The Effects of Union Contracts on the Firm’s Financial Structure

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Abstract

This paper examines the effects of union contracts on the firm’s capital structure. We consider one-stage and two-stage models, as well as wage and wage/employment contracts. We show that, for all Pareto efficient bargaining solutions, a higher debt reduces the expected tax bill, but increases the expected cost of labour contracts. This trade-off determines the optimal capital structure. We also show that a stronger union tends to increase the amount of equity used.
1 Introduction

The purpose of this paper is to examine interactions of contracts in labour and financial markets and to show that such interactions can determine the firm’s capital structure. Specifically, we consider a firm that makes production and financial decisions and in addition, has to negotiate a contract with workers. The wage contract may be signed directly with workers, or with a workers’ union. The discussion focuses on risk sharing, rather than incentives, considerations. We show that for all efficient contracts, a higher debt reduces the expected tax bill, but increases the expected cost of the labour contract. This trade-off between the effects of debt on the cost of labour and the tax bill determines the optimal capital structure. We also show that a stronger union results in a lower debt/equity ratio.

2 The Model

2.1 Production and Legal Structure

Consider a firm whose revenues, \( R \), are given by the function

\[
R = R(K, L, \theta)
\]

where \( K \) and \( L \) are capital and labour inputs used in production and \( \theta \) is a random variable, capturing the uncertainty facing the firm. It is convenient to define \( \theta \) such that \( \frac{\partial R}{\partial \theta} > 0 \) (i.e., better states are represented by higher values of \( \theta \)). We assume that the random variable \( \theta \), is distributed over the interval \([0, 1]\) according to the density function \( g(\theta) \) and that \( R(K, L, \theta) \geq 0 \), for all \( K, L, \theta \), with \( R(0, L, \theta) = R(K, 0, \theta) = 0 \).

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1For a general discussion of the literature on capital structure, see Copeland and Weston (1988) and references therein. For examples of studies that consider real and financial decisions, see Dotan and Ravid (1985), Brander and Lewis (1986), (1988), Brander and Spencer (1989), Appelbaum, (1993a), (1993b), Ravid (1988), provides a useful survey of this literature.

2Since the focus is on contracts with a union, or a "general" work force (rather than contracts with individual "managers"), we do not introduce incentive considerations. It is, of course, possible to add these considerations into the model.

3It has been also been shown in the literature that the firm’s capital structure can be used as a strategic tool (as a pre-commitment instrument) in the union/firm bargaining game. This will increase the incentive for debt financing, thus increasing the debt/equity ratio.
The firm makes its decisions at the beginning of the period, before uncertainty is resolved\(^4\). It can finance its capital using equity, \(e\), or debt, \(b\) (or both). To the extent that it uses debt, the interest rate is determined in negotiations with debtholders. The firm faces a union with which it negotiates a labour contract. In this section we assume that both financial and labour contracts are determined at the beginning of the period, prior to the resolution of uncertainty. In the following section we will consider a two-stage problem. We also assume that the distribution of \(\theta\) is common knowledge and that all actions are observable\(^5\).

At the end of the period, the state of the world \(\theta\) is observed by all agents. At this date, given its choice of debt, equity (where the price of \(K\) is normalized to one, so that \(K = b + e\)) and given the realized value of \(\theta\), the firm faces claims from debtholders and workers in the amount of \((1+r)b+wL\), where \(r\) and \(w\) are the contracted wage and interest rates respectively\(^6\). At this point it also faces a tax claim from the government.

If the firm’s terminal assets, \(R(K, L, \theta) + K\), are insufficient to meet its obligations in full, it defaults and goes into bankruptcy\(^7\). Given limited liability of equity holders, bankruptcy occurs if \(R(K, L, \theta) + K < wL + (1+r)b\), i.e., when terminal net worth is negative. In the case of bankruptcy, the firm’s terminal assets are distributed among claimants in accordance with priority rules established in the bankruptcy law. For example, according to the bankruptcy law in the U.S. and Canada (See Altman (1983), Willes (1985)), secured creditors receive the saleable value of the assets which are subject to security. If the value of the security is insufficient to satisfy the claims, the secured creditor is entitled to claim the remainder as an unsecured creditor. Unsecured assets are distributed, according to the U.S. and Canadian laws in the following order: (i) administrative costs, (ii) taxes, (iii) wages and rents, (iv) unsecured

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\(^4\)Defining the problem in this way enables us to avoid the moral hazard problems that arise if the firm’s debt structure had to be chosen first. These problems are discussed, for example, in the papers mentioned in footnote 2.

\(^5\)Since the effects of asymmetric information on the firm’s capital structure have been discussed in the literature extensively (see Jensen and Meckling (1976), Leland and Pyle (1977), Harris and Raviv (1985), Darrough and Stoughton (1986)) and since our focus is on the interaction of real and financial markets, we do not consider these issues here.

\(^6\)The wage rate could be thought of as the “full”, or “effective” wage, i.e., it could take into account various fringe benefits, pensions etc., which workers receive as part of their employment package and which cannot be insured through existing institutions.

\(^7\)For simplicity, and without affecting any of our results, we assume zero depreciation. We also do not consider the effects of bankruptcy costs, which have been extensively discussed in the literature.
creditors, (v) equity holders. If several claimants have the same priority, they are paid on a pro-rata basis. We assume that debtholders are secured creditors. The government’s tax claims may be positive, or negative, depending on whether the firm made profits or losses. Equity holders, are residual claimants.

2.2 Debtholders

Given that debtholders are first claimants (where their claim can be applied against \( R + K \))\(^9\), their receipts are

\[
b(\theta) = \begin{cases} 
(1 + r)b & \text{if } \theta \geq \theta^1 \\
R + K & \text{if } \theta \leq \theta^1 
\end{cases}
\]  

(2)

where \( \theta^1 \) is the “debt-default” state, which solves

\[
R(K, L, \theta^1) + K - (1 + r)b = 0.
\]  

(3)

We assume that debtholders are risk neutral and face an opportunity cost rate of return, \( s \). In a competitive capital market, equilibrium requires that the net value of debt is zero, i.e.,

\[
D(K, L, b, r) \equiv \int_0^{\theta^1} \{R + K\}g(\theta)d\theta + \int_{\theta^1}^1 (1 + r)b g(\theta)d\theta - (1 + s)b = 0
\]  

(4)

This “supply” condition can be solve for the contractual rate of interest required to induce debtholders to supply loans as

\[
r = r(K, L, b).
\]  

(5)

The effect of a change in capital structure (for a given \( K \)) on \( r \) can be obtained from (4) as:

\[
\frac{\partial r}{\partial b} \equiv r_b = \left\{ \int_0^{\theta^1} [R + K]g(\theta)d\theta \right\} \left\{ \int_{\theta^1}^1 b^2 g(\theta)d\theta \right\}^{-1} > 0
\]  

(6)

indicating that an increase in the debt/equity ratio will increase the equilibrium rate of interest.

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\(^8\)In practice the bankruptcy process is more complicated and may involve deviations from absolute priority rules. The payoffs in bankruptcy are often the outcome of a bargaining process among the parties. The bankruptcy law provides the guidelines that govern this bargaining process. The constraints imposed by the rules and equity’s relative strength in the in the bargaining process (e.g. “agenda power”), determine actual payoffs. The payoffs in this paper can, therefore, be viewed as approximations to the payoffs of these more complicated bargaining processes.

\(^9\)Other types of securities, such as specific liens and mortgages yield similar results.
2.3 Workers

We assume that workers are unionized and the union has a membership of $\bar{L}$. We assume, in this section, that both wage and employment levels are determined in union-firm bargaining. Given a wage and employment contract $(w, L)$, and given that in case of bankruptcy workers receive their payment after debtholders, the actual receipts of each worker will be

$$n(\theta) = \begin{cases} 
\frac{w}{R + K - (1+r)b} & \text{if } \theta \geq \theta^0 \\
0 & \text{if } \theta^1 \leq \theta \leq \theta^0 \\
\bar{\theta} & \text{if } \theta \leq \theta^1
\end{cases}$$  \hspace{1cm} (7)

where the bankruptcy state, $\theta^0$, solves

$$R(K, L, \theta^0) + K - (1+r)b - wL = 0 .$$  \hspace{1cm} (8)

From equations (3) and (8) it is clear that $\theta^0 \geq \theta^1$, for all $wL \geq 0$.

We assume that the union’s utility function is given by

$$U = U[n(\theta), L, \bar{L}]$$  \hspace{1cm} (9)

where $U$ is increasing in $(n, L, \bar{L})$ and concave in $n^{10}$. We denote: $U_n \equiv \frac{\partial U(n(\theta), L)}{\partial n}$. The union utility function in (9) is very general and has been used extensively in the literature$^{11}$. Given workers’ receipts the union’s expected utility is given by:

$$E[U(n(\theta), L)] = \int_{\theta^1}^{\theta^0} U(w(\theta), L) g(\theta) d\theta + \int_{\theta^0}^{1} U(w; L) g(\theta) d\theta$$  \hspace{1cm} (10)

where, for the sake of notational convenience, we define: $w(\theta) \equiv [R + K - (1+r)b]/L$ and where we assumed that $U(0, L) = 0$ for all $L$.

2.4 The Firm:

The firm’s tax liabilities are determined by its operating profits. It is well recognized, however, that the treatment of corporate profits and losses is not symmetric, in that profits

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$^{10}$For the sake of notational convenience, and since we are not concerned with the determination of union membership, we drop $\bar{L}$ in the function $U$, for the rest of the paper.

$^{11}$See for example, Oswald (1982), Farber (1986), Svejnar (1986), Anderson and Devereux (1989). Often the utility function was taken as $U(w, L) = LU(w) + (\bar{L} - L)U(\bar{w})$, where $w$ is the (certain) wage, $\bar{w}$ is the opportunity cost wage rate and $U$ is concave in $w$. This of course is a special case of the general function in (9).
usually attract immediate taxes, whereas losses do not attract immediate payment from the tax authorities\textsuperscript{12}. If we define $\theta^t$ by

$$ R(K, L, \theta^t) - rb - wL = 0 $$

(11)

then, the firm’s tax bill is given by

$$ T(\theta) = \begin{cases} 
    t[R - rb - wL] & \text{if } \theta \geq \theta^t \\
    \gamma t[R - rb - wL] & \text{if } \theta \leq \theta^t 
\end{cases} $$

(12)

where $t$ is the tax rate and the parameter $0 \leq \gamma \leq 1$ reflects the extent to which the effective treatment of losses and profits is asymmetric. The greater the asymmetry, the smaller the value for $\gamma$. Thus, if no loss offset is allowed then $\gamma = 0$, whereas if immediate full loss offset is possible, then $\gamma = 1$. In general, $\gamma$ is expected to lie between zero and one.

Given the claims by debtholders, workers and the government, under limited liability, equity holders receive

$$ e(\theta) = \begin{cases} 
    [R + K - (1 + r)b - wL] - t[R - rb - wL] & \text{if } \theta \geq \theta^t \\
    [R + K - (1 + r)b - wL] - \gamma t[R - rb - wL] & \text{if } \theta^0 \leq \theta \leq \theta^t \\
    -\gamma t[R - rb - wL] & \text{if } \theta \leq \theta^0. 
\end{cases} $$

(13)

Assuming that equity holders are also risk neutral and face the same opportunity cost rate of return, $s$, the net value of equity, $V$, is

$$ V = (1 + s)^{-1}\{E[e(\theta)] - (1 + s)e\}. $$

(14)

If we substitute the capital market equilibrium requirement (4), into (14), we get the net value of equity (subject to (4)) as

$$ V = (1 + s)^{-1}\{\int_0^1[R(K, L) - sk]g(\theta)d\theta - E[N(\theta)] - E[T(\theta)]\}. $$

(15)

where $E(N(\theta)) = LE[n(\theta)]$ is the expected total cost of labour. It is useful to note that the net value of the firm, $VF$, is the sum of the net values of equity and debt; $VF = (1 + s)^{-1}\{E[e(\theta)] - (1 + s)e\} + \{E[b(\theta)] - (1 + s)b\}$. But, using (2) and (13) we get that: $VF = V$.

In other words, firm value maximization and equity value maximization are identical, in this case. The net value of the firm is, therefore, simply the expected value of revenues minus expected costs of labour, taxes and capital.

\textsuperscript{12}This is a typical assumption in the finance literature. For a general discussion of these tax asymmetries see Auerbach (1983), (1986), Cooper and Franks (1983), Jog and Mintz (1989), Appelbaum and Katz (1986), (1987).
3 Efficient Capital Structure:

We assume that investment, capital structure and contracts with debtholders and the union (the optimal values of $K, b, e, r, w, L$) are all determined at the same time, at beginning of the period. An alternative structure, with a two stage decision process, will be examined in the next section.

Instead of looking at a specific bargaining solution, we simply assume that the optimal values of $w, L, K, b$ and $e$ are Pareto efficient. In other words, we assume that the outcome is on the “contract curve”. In the following, we refer to a capital structure that satisfies the Pareto efficiency conditions, as efficient. The contract curve (efficient values of $w, L, K, b$ and $e$) can be obtained by the solution to the problem$^{13}$:

$$\text{Max}_{(w, L, K, b)} \{V(K, L, b, w) : E[U(n(\theta), L)] \geq U^0, \ r = r(K, L, b), \ b \leq K\} \equiv H(U^0). \quad (16)$$

Problem (16) can be solved in two steps. First, for any given $K$ and $L$, we solve for the efficient $w$ and $b$, i.e., we solve the problem

$$\text{Max}_{(w, b)} \{V(K, L, b, w) : E[U(n(\theta), L)] \geq U^0, \ r = r(K, L, b), \ b \leq K\} \equiv J(K, L, U^0) \quad (17)$$

Then, we obtain efficient values of $K$ and $L$ by solving the problem:

$$\text{Max}_{(K, L)} J(K, L, U^0) \equiv H(U^0). \quad (18)$$

Since the objective of the paper is to explain the choice of capital structure, we will focus on problem (17)$^{14}$. But, since $w$ and $b$ only affect the wage and tax bills (they only appear in $E[N(\theta)]$ and $E[T(\theta)]$), problem (17) is equivalent (yields the same solution for $b$ and $w$) as the minimization of the sum of the expected costs of labour and taxes, given the required level of expected utility, $U^0$. In other words, we can find the efficient values of $b$ and $w$ by solving the equivalent problem:

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$^{13}$Note that since $e$ does not appear explicitly in $V$ or in $E(U)$ (it only appears (implicitly) in $K$), we can simply maximize with respect to $w, L, K, b$ and then solve for $e$ from the constraint $K = e + b$.

$^{14}$This does not mean that we are assuming that $K$ and $L$ are fixed. It simply means that for any $K$ and $L$, the efficient choices of $w$ and $b$ must satisfy the optimality conditions corresponding to problem (17).
\[ \text{Min}_{(w,b)} \{(1+s)^{-1}(E[N(\theta)] + E[T(\theta)]): E[U(n(\theta), L)] \geq U^0, } \]
\[ r = r(K, L, b), \ b \leq K \} \]  \hspace{1cm} (19)

If we substitute the constraint, \( r = r(K, L, b) \), directly, then the Lagrangean corresponding to problem (19) can be written as:

\[ \mathcal{L}(K, L, b, w) \equiv (1+s)^{-1} \{ E[N(\theta)] + E[T(\theta)] \} + \lambda \{ E[U^0 - U(n(\theta), L)] \} + \phi(b - K) \]

where \( \lambda \) and \( \phi \) are the Lagrangean multipliers corresponding to the constraints \( E[U(n(\theta), L)] \geq U^0 \) and \( b \leq K \), respectively. The Kuhn-Tucker conditions are, therefore, given by:

\[ \frac{\partial \mathcal{L}}{\partial b} = (1+s)^{-1} \left[ \frac{\partial E[N(\theta)]}{\partial b} + \frac{\partial E[T(\theta)]}{\partial b} \right] - \lambda \frac{\partial E[U(n(\theta)]}{\partial b} + \phi \geq 0, \ \frac{\partial \mathcal{L}}{\partial b} b = 0, \ b \geq 0 \]  \hspace{1cm} (20)

\[ \frac{\partial \mathcal{L}}{\partial w} = (1+s)^{-1} \left[ \frac{\partial E[N(\theta)]}{\partial w} + \frac{\partial E[T(\theta)]}{\partial w} \right] - \lambda \frac{\partial E[U(n(\theta)]}{\partial w} \geq 0, \ \frac{\partial \mathcal{L}}{\partial w} w = 0, \ w \geq 0 \]  \hspace{1cm} (21)

\[ \frac{\partial \mathcal{L}}{\partial \phi} = b - K \leq 0, \ \frac{\partial \mathcal{L}}{\partial \phi} \phi = 0, \ \phi \geq 0 \]  \hspace{1cm} (22)

\[ \frac{\partial \mathcal{L}}{\partial \lambda} = U^0 - E[U(n(\theta), L)] \leq 0, \ \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \ \lambda \geq 0 \]  \hspace{1cm} (23)

The efficient values of \( b \) and \( w \) solve the Kuhn-Tucker conditions (20), (21), (22) and (23). What can be said about the nature of this solution? In particular, will it be an interior solution with respect to \( b \)? Since \( \frac{\partial E[N(\theta)]}{\partial b} < 0, \ \frac{\partial E[T(\theta)]}{\partial b} < 0 \) and \( \frac{\partial E[U(n(\theta)]}{\partial b} < 0 \) (see appendix), it follows that the direct effect (holding everything else constant) of an increase in debt is to decrease both the tax bill and the cost of labour, but to increase the cost of achieving the required level of expected utility. What will the overall effect be? Assuming that the solution yields a strictly positive wage rate (which is the only sensible solution), we can solve for \( \lambda \) from the condition \( \frac{\partial \mathcal{L}}{\partial w} = 0 \). Using this in (20) we get\textsuperscript{15}:

\[ \frac{\partial \mathcal{L}}{\partial b} = A + B + \phi \geq 0, \ \frac{\partial \mathcal{L}}{\partial b} b = 0, \ b \geq 0 \]  \hspace{1cm} (24)

\textsuperscript{15}See appendix.
where
\[ A \equiv (1 + s)^{-1}(1 + r + b r_b) \int_{\theta_1}^{\theta_0} \left\{ \frac{U_n(w(\theta), L)}{U_n(w, L)} - 1 \right\} g(\theta) d\theta > 0 \tag{25} \]
and
\[ B \equiv -t(1+s)^{-1} \left\{ \gamma G(\theta^b) + |1 - G(\theta^b)| \right\} \left( r + b r_b + \int_{\theta_1}^{\theta_0} \frac{U_n(w(\theta), L)(1 + r + b r_b)}{|1 - G(\theta^b)| U_n(w, L)} g(\theta) d\theta \right) < 0 \tag{26} \]
Condition (24) can be interpreted as follows. An increase in debt effects the expected cost of labour, both directly and indirectly. For a given wage, the direct effect is to reduce the expected cost of labour \( \partial E[N(\theta)]/\partial b < 0 \). But, since an increase in debt increases the risk to workers, the wage rate will have to go up (for any given level of expected utility), therefore increasing the wage bill. What will be the overall effect on the wage bill? Given the concavity of the union’s utility function (and the risk neutrality of the firm), an increase in debt requires a higher risk premium to compensate for the increased risk. The indirect effect, therefore, always dominates. The overall effect of an increase in \( b \) on the wage bill is given by \( A \) in equation (25). As for the effects on the tax bill, we note that an increase in debt decreases the expected tax bill for two reasons: (i) interest costs are deductible, (ii) the higher wage which is needed to compensate for the greater risk is also deductible. The overall effect of an increase in \( b \) on the tax bill is given by \( B \) in equation (26). Thus, condition (24) says that there is a trade-off between the overall effects of an increase in debt on the wage and tax bills. It is this trade-off, that determines the efficient debt/equity ratio for any given \( K, L \) and \( U^0 \). In general, we can expect an interior solution for the efficient capital structure, \( b \), i.e., for any given \( K > 0 \) we will have \( b + e = K, \ 0 < b < K, \ 0 < e < K \). While the tax benefits of debt financing are well known\(^{17} \), the cost of debt financing due to higher costs of labour contracts has not been previously recognized.

To better understand this result, it is useful to consider the two special cases when there are no taxes and when the utility function exhibits risk neutrality. If there are no taxes, we

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\(^{16}\)The sign of \( A \) is obtained as follows. The concavity of the utility function in \( n \) implies that \( U_n(w(\theta), L) > U_n(w, L) \), since for all \( \theta^1 \leq \theta \leq \theta^0 \) we have \( w(\theta) \equiv R + K - (1 + r)\tilde{b}/L \leq w \). In addition, it was shown above in (6) that \( r_b > 0 \).

\(^{17}\)See Copeland and Weston (1988).
have from (26) that $B = 0$, so that $\frac{\partial C}{\partial b} = A + \phi > 0$, and therefore, $b = 0$. Since in this case, debt financing increases the cost of the labour contract, but does not provide tax benefits, the firm will be fully equity financed. On the other hand, if the union utility function is linear, we have from (25) that $A = 0$, so that $\frac{\partial C}{\partial b} = B + \phi$. But since $B < 0$ and in addition the Kuhn-Tucker condition requires that $B + \phi \geq 0$, we must have that $\phi > 0$. Thus, it follows from condition (22) that $b = K$. In this case, debt financing reduces the tax bill, but does not affect the cost of the labour contract. The firm will, therefore, be fully debt financed. Finally, in the case when there are no taxes and the union’s utility function is linear, we have $\frac{\partial C}{\partial b} - \frac{\partial C}{\partial b} \equiv 0$ (identically zero), which implies that the value of the firm’s is independent of its capital structure; the standard Modigliani-Miller result.

Before we conclude this section, let us examine what happens if consider a specific bargaining solution, instead of assuming only Pareto efficiency. For example, we can consider the generalized Nash bargaining solution examined in Svejnar (1986). As shown in Svejnar (1986) the generalized Nash bargaining solution can be obtained as the solution to the problem:

$$Max_{(w, L, K, b)} \{[V(K, L, b, w)^{1-\beta}(E[U(n(\theta), L)] - U^+)^\beta] : r = r(K, L, b)\}$$

(27)

where $\beta$ characterizes the relative bargaining strength of the union and $U^+$ is the union’s opportunity cost (possibly certain) utility. In the standard Nash bargaining solution $\beta = 1/2$. Again, if we focus on the choice of $b$, we can solve the problem in

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18 In this case $A = B = 0$, so that the Kuhn-Tucker condition becomes $\frac{\partial C}{\partial b} = \phi \geq 0, \frac{\partial C}{\partial b} b = 0, b \geq 0$. We prove that $\phi$ must be zero by contradiction. Suppose $\phi > 0$. Then, from (22) it follows that $b = K > 0$. But if $\phi > 0$, it must be that $\frac{\partial C}{\partial b} = \phi > 0$, so that we must have $b = 0$, which is a contradiction. Thus, we must have $\phi = 0$. 

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two stages; first we choose \( \beta \) given \( w, L, K \) and then, \( w, L, K \) are chosen. Thus, we can solve the problem:

\[
Max_{(b)} \{ [V(K, L, b, w)^{1-\beta}(E[U(n(\theta), L)] - U^+)\beta] \ : \ r = r(K, L, b) \}
\]

which yields the Kuhn-Tucker condition:

\[
Z \equiv \frac{\beta}{(1-\beta)} \frac{\partial E[U(n(\theta)]}{\partial b} V(\cdot) - \left[ \frac{\partial E[N(\theta)]}{\partial b} + \frac{\partial E[T(\theta)]}{\partial b} \right] (E[U(n(\theta)] - U^+) \leq 0
\]

Since we showed above that \( \frac{\partial E[N(\theta)]}{\partial b} < 0 \), \( \frac{\partial E[T(\theta)]}{\partial b} < 0 \) and \( \frac{\partial E[U(n(\theta)]}{\partial b} < 0 \), it follows from condition (29), that in general (assuming a bargain is reached), there will be an interior solution. Furthermore, condition (29) can be used to obtain the effects of an increase in the union’s strength, on capital structure. Assuming that there is an interior solution, we get:

\[
\frac{\partial b}{\partial \beta} = -\frac{1}{(1-\beta)^2} \frac{\partial E[U(n(\theta)]/\partial b}{\partial Z(b, \beta)/\partial b} V(\cdot) < 0
\]

since \( \partial Z(b, \beta)/\partial b < 0 \) (from the second order condition) and \( \partial E[U(n(\theta)]/\partial b < 0 \). In other words, a stronger union (a higher value for \( \beta \)) increase the relative cost of debt financing to the firm, thus reducing the amount of debt used.

4 Two Stage Model:

In the previous section all decisions were made at the same time; at the beginning of the period. In this section we consider a two stage decision model, in which the labour contract is determined in a second stage. All decisions, however, are still made before uncertainty is resolved. The two types of contracts we consider are: wage and wage/employment contracts. Both are commonly examined in the literature\(^{19}\). We look at two possible cases: (i) the firm chooses the level of employment at the beginning of the period and then a wage contract is negotiated with the union (ii) both the wage and the level of employment are determined in a contract in the second stage.

\(^{19}\)See for example, Oswald (1985) Farber (1986), Anderson and Devereux ((1989).
4.1 Wage Contract:

We assume that equity holders choose investment, capital structure and the level of employment at the beginning of the period\textsuperscript{20}. At this stage, also the interest rate is determined in a contract with debtholders. We refer to this as stage 1. In stage 2, given these choices, but before uncertainty is resolved, equity holders negotiates a wage contract with the union. The net value of equity in the second stage (given $K, L, b, r$) is given by:

\[
V = (1 + s)^{-1} \left[ \int_{\theta_0}^{1} \left[ R(K, L) - (1 + r)b - wL]g(\theta)d\theta - E[T(\theta)] - (1 + s)c \right] \right].
\]  

(31)

Hence, in the second stage, the wage contract is obtained by the solution to the problem:

\[
Max_{(w)} \{ V : E[U(n(\theta), L)] \geq U^0 \}
\]  

(32)

Obviously\textsuperscript{21}, for any given $U^0$, the solution to problem (32) must be such that $E[U(n(\theta), L)] = U^0$. Thus, for any given $U^0$, we can define the wage which solves the equation $E[U(n(\theta), L)] = U^0$, as: $w^0(K, L, b, r, U^0)$. This implies that:

\[
\begin{align*}
\frac{\partial w^0}{\partial b} &= -\frac{\partial E[U(n(\theta))]}{\partial b} = \frac{(1 + r) \int_{\theta_0}^{1} U_n(w(\theta), L)g(\theta)d\theta}{L_{n}(w, L)[1 - G(\theta^0)]} > 0 \\
\frac{\partial w^0}{\partial r} &= -\frac{\partial E[U(n(\theta))]}{\partial r} = \frac{b \int_{\theta_0}^{1} U_n(w(\theta), L)g(\theta)d\theta}{L_{n}(w, L)[1 - G(\theta^0)]} > 0
\end{align*}
\]  

(33)

In other words, the second stage contracted wage must be higher if debt, or the interest rate are higher.

In the first stage, equity holders choose $K$, $L$, and $b$, to maximize the net value of the firm\textsuperscript{22}, given the optimal second stage wage contract in (32), i.e., they solve the problem:

\[
Max_{(K, L, b)} \{ (1 + s)^{-1} \left[ E[R(K, L)] - sk - E[N(\theta)] - E[T(\theta)] \right] \}
\]

subject to: \( r = r(K, L, b), \ w^0 = w^0(K, L, b, r(K, L, b)), \ b \leq K \}.

\]  

(34)

\textsuperscript{20}In order to conserve on notation, we use the same notation as in the previous model.

\textsuperscript{21}The Kuhn-Tucker condition for problem (32) is given by: $V^* = \lambda \partial E(U(\cdot))/\partial w \leq 0$, $[V^* - \lambda \partial E(U(\cdot))/\partial w]w = 0$, $w \geq 0$, where $\lambda$ is the Lagrangean corresponding to problem (32). Assuming that $w > 0$, we must have $V^* - \lambda \partial E(U(\cdot))/\partial w = 0$. Since $V^* < 0$, this means that we must have $\lambda > 0$.

\textsuperscript{22}Again, as in the previous section, given the capital market equilibrium condition (4), the net value of equity (subject to (4)) is the same as the net value of the firm. See Brender and Lewis (1986), for a discussion of a similar two stage problem.
Since the objective of the paper is to explain the choice of capital structure, we focus only on the choice of $b^{23}$. Furthermore, since $w, r$ and $b$ only affect the wage and tax bills (they only appear in $E[N(\theta)]$ and $E[T(\theta)]$), problem (34) is equivalent (yields the same solution for $b$) as the minimization of the sum of the expected costs of labour and taxes. In other words, we can find the optimal value of $b$ by solving the equivalent problem:

$$\text{Min}_{(b)}\{(1 + s)^{-1}(E[N(\theta)] + E[T(\theta)]): \quad r = r(K, L, b), \quad w^0 = w^0(K, L, b, r(K, L, b), \quad b \leq K}\}$$

(35)

The corresponding Kuhn-Tucker condition is given by:

$$\frac{\partial \mathcal{L}}{\partial b} = (1 + s)^{-1} \left[ \frac{\partial E[N(\theta)]}{\partial b} + \frac{\partial E[T(\theta)]}{\partial b} \right] \phi + \phi \geq 0, \quad \frac{\partial \mathcal{L}}{\partial \phi} b = 0, \quad b \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = b - K \leq 0, \quad \frac{\partial \mathcal{L}}{\partial \phi} \phi = 0, \quad \phi \geq 0$$

(36)

where $\mathcal{L}$ is the Lagrangean function. Will this yield an interior solution? Using (33) and (6) we obtain:

$$\frac{\partial E[N(\theta)]}{\partial b} = (1 - G(\theta^r))^{-1} \int_{\theta^r}^{\theta^0} \left\{ \frac{U_n(w(\theta), L)}{U_n(w, L)} - 1 \right\} g(\theta)d\theta > 0$$

$$\frac{\partial E[T(\theta)]}{\partial b} = -t \left\{ \gamma G(\theta^r) + [1 - G(\theta^r)] \right\} \left( r + L \frac{\partial w^0}{\partial b} + (b + \frac{\partial w^0}{\partial r}) \frac{\partial r}{\partial b} \right) < 0$$

Thus, as was the case in the previous section, there is a trade-off between the overall effects of an increase in debt on the wage and tax bills, so that in general there will be an interior solution for $b$.

### 4.2 Wage and Employment Contract:

In this section we assume that equity holders choose investment and capital structure (hence also an interest rate contract with debtholders) at the beginning of the period and then, in the second stage, they negotiate a wage and employment contract with the union. The net value of equity in the second stage (given $K, b, r$) is the same as in (31). Hence, in

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23 The same as before, problem (35) can be solved in two steps. First, for any given $K$ and $L$, we maximize the net value of the firm with respect to $b$. Then, given the optimal value for $b$, we solve for the optimal values for $K, L$. 

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second stage, the Pareto efficient wage/employment contract is obtained by the solution to the problem:

\[
Max_{(w, L)} \{ V : E[U(n(\theta), L)] \geq U^0 \}
\]

Assuming an interior solution for both \( L \) and \( w \), the Kuhn-Tucker conditions for problem (38) are given by:

\[
\begin{align*}
\frac{\partial L(K, L, b, w, r)}{\partial L} &= 0 \\
\frac{\partial L(K, L, b, w, r)}{\partial w} &= 0 \\
\frac{\partial L(K, L, b, w, r)}{\partial \lambda} &= 0
\end{align*}
\]

where \( L \) is the Lagrangean function and \( \lambda \) is the Lagrangean multiplier.

Conditions (39) can be solved to obtain the optimal wage/employment contract. If we denote the optimal solutions as: \( L^0(K, b, r, U^0) \), \( w^0(K, b, r, U^0) \), then, in the first stage, equity holders choose \( K \) and \( b \), to maximize the net value of the firm, given these optimal values, i.e., they solve the problem

\[
Max_{(K, b)} \{(1 + s)^{-1} \{ E[R(K, L)] - sk - E[N(\theta)] - E[T(\theta)] \} \text{ subject to:} \\
r = r(K, L, b), \quad L = L^0(K, b, r, U^0), \quad w = w^0(K, b, r, U^0) \quad b \leq K \}.
\]

Since under this contract \( L \) depends on \( b \) (and \( r \) ), we now have to take into account the effects of the contract on expected revenues, in addition to its effects on the cost of labour and taxes. The optimal capital structure, for any given value of \( K \), is now obtained from the Kuhn-Tucker condition:

\[
\frac{\partial L}{\partial b} = (1 + s)^{-1} \left[ \frac{\partial E[R(K, L^0)]}{\partial L} \frac{\partial L^0}{\partial b} - \frac{\partial E[N(\theta)]}{\partial b} - \frac{\partial E[T(\theta)]}{\partial b} \right] + \phi \geq 0 ,
\]

\[
\frac{\partial L}{\partial b} b = 0, \quad b \geq 0
\]

\[
\frac{\partial L}{\partial \phi} = b - K \leq 0, \quad \frac{\partial L}{\partial \phi} \phi = 0, \quad \phi \geq 0
\]

To solve this problem we have to able to derive the comparative statics results for the effects of a change in \( b \) on the wage/employment contract, taking into account that the interest rate
itself depends on $L$ and $b$. In other words, we have to get the effects of a change in $b$ on $L$, $w$ and $r$, by using the four equations given by conditions (39) and $r = r(K, L, b)$. Unfortunately, it is impossible to obtain these comparative statics results without imposing further restrictions on the model (utility, revenue, or distribution functions). However, since now a change in $b$ affects both $w$ and $L$, it will affect the wage and tax bills for two reasons. Moreover, it will also affect expected revenues, through its effect on $L$. Consequently, in general, we can expect the net value of the firm to be affected by its capital structure.

Before we conclude it is important to make the following points. First, the practical significance of the effects discussed in the paper depends on the importance of the claims by the unionized workers, which in turn, depends on the nature of the firm under discussion. For example, wage claims by workers may not seem significant by themselves. But, if we recognize that the contract with workers usually includes various fringe benefits and a sizable pension plan which will also be subject to risk, the magnitude of this claim may become significant\(^{24}\).

This becomes even more important if, in addition, workers also acquire firm specific human capital\(^ {25}\).

## 5 Conclusion

The purpose of this paper is to examine the effects of union contracts on the firm’s capital structure. Specifically, we consider the a firm that negotiates a labour contract with its union and show that, for all Pareto efficient bargaining solutions, a higher debt reduces the expected tax bill, but increases the expected cost of labour contracts. This trade-off determines the optimal capital structure. We also show that a stronger union tends to increase the amount of equity used.

\(^{24}\)In fact, the importance of this problem and the lack of wage insurance markets has, recently, led the Province of Ontario to consider the introduction of such a programme in the labour market.

\(^{25}\)In fact, the model above can be amended to account for firm specific human capital. For example, we can take the firm’s revenue function as $R(K, L, I, \theta)$, where $I$ is the workers’ investment in firm specific human capital. Taking a union utility function as $U(n(\theta) - I, L, \hat{L})$, the model becomes very similar to the one we examined.
6 Appendix:

I. From (7) (12) and (10) we get:

\[
\frac{\partial E[N(\theta)]}{\partial b} = -(1 + r + br_b) \int_{\theta^0}^{\theta^1} g(\theta) d\theta < 0
\]

\[
\frac{\partial E[T(\theta)]}{\partial b} = -(r + br_b) t \left\{ \gamma G(\theta^t) + [1 - G(\theta^t)] \right\} < 0
\]

\[
\frac{\partial E[U(n(\theta))]}{\partial b} = -(1 + r + br_b) \int_{\theta^0}^{\theta^1} U_n(w(\theta), L) g(\theta) d\theta < 0
\]

II. The condition \( \frac{\partial \mathcal{L}}{\partial w} = 0 \) can be written as:

\[
\frac{\partial \mathcal{L}}{\partial w} = (1 + s)^{-1} \mathcal{L} \left( [1 - G(\theta^0)] - t \left\{ \gamma G(\theta^t) + [1 - G(\theta^t)] \right\} \right) - \lambda U_n(w, L) (1 - G(\theta^0))
\]

where \( G \) is the cumulative distribution function of \( \theta \), so that for example, \( G(\theta^0) = \int_0^{\theta^0} g(\theta) d\theta \).

This can be solved for \( \lambda \) to obtain:

\[
\lambda = \frac{L(1 + s)^{-1} \left( [1 - G(\theta^0)] - t \left\{ \gamma G(\theta^t) + [1 - G(\theta^t)] \right\} \right)}{U_n(w, L) [1 - G(\theta^0)]}
\]

Plugging this into (20) we get: \( \frac{\partial \mathcal{L}}{\partial w} = A + B + \phi \), where \( A \) and \( B \) are defined in equations (25) and (26).

7 References


European Economic Review, 37, 1185-1196.


