_{i-i} z'_{-i}\} > 0,
$$

and

$$
\frac{dg_{h}}{d\delta} = - u''_{i-i} \lambda_g^2 \sigma [z' f''_{h} (z' - z'_{h}) + f'_h z'_{h}] \leq 0.
$$

### 4.3 Socially Motivated Philanthropy Model

Finally, for $j = SP$, we have that

$$
a_{1g_{i}}^{SP} = u''_{i-i} + \lambda^2 s''_{i-i} \left( \frac{1}{\vartheta} - \frac{\beta}{\varpi} \right) \left( \frac{1}{\vartheta} - \frac{\sigma \beta}{\varpi} \right) \varphi^2 < 0,
$$

$$
a_{1g_{h}}^{SP} = - \lambda^2 s''_{i-i} \left( \frac{1}{\vartheta} - \frac{\beta}{\varpi} \right) \left( \frac{1 - \sigma}{\varpi} \beta \varphi^2 \geq 0,
$$

$$
a_{2g_{i}}^{SP} = - \lambda^2 s''_{i-i} \left( \frac{1 - \beta}{\varpi} \varphi \geq 0,
$$

$$
a_{2g_{h}}^{SP} = u''_{h} + \lambda^2 s''_{h} \left( 1 - \frac{\beta}{\varpi} \right) \left[ 1 - \frac{(1 - \sigma) \beta}{\varpi} \right] < 0,
$$

$$
a_{i_{i}}^{SP} = - u''_{i-i} > 0,
$$

$$
a_{i_{\lambda}}^{SP} = - \left( \frac{1}{w_i - \frac{\beta}{\varpi}} \right) \left[ s''_{i} + \lambda \left( \frac{g_i}{w_i} - \frac{\beta \varphi}{w_i} \right) w_i s''_{i} \right] w_i < 0,
$$

$$
a_{i_{\sigma}}^{SP} = \lambda \left( \frac{\beta}{\varpi} \right) \left[ (1 - \vartheta) s''_{i} + \lambda (g_i - \vartheta g_h) \left( \frac{1}{w_i} - \frac{\beta}{\varpi} \right) w_i s''_{i} \right] w_i > 0,
$$

$$
a_{1_{i}}^{SP} = u''_{i-i} < 0 \quad \text{&} \quad a_{2_{i}}^{SP} = 0,
$$

$$
a_{i_{\sigma}}^{SP} = \lambda \left( \frac{1 - \sigma \beta}{\varpi} \right) \left[ s''_{i} + \lambda \left( \frac{1}{w_i} - \frac{\beta}{\varpi} \right) \varphi s''_{i} \right] > 0 \quad \text{&} \quad a_{2_{i}}^{SP} = - \lambda \left( \frac{\sigma \beta}{\varpi} \right) \left[ s''_{i} + \lambda \left( \frac{1 - \beta}{\varpi} \right) s''_{i} \right] < 0,
$$

and

$$
a_{i_{\beta}}^{SP} = \lambda \left( \frac{w_i}{\varpi} \right) \left[ s''_{i} + \lambda \left( \frac{1}{w_i} - \frac{\beta}{\varpi} \right) w_i s''_{i} \right] > 0,
$$

for $i = l, h$. Hence,

$$
D_{i}^{SP} = u''_{i-i} u''_{h} + \lambda^2 u''_{i-i} s''_{h} \left( \frac{1 - \beta}{\varpi} \right) \left[ 1 - \frac{(1 - \vartheta) \beta}{\varpi} \right] + \lambda^2 u''_{h} s''_{i} \left( \frac{1}{\vartheta} - \frac{\beta}{\varpi} \right) \left( \frac{1}{\vartheta} - \frac{\sigma \beta}{\varpi} \right) \varphi^2 +
$$
\[ + \lambda h_s^l h_p^l (1 - \beta) \left( \frac{1}{\varphi} - \frac{\beta}{\varpi} \right) (1 - \frac{\beta}{\varpi}) \vartheta > 0. \]

For the numerical example of the socially motivated philanthropy model, which is based on the assumption that \( V_i = \ln (1 + c_i) + \ln (1 + s_i) \) and \( \vartheta = \delta \) (that is, ability-to-pay matters in social considerations), where \( c_i = w_i - g_i - t > 0 \) and \( s_i = w_i \left( \frac{g_i}{w_i} - \beta \frac{\varpi}{\varpi} \right) \) \( w_i > 0 \), for \( i = l, h \), we have that

\[
D^{SP} = \frac{2 \lambda^2}{\varpi^2} \left[(2\varpi - \delta \beta)(\varpi - \beta) + \varpi \sigma \beta (1 - \delta) \right] > 0,
\]

\[
\frac{dg_l}{dt} = \frac{dg_h}{ht} = -\lambda^2 \left[(2\varpi - \delta \beta)(\varpi - \beta) + \varpi \sigma \beta (1 - \delta) \right] = -\frac{1}{2},
\]

\[
\frac{dg_l}{d\beta} = -\delta \lambda^2 \left[2(2\varpi - \beta) (\delta - g_l) + 2\varpi (1 - t - \varpi) - \beta (1 - t - \varpi) \right] < 0,
\]

\[
\frac{dg_h}{d\beta} = -\lambda^2 \left[2(2\varpi - \beta) (1 - g_h) + 2\varpi (1 - t - \varpi) - \delta \beta (1 - t - \varpi) \right] < 0,
\]

\[
\frac{dg_l}{d\lambda} = \frac{2(2\varpi - \beta) + \beta \left[ \delta + \sigma (1 - \delta) \right]}{\varpi D^{SP}} > 0,
\]

\[
\frac{dg_h}{d\lambda} = \frac{\varpi (2 - \beta) + \beta (1 - \delta)}{\varpi D^{SP}} > 0,
\]

\[
\frac{dg_l}{d\sigma} = -\beta \delta \lambda^2 \left\{ -g_l + \delta g_h + (1 - \delta) \right\} (2\varpi - \beta) + \frac{2\varpi - \beta (2 - \sigma)}{\varpi^3 D^{SP}} (1 - \delta) c_l + \beta (1 - \sigma) (1 - \delta) c_h < 0,
\]

where

\[
-g_l + \delta g_h + (1 - \delta) = \frac{\lambda (1 - \delta) \left\{ [\varpi + \lambda (1 + t) (\varpi - \beta)] (\varpi - \beta) + [\varpi + (\varpi - \delta \beta) \lambda + \varpi t] (1 - \beta) \varpi - \varpi \beta \right\}}{\varpi D^{SP}} > 0,
\]

\[
\frac{dg_h}{d\sigma} = -\beta \lambda^2 \left\{ -g_l + \delta g_h + (1 - \delta) \right\} (2\varpi - \beta \delta) + \frac{2\varpi - \beta \delta (1 + \sigma)}{\varpi^3 D^{SP}} (1 - \delta) c_h + \beta \delta \varpi (1 - \delta) c_l < 0;
\]

\[
\frac{dg_l}{d\delta} = \frac{\lambda^2 \beta (1 - \delta) \left\{ -(1 - t - \varpi) [(\varpi - \beta) + (1 - \beta) \varpi] - 2(\varpi - \beta) (\delta - g_l) + \beta \sigma (g_l - \delta g_l) \right\}}{\varpi^3 D^{SP}} + \left\{ \frac{2\lambda^2 (\varpi - \beta) (\varpi - \delta \beta)}{\varpi^2 D^{SP}} + \frac{\lambda^2 \beta \sigma (\varpi - \delta \beta)}{\varpi^2 D^{SP}} \right\}
\]

through social approval (<0)

\[
= \frac{1}{2} \left\{ \sigma (1 - \delta) [4 - \beta (3 + \delta)] + (1 - \beta) (2 + \delta \beta) - \sigma^2 (1 - \delta)^2 (2 - \beta) \right\} > 0,
\]

where \( (1 - \beta) > \sigma (1 - \delta) > \sigma^2 (1 - \delta)^2 \), and

\[
\frac{dg_h}{d\delta} = \frac{\lambda^2 \beta \sigma \left\{ (1 - t - \varpi) [(\varpi - \delta \beta) + (1 - \beta) \varpi] + 2(\varpi - \delta \beta) (1 - g_h) + \beta (1 - \sigma) (g_l - \delta g_h) \right\}}{\varpi^3 D^{SP}} + \left\{ \frac{\lambda^2 \beta \sigma (\varpi - \delta \beta)}{\varpi^2 D^{SP}} \right\}
\]

through social approval (>0)

\[
= \frac{1}{2} \left\{ \sigma (1 - \delta) [4 - \beta (3 + \delta)] + (1 - \beta) (2 + \delta \beta) - \sigma^2 (1 - \delta)^2 (2 - \beta) \right\} > 0.
\]
For the effects of the various parameters of the model on the contribution gap (i.e., $g_h - g_l$), we have that

$$\frac{d (g_h - g_l)}{dt} = 0,$$

$$\frac{d (g_h - g_l)}{d\beta} = -\frac{2\lambda^2 \left[ (\bar{w} - \delta \beta) (1 - g_h) - \delta (\bar{w} - \beta) (\delta - g_l) \right]}{\bar{w}^2 D^{SP}}.$$

which is negative as $[(1 - g_h) - (\delta - g_l)] = \frac{2\lambda(1-\delta)\beta}{\bar{w}^2 D^{SP}} > 0$,

$$\frac{d (g_h - g_l)}{d\lambda} = \frac{2\beta (1 - \delta)}{\bar{w} D^{SP}} > 0,$$

$$\frac{d (g_h - g_l)}{d\sigma} = -\frac{\beta (1 - \delta)^2 \left[ \sigma \lambda (\bar{w} - \delta \beta)^2 + \lambda \delta (1 - \sigma) (\bar{w} - \beta)^2 + (\bar{w}^2 - \delta \beta^2) + \bar{w}^2 (1 - \beta) \right]}{\lambda [(\bar{w} - \beta) (2\bar{w} - \delta \beta) + \beta \sigma (1 - \delta) \bar{w}]^2} - \frac{\beta (1 - \delta)^2 (\bar{w} - \beta) (\bar{w} - \delta \beta) \bar{w} [(\bar{w} - \beta) (2\bar{w} - \delta \beta) + \beta \sigma (1 - \delta) \bar{w}]^2}{[(\bar{w} - \beta) (2\bar{w} - \delta \beta) + \beta \sigma (1 - \delta) \bar{w}]^2} < 0,$$

and

$$\frac{d (g_h - g_l)}{d\delta} = \frac{(\bar{w} - \beta) (\bar{w} - \delta \beta)}{\sigma (1 - \delta) [4 - \beta (3 + \delta)] + (1 - \beta) (2 + \delta \beta) - \sigma^2 (1 - \delta)^2 (2 - \beta)} > 0.$$

When $\vartheta = 1$, however,

$$-1 < \frac{d (g_h - g_l)}{d\delta} = -\frac{(1 - \beta)}{(2 - \beta)} < 0.$$
References


Fig. 1b. Giving in Canada as a percentage of household income by income level for 2007 and 2010. Data from Martin Turcotte (2012), available at http://www.statcan.gc.ca/pub/11-008-x/2012001/article/11637-eng.htm.
Fig. 2a. Public Goods Philanthropy Model: Impact of $\lambda_g$ on the Crowding Out Effect. An increase in $\lambda_g$ from 0.7 to 0.9 results in more crowding out when $\lambda_t = 0.3$ and in less crowding out when $\lambda_t = 0.5$.

Fig. 2b. Public Goods Philanthropy Model: Impact of $t$ on the Crowding Out Effect. As $t$ increases, the crowding out effect increases when $\alpha = 1/4$ and decreases when $\alpha = 3/4$, where $1 - \alpha$ reflects the strength of philanthropic motives.
Fig. 3a. Impact Philanthropy Model: The Case of More than Complete Crowding Out.

Fig. 3b. Impact Philanthropy Model: Trust-Dependent Crowding Out under Equality of Trust Parameters.
Fig. 4. Reaction functions and Nash equilibria for $\lambda = 1$, $\beta = 0.3$, $\sigma = 0.7$, and $t = 0$ but different $\delta$ values: 0.56, 0.66 (+), and 0.46 (−). As $\delta$ decreases, low-income donors are more likely to contribute more than high-income donors.

Fig. 5. Reaction functions and Nash equilibria for $\lambda = 1$, $\beta = 0.3$, $\delta = 0.6$, and $t = 0$ but different $\sigma$ values: 0.74, 0.84 (+), and 0.64 (−). As $\sigma$ increases, low-income donors are more likely to contribute more than high-income donors.
Fig. 6. Reaction functions and Nash equilibria for $\lambda = 1, \sigma = 0.7, \delta = 0.6,$ and $t = 0$ but different $\beta$ values: 0.31, 0.36 (+), and 0.26 (−). As $\beta$ increases, low-income donors are more likely to contribute more than high-income donors.

\[
\begin{align*}
\{ & E: g_l = g_h = 0.13 \\
& E^{(+)}: g_l = 0.08; g_h = 0.03 \\
& E^{(-)}: g_l = 0.17; g_h = 0.21
\end{align*}
\]

Fig. 7. Reaction functions and Nash equilibria for $\beta = 0.3, \sigma = 0.7, \delta = 0.6,$ and $t = 0$ but different $\lambda$ values: 0.95, 1.00 (+), and 0.90 (−). As $\lambda$ decreases, low-income donors are more likely to contribute more than high-income donors.

\[
\begin{align*}
\{ & E: g_l = g_h = 0.10 \\
& E^{(+)}: g_l = 0.135; g_h = 0.143 \\
& E^{(-)}: g_l = 0.06; g_h = 0.05
\end{align*}
\]
Fig. 8. Reaction functions and Nash equilibria for $\lambda = 1, \beta = 0.39, \sigma = 0.2$, and $t = 0$ but different $\delta$ values: 0.705, 0.855 (+), and 0.555 (−). As $\delta$ increases, low-income donors are more likely to contribute more than high-income donors.

Fig. 9. Equilibrium contribution levels as functions of $\delta$ for $\lambda = 1, t = 0, \sigma = 0.7$, and $\beta = 0.25$ or $\beta = 0.30$. As $\delta$ increases, both $g_l$ and $g_h$ increase. As $\beta$ increases, both $g_l$ and $g_h$ decrease and the $\delta$ value below which $g_l > g_h$ increases. For interior solutions, $\delta > 0.27$ when $\beta = 0.25$ (not shown) and $\delta \geq 0.41$ when $\beta = 0.30$ (shown).
Fig. 10. Equilibrium contribution levels as functions of $\lambda$ for $\delta = 0.7$, $t = 0$, $\beta = 0.32$, and $\sigma = 0.4$ or $\sigma = 0.6$ or $\sigma = 0.8$. As $\lambda$ increases, both $g_l$ and $g_h$ increase. As $s$ increases, both $g_l$ and $g_h$ decrease and the $\lambda$ value below which $g_l > g_h$ increases. The minimum $\lambda$ value for interior solutions increases from 0.79 to 0.86 when $\sigma$ increases from 0.4 to 0.8.

Fig. 11. Equilibrium contribution levels as functions of $s$ for $\lambda = 0.9$, $t = 0$, $\beta = 0.3$, and $\delta = 0.6$ or $\delta = 0.7$. As $\sigma$ increases, both $g_l$ and $g_h$ decrease. As $\delta$ increases, both $g_l$ and $g_h$ increase, the $\sigma$ value above which $g_l > g_h$ increases, and the $\beta$ value such that $g_l = g_h$ (i.e., $\beta^*$) decreases (increases) at low (high) $\sigma$ values. When $\delta = 0.6$, interior solutions require that $\sigma < 0.84$. 
Fig. 12a. Difference between shares of income donated by high- and low-income donors as a function of $\delta$ for $\lambda_g = 0.9, \lambda_t = 0.8, \sigma = 0.7, \beta = 0.2, \vartheta = 1$, and $t = 0$. 

Fig. 12b. Difference between shares of income donated by high- and low-income donors as a function of $\delta$ in the socially motivated philanthropy model for $\lambda_g = 0.9, \lambda_t = 0.8, \sigma = 0.7, \beta = 0.2, \vartheta = \delta$, and $t = 0$. 

$g_h - \frac{g_t}{\delta}$
Table 1. NUMERICAL EXAMPLE: Summary of the effects of changes in the parameters of the model on the two contribution levels and their difference.

<table>
<thead>
<tr>
<th>Socially Motivated Philanthropy Model ($\vartheta = \delta$)</th>
<th>Parameters</th>
<th>$t$</th>
<th>$\sigma$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\lambda$</th>
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</thead>
<tbody>
<tr>
<td>Contributions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_l$</td>
<td></td>
<td>$-\frac{1}{2} &lt; 0$</td>
<td>$-\frac{\beta(1-\delta)}{2(2-\beta)} &lt; 0$</td>
<td>$E_l &lt; 0^*$</td>
<td>$\frac{2(1-\beta)+\sigma\beta}{2(2-\beta)} &gt; 0$</td>
<td>$\frac{1}{2(1-\beta)^2} &gt; 0$</td>
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<tr>
<td>$g_h$</td>
<td></td>
<td>$-\frac{1}{2} &lt; 0$</td>
<td>$-\frac{\beta(1-\delta)}{2(2-\beta)} &lt; 0$</td>
<td>$E_h &lt; 0^*$</td>
<td>$\frac{\sigma\beta}{2(2-\beta)} &gt; 0$</td>
<td>$\frac{1}{2(1-\beta)^2} &gt; 0$</td>
</tr>
<tr>
<td>$g_h - g_l$</td>
<td></td>
<td>no change</td>
<td>no change</td>
<td>$-\frac{1-\delta}{(2-\beta)^2} &lt; 0$</td>
<td>$-\frac{1-\beta}{2-\beta} &lt; 0$</td>
<td>no change</td>
</tr>
<tr>
<td>$g_l - g_h &gt; 0$</td>
<td></td>
<td>no effect</td>
<td>more likely at high $\sigma$ values</td>
<td>more likely at high $\beta$ values</td>
<td>more likely at either low or high $\delta$ values</td>
<td>more likely at low $\lambda$ values</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Socially Motivated Philanthropy Model ($\vartheta = 1$)</th>
<th>Parameters</th>
<th>$t$</th>
<th>$\sigma$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\lambda$</th>
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<tbody>
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<td></td>
</tr>
<tr>
<td>$g_l$</td>
<td></td>
<td>$-\frac{1}{2} &lt; 0$</td>
<td>$-\frac{\beta(1-\delta)}{2(2-\beta)} &lt; 0$</td>
<td>$E_l &lt; 0^*$</td>
<td>$\frac{2(1-\beta)+\sigma\beta}{2(2-\beta)} &gt; 0$</td>
<td>$\frac{1}{2(1-\beta)^2} &gt; 0$</td>
</tr>
<tr>
<td>$g_h$</td>
<td></td>
<td>$-\frac{1}{2} &lt; 0$</td>
<td>$-\frac{\beta(1-\delta)}{2(2-\beta)} &lt; 0$</td>
<td>$E_h &lt; 0^*$</td>
<td>$\frac{\sigma\beta}{2(2-\beta)} &gt; 0$</td>
<td>$\frac{1}{2(1-\beta)^2} &gt; 0$</td>
</tr>
<tr>
<td>$g_h - g_l$</td>
<td></td>
<td>no change</td>
<td>no change</td>
<td>$-\frac{1-\delta}{(2-\beta)^2} &lt; 0$</td>
<td>$-\frac{1-\beta}{2-\beta} &lt; 0$</td>
<td>no change</td>
</tr>
</tbody>
</table>

$^* E_l = (1-\sigma)(1-\delta)/(2-\beta)^2 - 1/[2\lambda(1-\beta)^2] < 0; E_h = -\sigma(1-\delta)/(2-\beta)^2 - 1/[2\lambda(1-\beta)^2] < 0.$

<table>
<thead>
<tr>
<th>Public Goods Philanthropy Model</th>
<th>Parameters</th>
<th>$t$</th>
<th>$\sigma$</th>
<th>$\lambda_t$</th>
<th>$\delta$</th>
<th>$\lambda_g$</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_l$</td>
<td></td>
<td>$-\frac{\lambda_t+\lambda_l}{2\lambda_y} &lt; 0$</td>
<td>$\frac{1-\delta}{2} &gt; 0$</td>
<td>$\frac{t}{2\lambda_y} &lt; 0$</td>
<td>$\frac{2-\sigma}{2} &gt; 0$</td>
<td>$\frac{1+\lambda_t}{2} &gt; 0$</td>
</tr>
<tr>
<td>$g_h$</td>
<td></td>
<td>$-\frac{\lambda_t+\lambda_l}{2\lambda_y} &lt; 0$</td>
<td>$\frac{1-\delta}{2} &gt; 0$</td>
<td>$\frac{t}{2\lambda_y} &lt; 0$</td>
<td>$\frac{\sigma}{2} &lt; 0$</td>
<td>$\frac{1+\lambda_t}{2} &gt; 0$</td>
</tr>
<tr>
<td>$g_h - g_l$</td>
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<td>no change</td>
<td>no change</td>
<td>$-1$</td>
<td>no change</td>
</tr>
</tbody>
</table>