

# The Effects of Population Diversity on Productivity, Competitiveness and Employment

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Revised: May 1999

## **Abstract**

The purpose of this paper is to measure some of the economic effects of greater population diversity. Theoretically, the effects of greater diversity depend on trade-offs between dynamic and static efficiency and on the importance of informational networks. In general, the effects are ambiguous and need to be investigated empirically. To measure these effects, I estimate a production model for the manufacturing industry in Ontario and find that increased diversity, increased labour and production efficiency and had a positive effect on the demand for labour, total factor productivity and competitiveness.

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# 1 Introduction

Changing immigration patterns, combined with declining fertility, have reshaped Canada over the years and turned it into a culturally, ethnically and racially diverse society. Whereas at the time of Confederation, people of British or French origin comprised about 90% of the population, in 1986 their share declined to about 67%<sup>1</sup>. In addition to these, self-proclaimed, “two founding nations” and the aboriginal people, a large new group of diverse ethnic minorities has become an important factor in Canadian society. Given current immigration patterns and given global political/economic conditions, this diversity is expected to increase<sup>2</sup>. The implications of this increasing diversity are particularly important in Ontario, where the largest concentration of people of ethnic origins other than aboriginal, English and French<sup>3</sup>, and 50% of visible minorities can be found.

In addition to the increase in diversity and perhaps as a result of this, there has also been an increase in the “demand” for some form of ethnic identification<sup>4</sup>. This resurgence in “ethnicity” has defied earlier predictions that assimilation may diminish cultural diversity. Thus, not only is Canadian society increasingly diverse, it has also shown the desire to remain so.

The changing character of Canadian society has led to extensive public and academic discussions of the various sociological, psychological, legal and political aspects of population diversity and multiculturalism<sup>5</sup>. Thus far, however, there have not been discussions of the economic aspects.

The purpose of this paper is to measure some of the economic effects of greater population diversity. Theoretically, the effects of greater diversity depend on trade-offs between dynamic and static efficiency and on the importance of informational networks. In general, the effects are ambiguous and need to be investigated empirically.

In the empirical analysis I examine the effects of population diversity on economic performance in Ontario, as captured by standard measures, such as efficiency, productivity and

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<sup>1</sup>See: Census of Canada, 1986.

<sup>2</sup>See, for example, Samuel (1988).

<sup>3</sup>Four million in 1986.

<sup>4</sup>For evidence of this resurgence in ethnicity, see Herberg (1989), Driedger (1989).

<sup>5</sup>For example, see Fleras and Elliott (1991), Berry (1991).

competitiveness. First, I propose and calculate two alternative diversity indices to measure changes in diversity in Ontario during the period 1962-1988. I find that diversity has increased from the mid 1960's to the late 1970's, but has become relatively stable after the early 1980's. Using these diversity indices, I estimate a cost function/productivity model and find that increased diversity had a significant positive effect on labour and production efficiency. Consequently, it also had a positive effect on the demand for labour and competitiveness. I also examine the contribution of diversity to the rate of growth in total factor productivity and find that although the absolute contribution was small, the relative contribution was significant.

## 2 Diversity and Flexibility and Efficiency in Production

Every society is composed of many individuals with different characteristics. These characteristics include beliefs, outlook, behaviour, language, education, abilities, experience, traditions etc., and define what we may think of as the individual's "human and cultural capital" (HCC).

In most societies individuals identify themselves with, and are identified by society as members of, particular "groups." These "groupings" reflect a certain sense of "belonging" that is often based on some of the individuals' underlying characteristics. Although the distribution of some of the HCC may be similar across groups, the distribution of others may not. Each group is distinct in terms of some elements (or the mix) of its members' HCC, reflecting its unique heritage. Since groups within society have something unique about their heritage, individuals within these groups have some elements, and consequently combinations of HCC, which are distinct<sup>6</sup>. A greater variety of distinct groups, therefore, tends to increase the number of "types" of labour inputs available in the economy.

Standard results in the theories of production under uncertainty, options and information networks, can give us an idea how greater diversity may affect economic performance and

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<sup>6</sup>This is not to say that individuals within groups are the same; they are different (have different distributions of HCC characteristics), but they have something in common, which binds them together in the first place.

productivity. The arguments are quite simple and well known.<sup>7</sup> Greater diversity has both benefits and costs. Greater diversity makes the economic system more flexible by making it easier to adjust to changes in an uncertain environment.<sup>8</sup> This is what we may call dynamic efficiency.<sup>9</sup>

It is also useful to think of diversity as providing “options.” Options are valuable because they can be exercised when it is beneficial to do so. Similarly, diversity is valuable because it provides the flexibility to “exercise options”, by utilizing particular HCC characteristics, if and when circumstances call for it. Hence, the benefits from diversity are due to the flexibility that results from the ability to exercise the options.

Greater diversity, however, may be costly, since it could decrease static efficiency. For any given set of circumstances, static production efficiency increases if the cost of production decreases. Greater input diversity does not necessarily increase static efficiency. First, the production process may involve economies of scale (in the production of input characteristics) and consequently, potential gains from input specialization may be lost as a result of an increase in diversity<sup>10</sup>. Second, additional inputs may not reduce the cost of production because they are too expensive. Third, the characteristics proportions of additional inputs may be “too different” than what is efficient, so they may not be useful and hence will not reduce production costs. Thus, on a theoretical level, it is not clear whether greater variety

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<sup>7</sup>These are closely related to the literature on flexibility versus efficiency. See for example, Fuss and McFadden (1978b), Kokinov (1994), Adler Goldoftas and Levine (1999) and Piore and Sable (1984) (for a theory of flexibility).

<sup>8</sup>Changing market conditions lead to changes in production decisions. As a result, desired human capital characteristics and the corresponding efficient input combination needed to obtain these characteristics change as well. For any set of circumstances there corresponds a “best” structure of human capital and consequently an optimal input combination. Conversely, any human capital (input) structure may be optimal under a specific set of circumstances. Thus, an existing structure is optimal only if we are “lucky” and very specific circumstances happen to occur. Under most circumstances, however, other structures are desirable. Greater diversity makes it easier, or cheaper, to obtain these other efficient human capital (and input) structures.

<sup>9</sup>The dynamic benefits of diversity can be interpreted in terms of the value of information and learning. Information is valuable only if there is sufficient flexibility to allow for adjustments. Greater flexibility means that adjustments are easier to make, so that information on new conditions is more valuable. When adjustments are impossible (or very costly), new information is not very valuable, because it cannot be used.

<sup>10</sup>Whether there is an increase in efficiency depends on the existence of economies of scope and scale. Economies of scope arise when it is beneficial to combine and diversify activities. Economies of scale arise when it is beneficial to “deepen,” rather than spread activities. For a “given pool” of inputs, efficiency increases with diversity, when economies of scope dominate economies of scale. See Baumol et al. (1982), for a discussion.

increases static production efficiency. This has to be investigated empirically.

The overall effects of diversity depend on many factors, including market and cost conditions, prices, etc. In particular, it depends on the stability of the environment. If conditions are relatively stable, gains from dynamic flexibility are small. On the other hand, within a constantly changing environment there is a much greater potential for dynamic efficiency gains. In such an environment, new conditions and information have to be constantly incorporated and adjusted to. But adjustments are possible only if there is some flexibility. Thus, dynamic flexibility is of greater importance within a constantly changing, more uncertain environment<sup>11</sup>.

In recent years, as a result of the reduction in trade barriers and the consequent increasing importance of international trade, world economies have become closely integrated and globalized. In such an environment, conditions within one economy are more likely to be affected by changing conditions in the rest of the world. As a result, individual economies are more susceptible to “imported” uncertainty and instability; changing conditions abroad are more likely to have spillover effects within local economies. This is particularly true for a country like Canada, whose economy is strongly dependent on international markets. Consequently, potential gains from flexibility are high in the context of the Canadian economy.

The discussion above assumes that the type, amount and quality of information that is available to economic units is exogenous. In other words, information was taken to be given and economic decisions simply reacted to changing conditions. The information available to economic units, however, is neither exogenous, nor independent of the composition of the population. In general, the availability and nature of information will depend on the degree of diversity in the economy. Specifically, in a society that is composed of individuals with diverse backgrounds, it is likely that “more, better and faster” world-wide information will be available. These informational advantages follow from an improved ability to “import foreign information” and “export local information”, due to the existence of “ethnic trading”, or “ethnic informational” networks<sup>12</sup>. This better access to information makes economic decisions

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<sup>11</sup>This conclusion is clear given the portfolio, or option interpretations; greater uncertainty increases the importance of portfolio diversification and the value of an option.

<sup>12</sup>For a discussion of ethnic trading and informational networks, see Curtin (1984), Landa (1981), (1993).

more efficient, hence improving performance. It also endows the economy with a “comparative advantage” in the “market for information”, enabling it to exploit new opportunities and to specialize in the use and provision of information. The existence of ethnic trading, or informational networks also helps reduce investment and trading risks, again introducing new/more/better economic opportunities.

Thus, in addition to increasing the flexibility of production processes, increased diversity yields informational advantages which improve performance by making existing economic activities more efficient, as well as by introducing new opportunities.

### 3 Diversity and Productivity

In this section I discuss a framework that can be used to estimate the importance of diversity in explaining the performance of the economy. I develop an aggregate production model<sup>13</sup> for the manufacturing sector in Ontario and use it to identify the direct and indirect effects of increased diversity on production costs, labour productivity and total factor productivity. The model is also used to examine the effects of increased diversity on the demand for labour, employment levels, average costs of production and prices.

Diversity has been shown to affect both static and dynamic efficiency. Consequently, we can expect it to be one of the determinants of changes in production conditions over time. The most natural framework for analyzing efficiency patterns over time is the standard factor productivity model<sup>14</sup>. We amend the model to allow for the effects of diversity.

Since increased diversity is likely to affect the efficiency of the production process through its effect on the productivity of labour, rather than other inputs, we take the effect of diversity to be of the “labour augmenting” type. A “labour augmenting” effect, to the extent that such an effect indeed exists, means that an increase in diversity increases labour’s efficiency.<sup>15</sup>

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<sup>13</sup>Using an aggregate cost function.

<sup>14</sup>See for example Denny, Fuss and Waverman (1981), Jorgenson et al. (1987), Appelbaum (1991), Appelbaum and Berechman (1991), for a discussion of the measurement of total factor productivity.

<sup>15</sup>A standard way to capture this labour augmentation is to define labour in terms of “efficiency units” as:  $l^* = le^{av}$ , where  $l$  is the total number of units of labour,  $v$  is a measure of diversity and  $a$  is the rate of labour productivity growth due to increased diversity. Another way of thinking about this effect is that an increase in diversity increases the number of efficiency units of labour. An increase in diversity, therefore, has the same effect as an increase in the number of *actual* units of labour used.

Labour augmenting effects are, therefore, endogenously determined by changes in diversity. Interestingly, in contrast with this formulation, in most productivity studies<sup>16</sup> these effects are simply treated as exogenously determined by a time trend.<sup>17</sup>

The extent to which a labour augmenting diversity effect is significant is, of course, an empirical issue that needs to be investigated. Specifically, empirical analysis would confirm whether (and the degree to which) diversity is important in explaining labour efficiency, total factor productivity, competitiveness and the demand for labour.<sup>18</sup> This will be undertaken in the following section.

Applying the standard productivity approach, I first look at the rate of growth in the cost of production over time. It can be easily shown<sup>19</sup> that the rate of growth in total costs,  $RGTC$ , is explained by four factors: (i) changes in the scale of production, (ii) changes in input prices, (iii) changes in diversity and (iv) technical change. We write this as:

$$RGTC = \text{scale effect} + \text{input prices effect} + \text{diversity effect} + \text{technical change} \quad (1)$$

The contributions of these factors to the rate of growth in total costs depend on the growth rates of the underlying variables<sup>20</sup> and the sensitivity of costs with respect to these variables (their elasticities).

The rate of growth in total costs can be decomposed further to obtain the rate of growth in average costs. I define the rate of growth in average costs,  $RGAC$ , as the rate of growth in total costs minus the rate of growth in output:

$$RGAC \equiv (\text{scale effect} - \text{rate of growth in output}) + \text{prices effect} + \text{diversity effect} + \text{technical change} \quad (2)$$

Since a reduction in average costs captures improved production efficiency, it also captures improvements in the degree of competitiveness. But, the rate of growth in input prices is

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<sup>16</sup>See references in footnote 14.

<sup>17</sup>For example,  $l^* = le^{at}$ , where  $t$  is time.

<sup>18</sup>Similarly, empirical analysis would also test the validity of the augmentation formulation itself.

<sup>19</sup>See Appendix.

<sup>20</sup>Namely: level of production, input prices, diversity and technical change.

exogenous to firms, so that it is unrelated to endogenous changes in production efficiency. Thus, when examining changes in production efficiency, we should look at contributions to the rate of growth in average costs, excluding the rate of growth in input prices. We define the rate of growth in production efficiency,  $RGPE$ , as the rate of growth in average costs, excluding the effects of changes in input prices:

$$\begin{aligned}
 RGPE \equiv RGAC - price\ effect &= (scale\ effect - rate\ of\ growth\ in\ output) \\
 &+ diversity\ effect + technical\ change
 \end{aligned}
 \tag{3}$$

$RGPE$  is equivalent to the standard measure of the rate of growth of total factor productivity ( $RGTFP$ ).<sup>21</sup> The decomposition in (3) shows that total factor productivity can be explained by changes in technical efficiency (technical change), scale of operations and diversity.

To be able to calculate the components of productivity in (3), we have estimate a complete production, or cost model. Once such a model is estimated, we can use its estimated parameters to calculate the rate of growth in total factor productivity and its components. In particular, we can calculate the effects of diversity on the rate of growth in total factor productivity.

Finally, it should be pointed out that the effects of diversity on the economy's production possibilities frontier and hence on its GNP are, implicitly, captured by the productivity analysis provided here. In other words, increased productivity, as captured by the reduction in average costs, is translated into an expanded production possibilities frontier and hence a higher GNP. It should also be pointed out that while, often, productivity growth itself increases immigration into the growing economy, it is not clear whether this, necessarily, translates into increased diversity.<sup>22</sup>

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<sup>21</sup>In fact, it can be shown that the rate of growth in production efficiency is equal to minus the rate of growth of total factor productivity:  $RGPE \equiv -RGTFP$ . See Ohta (1975), Appelbaum (1991), for a discussion of this result.

<sup>22</sup>It also possible that diversity, itself, tends to lead to even greater diversity. In both cases this can be addressed within a simultaneous system that takes into account the endogeneity of the process.



## 4 Empirical Application and Results

### 4.1 Econometric Model

I apply the model to the Ontario manufacturing industry. I consider a three input production process, where labour,  $l$ , capital,  $k$ , and intermediate goods (materials),  $m$ , are used to produce output,  $q$ . The prices of these inputs are  $w$ ,  $r_k$  and  $r_m$ , respectively. The wage rate includes the cost of all paid vacations. The data is for the period 1962-1988 and is described in detail in Appelbaum and Smith (1998).

First, let us consider possible measures of diversity. For example, suppose society is composed of  $n$  groups, whose population shares are given by  $\alpha_1, \alpha_2, \dots, \alpha_n$ . Any definition of diversity should capture the fact that both a greater variety of groups (a larger  $n$ ) and a less concentrated distribution of shares, increase diversity. One example of a possible measure of diversity is the standard entropy index,  $D_e$ , which measures the degree of “disorder,” or “dispersion” of a system and is commonly used in the physical sciences<sup>23</sup>. Concentration measures, which are frequently used in economics to measure industrial concentration, are also reasonable measures of diversity. For example, a commonly used measure is the Herfindahl index,<sup>24</sup>  $D_h$ , which is defined as:  $D_h \equiv \sum_{i=1}^n \alpha_i^2$ . Being a concentration index, rather than a diversity index, the Herfindahl index is inversely related to diversity, i.e., a smaller  $D_h$  indicates greater diversity. It is easy to show that this index can be written as  $D_h = \sigma + \frac{1}{n}$ , where  $\sigma$  is the variance of the distribution of shares. The Herfindahl index, therefore, satisfies the requirement that an increase in the number of groups, or a decrease in the variance of the distribution of shares, increase diversity in society<sup>25</sup>.

Both the entropy and Herfindahl indices capture the distributional aspects of diversity. Diversity, however, depends also on the strength of group identification. In other words, it is also a function of the “intensity” of “distinctiveness feelings” of the groups. If measures

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<sup>23</sup>For example, it is the fundamental concept in the second law of thermodynamics. The entropy index can be defined by:  $D_e \equiv \sum_{i=1}^n \alpha_i \log(1/\alpha_i)$ .

<sup>24</sup>See Scherer (1980), Tirole (1989).

<sup>25</sup>Another desired property of diversity measures could be symmetry (invariance of permutations of population shares between groups). These two indices are only examples of diversity measure. It is of course possible to provide other measures of diversity which also satisfy the properties mentioned above.

of these intensities are available, they can be used to adjust the measures of diversity<sup>26</sup>. For example, suppose the intensity measure for group  $i$  is given by  $\delta_i$ , where  $0 \leq \delta_i \leq 1$ . The above indices can be adjusted to account for the distribution of intensities. Specifically, each  $\alpha_i$  now gets a weight according to group intensity (or equivalently, each intensity gets a weight according to group share). Thus, if we have information on the distribution of groups by size and intensity, it is possible to construct intensity weighted diversity indices. Since I have not yet been able to obtain intensity information, I do not calculate these weighted indices in this study. I hope to do this in the future.

To construct the index of diversity,  $v$ , I need information on the distribution of population by ethnic origin. The use of data on the distribution of population by ethnic origin, however, is problematic for practical and theoretical reasons. First, there does not exist time-consistent data on population by ethnic origin. Second, the classification of ethnic groups for which data is given, changes through time. Furthermore, it seems that the list and definition of ethnic groups, are themselves endogenous variable, reflecting the relative importance and social attitudes toward the various groups at the time of the data collection. For example, certain groups, such as Latinos, Indo-Pakistani, Caribbean, etc. do not appear in the early lists of ethnic groups. Third, the inclusion of non-single origin groups makes it very difficult to determine an appropriate distribution. Finally, there is no information on inter-Provincial flows by ethnic origin<sup>27</sup>. As a result of these difficulties, I use data on immigration flows by country of origin to construct a proxy measure of diversity. Although the use of these flows does not give us the “stock” picture, it is nevertheless reasonable, since changes in population composition are mainly due to changes in immigration patterns. In this sense, by using flows figures, we only lose the effects of “initial conditions”. The use of country of origin data avoids the problems of multi-origin categories and the emergence of new reported categories over time. Hopefully, in the future, a more careful and complete index, which uses population distributions as well intensity of ethnic association measures, can be constructed.

The data on immigration flows are published by the Department of Manpower and Im-

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<sup>26</sup>Possible variables to capture intensity are: rates of use of heritage languages, enrolment in parochial schools, participation in ethnic education and community activities, etc.

<sup>27</sup>Or birth and death rates by ethnic origin.

migration, Canada. I look at sixteen groups: Africa, South America, Central America (including Mexico), U.S., U.K., Germany, Italy, Greece, Portugal/Spain, Poland, other Europe, Caribbean, China (including Hong-Kong), India/Pakistan, Australia/Australasia and other Asian. Using the immigration figures for these groups I calculate the two diversity measures given by  $1 - D_h$  (one minus the Herfindahl index) and the entropy index,  $D_e$ . These figures are reported in Table 1.

As Table 1 shows, diversity patterns over time are similar for both indices. In fact, the two measures are highly correlated, with a correlation coefficient of .987. Overall, both indices indicate an increase in diversity over time. Specifically, both measures indicate that immigration diversity has increased from the mid 1960's to the late 1970's. It has become relatively stable from the early 1980's and on. The early 1960's, 1979 and 1980 seem to be "outlier" years, where immigration diversity actually decreases.

Given these diversity indices I estimate the model as follows. I specify a translog functional form to represent the underlying cost function.<sup>28</sup> This is a very general specification and is the most frequently used in empirical studies in production theory<sup>29</sup>. From this cost function I obtain the cost share equations for labour, capital and materials. I estimate a full model consisting of the cost function and the share equations for labour and capital<sup>30</sup>, using both diversity indices<sup>31</sup>. Since the model is nonlinear in the parameters and involves cross equation constraints, I estimate it by the Nonlinear Iterative Seemingly Unrelated Regression method<sup>32</sup>. Upon convergence, this method is equivalent to the maximum likelihood technique.

The parameter estimates and the corresponding  $t$  values for the two models are given in Table 2. Table 2 also gives the  $R^2$  (goodness of fit) and D.W. (serial correlation) statistics. As Table 2 indicates both models seem to perform well in terms of their  $R^2$  and D.W. statistics.

The effect of diversity on the efficiency of labour is captured by the parameter  $a$ . In fact,  $a$  is the rate of labour productivity growth due to increased diversity<sup>33</sup> The important thing to

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<sup>28</sup>The underlying cost function can be written as:  $C(r, w, v, q, t)$ . If diversity has a "labour augmenting" effect, the cost function can be written as:  $C(r, w, v, q, t) \equiv C(r, we^{-av}, q, t)$

<sup>29</sup>See Fuss and McFadden (1978). The detailed descriptions of the functional form and the econometric model are given in the Appendix.

<sup>30</sup>The materials share equation is not estimated since it is implied by the other two.

<sup>31</sup>Since the models yield similar results, I do not always report all the results for both models.

<sup>32</sup>This is the iterative Zellner method. See Zellner (1962).

<sup>33</sup>Remember that we defined labour in efficiency units as:  $l^* = le^{av}$ .

notice in Table 2 is that the effect of diversity on labour efficiency is positive and statistically significant, using both diversity indices. Using the Herfindahl index I get  $\hat{a} = .6707$  with a  $t$  value of 4.8973 and using the entropy index I get  $\hat{a} = .4147$  with a  $t$  value of 5.585<sup>34</sup>. The conclusion, therefore, is that we cannot reject the hypothesis that diversity has a, statistically, significant effect on the efficiency of labour. The economic importance of this effect will be discussed below. I also tried to capture the efficiency effect of diversity using alternative specifications<sup>35</sup>. Regardless of which specification was used, an increase in diversity always had a statistically significant effect on labour efficiency. The specification I chose is very general, intuitive, and could not be rejected statistically.

## 4.2 The Effects of Increased in Diversity on Production Costs

Given the parameter estimates I calculate the sensitivity (elasticity) of production costs with respect to changes in diversity,  $\theta_v^h$  (for the Herfindahl model) and  $\theta_v^e$  (for the entropy model) and report them in Table 3<sup>36</sup>. These elasticities capture the effects of increased diversity on production costs, through their effect on the efficiency of labour. They give us the percentage change in costs due to a given percentage change in diversity. As Table 3 shows the diversity elasticity is between -.100 and -.139 for the entropy model and between -.151 and -.208 for the Herfindahl model. These figures should be interpreted as follows. When the elasticity is -.10388 (as in the entropy case in 1988), an increase in diversity (as measured by the entropy index) of, say 10%, will decrease total production costs by  $-.10388 \times 10 = -1.0388\%$ .

To better appreciate the magnitude of these figures, it is helpful to consider an example in which these effects are calculated in dollar terms. Thus, for example, suppose that in every year during the sample period, 1962-1988, diversity had increased by 10%. What effect would this have had, in dollar terms, on total production costs? These effects, for the sample period, are also reported in Table 3. An examination of Table 3 shows that a 10% increase in diversity, as captured by the entropy measure, would have reduced total production costs by 127 million to 1.36 billion dollars. A 10% increase in diversity, as captured by the Herfindahl measure,

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<sup>34</sup>Since the two indices are numerically different, the estimated parameters will also be different.

<sup>35</sup>Using alternative assumptions regarding the scale elasticity, the nature of technical change and using a non labour augmenting diversity effects.

<sup>36</sup>The diversity elasticity is given by  $\frac{\partial \ln C}{\partial \ln v}$ .

would have reduced total production costs by 200 million to 2.09 billion dollars.

### 4.3 The Effects of Increased Diversity on the Demand for Labour

Next, I examine the effects of an increase in diversity on the demand for labour. Since an increase in diversity makes labour more efficient, *ceteris paribus*, it will also tend to increase the demand for labour. The overall effect of an increase in diversity on the demand for labour is given by the sensitivity measure (elasticity) of the demand for labour with respect to  $v$ , denoted as  $\theta_v^l$ . These can be calculated using the parameter estimates for the translog cost function. I calculate these elasticities for the Herfindahl index case and report them in Table 4. As Table 4 shows, the labour demand elasticity with respect to diversity goes from a low of .0018 (in 1985) to a high of .0842 (in 1972). In all cases, however, the elasticity is positive, indicating that the demand for labour will, indeed, increase due to the increase in labour efficiency that follows from increased diversity.

### 4.4 The Effects of Increased Diversity on Prices

The Ontario manufacturing industry is perhaps best characterized by a market structure which is neither perfectly competitive, nor purely monopolistic. Decision rules for oligopolistic markets (lying between perfect competition and monopoly) are rather complicated and consequently, such industries are very difficult to model.

A possible way of modelling pricing in such industries is to assume that output price is determined by a markup factor. In other words, output price,  $P$ , is given by,  $P = AC(1 + \beta)$ , where  $AC$  is the average cost and  $\beta$  is the markup factor. Given a constant markup, we simply have to calculate the effect of a change in diversity on average cost. For any given level of output, the elasticity of average cost with respect to  $v$ , is the same as the elasticity of total cost with respect to  $v$ . These figures were reported in Table 3. Thus, for example, to the extent that prices are determined by markup pricing, a 10% increase in diversity (as measured by the Herfindahl index), would have decreased the price level by 1.5974% in 1988 (for the same level of output). Since competitiveness is generally determined by average costs, these elasticities also capture the effects on the degree of competitiveness.

We also calculate the elasticity of marginal costs with respect to diversity and report it

in Table 4. As can be seen in Table 4, these elasticities are between -.131 and -.190 (for the Herfindahl model). The fact that the elasticities are all negative indicates that increased diversity would decrease marginal production costs. Thus, for example, an increase in diversity of 10%, as given by the Herfindahl measure, would have decreased marginal costs by .13829% in 1988, for any given level of output. Comparing the effects of  $v$  on average and marginal costs we see that they are quite similar. These two effects give us a general idea of the range of price decreases, and hence the increase in competitiveness, due increased diversity.

## 4.5 The Measurement of Productivity

Having estimated the model, I use the parameter estimates to calculate the rate of growth in total factor productivity<sup>37</sup>. First, I calculate the measures of economies of scale (for the two alternative diversity measures)  $\theta_q^h$  and  $\theta_q^e$  and report them in Table 4. As Table 4 shows, technology has been characterized by an increasing degree of economies of scale over the sample period.<sup>38</sup>

Given the diversity and scale elasticities, I decompose the change in total costs into its components, as given in equation (1). Table 5 gives the percentage rates of growth in total cost and its components<sup>39</sup>. As this tables 6 indicates, total costs have increased throughout the sample period ( $RGTC$  is positive in every year). The growth in total costs, however, is mainly due to growth in input prices and output. The absolute contributions of changes in diversity and technical efficiency are rather small, but as was shown above, an increase in diversity increases labour productivity.

Next, I adjust the rate of growth in total costs by the rate of growth in output to get rate of growth in average costs. The rate of growth in average costs and its scale component as in equation (2) are given in Table 6<sup>40</sup>. As Tables 5 and 6 show, the most important component in explaining the rate of growth in average costs is the rate of growth in input prices. Table 6 also shows that due to economies of scale, output growth (when there is positive growth)

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<sup>37</sup>Since both the entropy and Herfindahl models yield similar productivity measures, I only report the results of the Herfindahl model.

<sup>38</sup>Note that the measure of economies of scale is inversely related to the scale elasticity.

<sup>39</sup>In all the calculations, I follow the standard convention and calculate all elasticity weights as the average of every two adjacent periods.

<sup>40</sup>The other three components are shown in Table 5.

reduces average costs.

Finally, I adjust the rate of growth in average costs to account for input price changes to obtain the rate of growth in production efficiency, which is the same as the negative of the rate of growth in total factor productivity, as in equation (3). The rates of growth in production efficiency are shown in Table 6<sup>41</sup>. As Table 6 shows, the rate of growth in “adjusted” average costs was negative in most periods, implying an increase in total factor productivity. Examining the contribution of diversity to rate of growth in total factor productivity we see that although the absolute contribution of changes in diversity is small, its contribution relative to the rate of growth in total factor productivity, which itself is a small number, is more significant<sup>42</sup>.

## 5 Conclusion

In this paper I provide an analysis of some of the economic benefits of population diversity. I estimate a cost function model for the manufacturing industry in Ontario and find that increased diversity, increased labour and production efficiency. I also find that increased diversity had a positive effect on the demand for labour, total factor productivity and competitiveness.

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<sup>41</sup>The other components of RGPE were shown in Table 5.

<sup>42</sup>The signs of the contributions of diversity (the signs of  $\theta_v \dot{v}$ ) during the sample period are explained as follows: whenever diversity increased (decreased), the corresponding increase (decrease) in labour productivity led to an increase (decrease) in rate of growth in total factor productivity. The fact that the signs of the contributions change over the period indicates that there have been both increases and decreases in  $v$  during the sample period.

## 6 Appendix

### 6.1 Productivity

We define technology by the production function

$$q = F(x, le^{av}, t) \equiv F(x, l^*, t). \quad (4)$$

where  $q$  is output,  $x$  is a vector of other inputs used in the production process  $t$  is a shift variable representing technical change,  $l$  is the total number of units of labour,  $v$  is a measure of diversity,  $a$  is the rate of productivity growth of labour due to increased diversity and  $l^* = le^{av}$  is labour in efficiency units.

Since firms minimize their production costs, we define the cost function corresponding to the production function  $F$ , as the minimum cost of producing an output level  $q$ , given the wage rate  $w$ , other input prices  $r$  and the measure of diversity,  $v$ . It can be written as

$$C(r, we^{-av}, q, t) \equiv C(r, w^0, q, t) \quad (5)$$

where  $w^0 \equiv we^{-av}$  is the wage per efficiency unit of labour.

As can be seen from the cost function, an increase in diversity changes the effective wage rate and will, therefore, have the same effect as a change in the actual wage rate. Specifically, if  $a > 0$ , diversity makes labour more efficient, hence reducing the effective wage rate and decreasing production costs.

To analyze changes in productivity, I look at the rate of growth in the cost of production over time. I define the proportionate rate of growth of a variable, say  $b$ , as  $\dot{b}$ <sup>43</sup>. From the cost function, the rate of growth in total costs, RGC, is then given by

$$RGC = \theta_q \dot{q} + \dot{W} + \theta_v \dot{v} + \theta_t \quad (6)$$

where  $\theta_q$ , and  $\theta_v$  are the elasticities of cost with respect to output and diversity, respectively<sup>44</sup>,  $\theta_t$  is the rate of technical change and  $\dot{W}$  is the rate of growth in the aggregate input price<sup>45</sup>.

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<sup>43</sup>In other words,  $\dot{b} \equiv \frac{\partial \log(b)}{\partial t}$ .

<sup>44</sup>The elasticities measure the sensitivity of cost with respect to these variables.

<sup>45</sup>Defined by:  $\dot{W} \equiv \sum_i s_i \dot{w}_i$ , where  $s_i$  is the cost share of input  $i$ , given by  $x_i w_i / C$ .



As can be seen in (6), the contributions of these four factors to the rate of growth in total costs depend on the growth rates of the underlying variables and their elasticities.

The rate of growth in total costs can be decomposed further to obtain the rate of growth in average costs. I define the rate of growth in average costs as:  $RGAC \equiv RGC - \dot{q}$ , i.e., the rate of growth in total costs minus the rate of growth in output. The rate of growth in average costs is given by

$$RGAC \equiv \dot{C} - \dot{q} = (\theta_q - 1)\dot{q} + \dot{W} + \theta_v \dot{v} + \theta_t \quad (7)$$

which indicates that average costs change due changes in input prices, output<sup>46</sup>, diversity and technical change.

Since a reduction in average costs captures improved production efficiency, it also captures improvements in the degree of competitiveness. But, the rate of growth in input prices is exogenous to firms, so that it is unrelated to endogenous changes in production efficiency. Thus, when examining changes in production efficiency, we should look at contributions to the rate of growth in average costs, excluding the rate of growth in input prices. We define the rate of growth in production efficiency, RGPE, as the rate of growth in average costs, excluding the effects of changes in input prices:

$$RGPE \equiv \dot{C} - \dot{q} - \dot{W} = (\theta_q - 1)\dot{q} + \theta_v \dot{v} + \theta_t \quad (8)$$

RGPE is equivalent to the standard measure of the rate of growth of total factor productivity (RGTFP). In fact, it can be shown that the rate of growth in production efficiency is equal to minus the rate of growth of total factor productivity:  $RGPE \equiv -RGTFP$ . This decomposition shows that total factor productivity can be explained by changes in technical efficiency, scale of operations and diversity. The contribution of diversity to productivity is captured by the term  $\theta_v \dot{v}$ , which is composed of the rate of growth in diversity ( $\dot{v}$ ) and the sensitivity of costs with respect to diversity, ( $\theta_v$ ).

To be able to calculate the components of productivity in (6), we have to estimate the scale elasticity,  $\theta_q$ , and the diversity elasticity,  $\theta_v$ . Given estimates of these elasticities we can calculate the rate of growth in total factor productivity and its components.

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<sup>46</sup>As long as we do not have constant returns to scale, that is, as long as  $\theta_q \neq 1$ .

## 6.2 Empirical Model

I assume that the industry's cost function can be approximated by the translog function:

$$\begin{aligned}
 \ln C &= a_0 + \sum_i a_i \ln p_i + \frac{1}{2} \sum_i \sum_j a_{ij} \ln p_i \ln p_j + \sum_i a_{iq} \ln p_i \ln q \\
 &\quad + a_q \ln q + \frac{1}{2} a_{qq} (\ln q)^2 + \sum_i a_{it} \ln p_i \\
 &\quad + a_t t + \frac{1}{2} a_{tt} t^2 \quad p_i, p_j = r_k, r_m, w^0
 \end{aligned} \tag{9}$$

Linear homogeneity in prices and symmetry imply the following parameter restrictions:

$$\sum_i a_i = 1, \sum_i a_{ij} = 0 \quad \text{all } j, \sum_i a_{iy} = 0, a_{ij} = a_{ji}, \sum_i a_{it} = 0 \tag{10}$$

Applying Shephard's Lemma to the translog cost function, we get the input cost share equations as

$$\begin{aligned}
 s_l &= a_l + a_{ll} \ln w^0 + a_{lk} \ln r_k + a_{lm} \ln r_m + a_{lq} \ln q + a_{lt} \\
 s_k &= a_k + a_{lk} \ln w^0 + a_{kk} \ln r_k + a_{km} \ln r_m + a_{kq} \ln q + a_{kt} \\
 s_m &= a_m + a_{lm} \ln w^0 + a_{km} \ln r_k + a_{mm} \ln r_m + a_{mq} \ln q + a_{mt}
 \end{aligned} \tag{11}$$

where  $s_l$ ,  $s_k$  and  $s_m$  are the shares of labour, capital and materials in total costs.

For the estimated system to be consistent with optimizing behaviour, we have to impose the linear homogeneity and symmetry restrictions. Furthermore, since cost shares sum to one, only two of the input share equations in (11) are independent. In the estimation we, therefore, drop the materials' share equation. The full model consists of the cost function and the labour and capital share equations.

For empirical implementation the model has to be imbedded within a stochastic framework.

To do this, we assume that equation (9), and the first two equations in (11) are stochastic due to "errors in optimization". We define the "optimization errors" in the cost and share equations at time  $t$ , as  $v_c(t), v_l(t), v_k(t)$ . We denote the column vector of disturbances at

time  $t$  as  $e(t) \equiv \{e_c(t), e_l(t), e_k(t)\}$  and assume that the vector of disturbances is identically and independently distributed with mean zero and non-singular covariance matrix  $\Omega$

$$E[e(s)e(t)] = \begin{cases} \Omega & \forall s, t \text{ if } s = t \\ 0 & \text{if } t \neq s \end{cases} \quad (12)$$

where  $\Omega$  is a  $3 \times 3$  positive definite matrix.

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Table 1: Measures of Diversity (normalized):  
 Herfindahl Index  $1 - D_h$ , Entropy Index  $D_e$

year	$1 - D_h$	$D_e$
1962	1.00000	1.00000
1963	0.97502	0.97167
1964	0.97443	0.97830
1965	0.95912	0.94407
1966	0.94420	0.95790
1967	0.98797	1.01888
1968	1.02791	1.07049
1969	1.04140	1.10318
1970	1.04717	1.11586
1971	1.06426	1.15197
1972	1.06274	1.15455
1973	1.06653	1.15488
1974	1.05847	1.13396
1975	1.05984	1.14136
1976	1.06200	1.14481
1977	1.05773	1.13941
1978	1.06593	1.15501
1979	1.01949	1.07275
1980	1.00066	1.04249
1981	1.04593	1.11501
1982	1.06205	1.14479
1983	1.06111	1.14942
1984	1.03840	1.10424
1985	1.03534	1.10346
1986	1.04404	1.11138
1987	1.05793	1.13668
1988	1.03901	1.09278

Table 2: Cost Function: Herfindahl (H) and Entropy (E) Index Models  
Parameter Estimates

H Model Parameters	Estimate	t-Statistic	E Model Parameters	Estimate	t-Statistic
$a_k$	.075452	16.1388	$a_k$	.077864	17.9986
$a_{kk}$	.034885	12.0304	$a_{kk}$	.034939	12.2174
$a_{kl}$	-.935617E-02	-4.47905	$a_{kl}$	-.931525E-02	-4.77736
$a$	.670790	4.89731	$a$	.414745	5.58526
$a_{kq}$	-.059441	-2.74049	$a_{kq}$	-.060028	-2.77766
$a_{kt}$	.249768E-02	3.11238	$a_{kt}$	.251268E-02	3.13848
$a_l$	.394978	13.6747	$a_l$	.354551	20.8498
$a_{ll}$	.172162	9.78822	$a_{ll}$	.179105	10.2567
$a_{lq}$	-.021983	-1.48429	$a_{lq}$	-.018958	-1.38105
$a_{lt}$	-.441760E-02	-9.84685	$a_{lt}$	-.455242E-02	-10.8951
$a_0$	.229165	4.19440	$a_0$	.134980	5.05982
$a_q$	.863213	20.4480	$a_q$	.884649	21.8666
$a_{qq}$	-.103136	-1.67693	$a_{qq}$	-.119421	-1.94630
$a_t$	-.335754E-02	-2.05098	$a_t$	-.250132E-02	-1.54344
$a_{tt}$	.268927E-03	2.44429	$a_{tt}$	.274603E-03	2.51435
Equation	H Model $R^2$	H Model D.W.	E Model $R^2$	E Model D.W.	
Labour	0.977889	1.04641	0.980642	1.2135	
Capital	0.878441	1.46142	0.879047	1.46619	
Cost	0.9999	1.933	0.9999	2.0259	



Table 3: Diversity Cost Elasticities ( $\theta_v^h, \theta_v^e$ ) and the Effects of Increased Diversity on Production Costs (in millions of dollars) ( $\mu^h, \mu^e$ )

year	$\theta_v^h$	$\theta_v^e$	$\mu^h$	$\mu^e$
1962	-0.18998	-0.11746	-206.50616	-127.68124
1963	-0.18414	-0.11346	-218.15854	-134.42274
1964	-0.18324	-0.11375	-239.67717	-148.77872
1965	-0.17884	-0.10884	-263.79382	-160.54240
1966	-0.17343	-0.10879	-289.55286	-181.62479
1967	-0.18323	-0.11683	-319.57590	-203.77258
1968	-0.19468	-0.12535	-356.55933	-229.58948
1969	-0.19375	-0.12690	-395.45105	-259.00854
1970	-0.20061	-0.13217	-417.42029	-275.01666
1971	-0.20413	-0.13661	-451.64899	-302.26511
1972	-0.20800	-0.13971	-498.27206	-334.69473
1973	-0.20808	-0.13932	-568.60883	-380.69214
1974	-0.18647	-0.12352	-644.27344	-426.76001
1975	-0.17917	-0.11930	-687.39642	-457.70569
1976	-0.18218	-0.12142	-782.63196	-521.62408
1977	-0.18092	-0.12050	-850.14545	-566.22986
1978	-0.17644	-0.11821	-955.92596	-640.43597
1979	-0.15920	-0.10357	-1032.66138	-671.83984
1980	-0.15539	-0.10009	-1104.51965	-711.46643
1981	-0.15860	-0.10454	-1295.89465	-854.15637
1982	-0.15856	-0.10567	-1349.65784	-899.49316
1983	-0.16079	-0.10769	-1443.37537	-966.69775
1984	-0.15138	-0.099534	-1577.44702	-1037.16907
1985	-0.15422	-0.10163	-1709.63171	-1126.60645
1986	-0.15941	-0.10492	-1824.82520	-1201.05396
1987	-0.16294	-0.10825	-1950.73425	-1295.90271
1988	-0.15974	-0.10388	-2098.20361	-1364.43811

Table 4: Diversity Elasticities of Labour Demand and Marginal Cost  
 $(\theta_v^l, \theta_v^{mc}, \text{H Model})$  and Scale Elasticities  $(\theta_q^h, \theta_q^e, \text{both models})$

year	$\theta_v^l$	$\theta_v^{mc}$	$\theta_q^h$	$\theta_q^e$
1962	0.073052	-0.17318	0.87796	0.89251
1963	0.069959	-0.16772	0.87584	0.88928
1964	0.068989	-0.16670	0.86864	0.88073
1965	0.066062	-0.16221	0.85039	0.86093
1966	0.061719	-0.15661	0.82796	0.83760
1967	0.066816	-0.16549	0.82151	0.83086
1968	0.074392	-0.17651	0.83448	0.84304
1969	0.071196	-0.17487	0.81315	0.82080
1970	0.078376	-0.18176	0.81953	0.82779
1971	0.079931	-0.18493	0.81736	0.82485
1972	0.084243	-0.18940	0.84251	0.84935
1973	0.083868	-0.19086	0.91319	0.91944
1974	0.058107	-0.16732	0.81507	0.81968
1975	0.046115	-0.15964	0.80010	0.80556
1976	0.050614	-0.16256	0.79844	0.80314
1977	0.049557	-0.16166	0.80969	0.81399
1978	0.039722	-0.15709	0.81239	0.81571
1979	0.018910	-0.14015	0.78928	0.79177
1980	0.016656	-0.13666	0.78782	0.79096
1981	0.0086570	-0.13884	0.78078	0.78343
1982	0.0027709	-0.13829	0.77282	0.77681
1983	0.0085228	-0.14066	0.77741	0.78053
1984	0.0066096	-0.13118	0.75809	0.75902
1985	0.0018308	-0.13411	0.75913	0.75932
1986	0.011232	-0.13891	0.75099	0.75058
1987	0.014613	-0.14212	0.74921	0.74857
1988	0.013687	-0.13917	0.74508	0.74354

Table 5: Rate of Growth in Total Costs and its Components:  
 $RGC, \theta_q \dot{q}, \dot{W}, \theta_v \dot{v}, \theta_t$

year	$RGC$	$\theta_q \dot{q}$	$\dot{W}$	$\theta_v \dot{v}$	$\theta_t$
1963	8.70421	7.66970	0.61378	0.47320	-0.052476
1964	9.65858	7.97123	1.73841	0.011092	-0.062161
1965	12.12605	8.13295	3.74562	0.28680	-0.039329
1966	12.01340	5.96017	5.76111	0.27605	0.016075
1967	4.01535	1.10671	3.64499	-0.80803	0.071675
1968	4.86767	5.03650	0.52530	-0.74885	0.054727
1969	10.70270	4.88879	6.01313	-0.25323	0.054014
1970	1.51282	-1.78409	3.32598	-0.10891	0.079840
1971	7.31941	5.22117	2.36532	-0.32761	0.060534
1972	7.54161	5.56393	1.95788	0.029492	-0.0096929
1973	13.28654	6.53340	7.04074	-0.074024	-0.21356
1974	24.90738	2.13912	22.74141	0.14957	-0.12271
1975	8.23017	-4.83828	12.92712	-0.023571	0.16490
1976	11.82390	4.86530	6.78822	-0.036928	0.20731
1977	10.06442	3.05158	6.76704	0.073238	0.17256
1978	13.39434	4.81361	8.56382	-0.13801	0.15492
1979	17.74673	2.70936	14.07240	0.74748	0.21748
1980	8.70000	-3.67681	11.77896	0.29331	0.30453
1981	14.25213	1.77031	12.81196	-0.69471	0.36457
1982	3.39654	-6.30719	9.51713	-0.24256	0.42916
1983	6.58096	5.63563	0.49110	0.014181	0.44005
1984	15.28150	10.07398	4.43378	0.33772	0.43602
1985	6.05497	4.25831	1.30278	0.045105	0.44877
1986	2.26006	2.88263	-0.94583	-0.13121	0.45448
1987	4.54071	1.21447	3.05917	-0.21303	0.48009
1988	8.46949	4.43840	3.24997	0.29111	0.49001

Table 6: Rates of Growth in Average Costs and Production Efficiency and Scale Contribution:  $RGAC$ ,  $RGPE$ ,  $(\theta_q - 1)\dot{q}$

year	$RGAC$	$RGPE$	$(\theta_q - 1)\dot{q}$
1963	-0.042175	-0.65596	-1.07669
1964	0.51976	-1.21865	-1.16758
1965	2.66379	-1.08184	-1.32931
1966	4.91097	-0.85014	-1.14227
1967	2.67344	-0.97155	-0.23519
1968	-1.21511	-1.74041	-1.04628
1969	4.76835	-1.24478	-1.04556
1970	3.69829	0.37231	0.40138
1971	0.94007	-1.42525	-1.15817
1972	0.83759	-1.12029	-1.14009
1973	5.84405	-1.19669	-0.90910
1974	22.43191	-0.30950	-0.33635
1975	14.22124	1.29412	1.15279
1976	5.73669	-1.05153	-1.22191
1977	6.26921	-0.49783	-0.74363
1978	7.45923	-1.10459	-1.12150
1979	14.36357	0.29117	-0.67379
1980	13.36273	1.58377	0.98592
1981	11.99495	-0.81701	-0.48687
1982	11.51601	1.99888	1.81228
1983	-0.68975	-1.18085	-1.63508
1984	2.16012	-2.27365	-3.04740
1985	0.44168	-0.86110	-1.35498
1986	-1.55768	-0.61185	-0.93512
1987	2.92163	-0.13754	-0.40461
1988	2.52903	-0.72094	-1.50206