A Keynesian Solution to Classical Unemployment

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Abstract

In a classical macroeconomic model, the real wage equals labor's marginal product and the real interest rate can fall no lower than the rate of investment. These rigidities may prevent labor market clearing. Economies with rapid labor supply growth, capital immobility and a low capital labor ratio will be prone to such `classical unemployment'. Downward flexibility in real wages restores full employment, lowers real interest rates and stimulates investment provided that firms also perceive that they are rationed in output sales. Such quantity constraints have been identified by Clower (1965) as a critical feature in Keynes (1936).

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1 Introduction

In the classical macroeconomic model, labor and capital produce output which can be consumed or added to the capital stock. The level and allocation of output are determined in markets with flexible prices and factors are rewarded according to their marginal products. The standard Keynesian criticism of this model is that employment and output may stay persistently below their classical levels because of demand constraints on output sales. The usual reasons cited for such `lack of effective demand' include price rigidities, dim investment prospects, low real wages and portfolio balance decisions holding interest rates above their `natural' level.

This paper presents a macroeconomic model in which most of these roles are reversed. Persistent unemployment may result because of the equality of the real wage and the marginal product of labor (`the first classical postulate' of Keynes, 1936). Moreover, an economy experiencing such `classical unemployment' will also have a low investment rate and a high real interest rate since the latter must exceed the former for profit to be positive. The source of these outcomes is not lack of effective demand for output but rather capital immobility between the consumption and capital sectors and an aggregate capital labor ratio which is `low' in a sense to be made precise. The model predicts that economies with rapid labor supply growth and a low capital stock (per capita) weighted heavily in the capital sector will be prone to classical unemployment.

The second principal result is that if real wages are °exible below labor's marginal product, then classical unemployment and its associated investment and interest rate rigidities can be eliminated. In the resulting full employment equilibrium, however, competitive ¯rms will be demand constrained; that is, rationed in their output sales. Because agent optimization in the face of rationing (or `quantity constraints') is fundamental to Robert Clower's in°uential interpretation of Keynes (Clower 1965), such a full employment equilibrium will be termed `Keynesian'.

The potential for classical unemployment and Keynesian full employment stands in sharp contrast with the conclusions of the disequilibrium literature (Barro and Grossman 1971, Benassy 1975, Malinvaud 1977) which emanated from Clower's paper. In that literature, price rigidities produce a taxonomy of non-Walrasian equilibria and, in the `Keynesian regime', quantity constraints on output sales coincide with an excess supply of labor. In the present model, such demand constraints are required to eliminate classical unemployment under price °exibility.[1] In the `classical regime' of that literature, the real wage is ¯xed at such a high level that while the usual ¯rst order conditions of ¯rms can be satis¯ed, households ¯nd themselves rationed on both output and labor markets. Classical unemployment in the present model cannot be alleviated by a lower real wage if labor's marginal product falls proportionately with it.

[1] Clower (1965) does not refer to price rigidities in his interpretation of Keynes.

At center stage in the analysis is the neoclassical theory of investment (Lucas 1967, Uzawa 1969, Purvis 1976) in which the acquisition of capital is spread out through time because of increasing marginal installation costs. In this paper, production of net output (i.e., gross output less that absorbed by installation or `adjustment' costs) displays constant returns to scale. If the real wage equals labor's marginal product then the marginal product of installed capital must be large enough to finance both the purchase and installation of new capital. Classical unemployment results because for any initial allocation of installed capital between the capital and consumption sectors, there is no assurance that this financing requirement can be met and firms still profitably absorb the full employment output of the capital sector.

This problem does not arise in the influential macro model with adjustment costs of Uzawa (1969). There, output is a homogeneous product of a one sector production function; it is `as if' installed capital can be instantly and costlessly shifted between sectors. In Mussa (1976), adjustment costs cause capital stock immobility between firms, but a firm can produce either output implying mobile, intra-firm capital. In Mussa (1977), there is an explicit two sector macro model with adjustment costs, but the capital sector employs no capital. As we shall see, this is a (more than) sufficient assumption to obviate the problem.

In this paper, a Keynesian solution is proposed. Real wages fall below labor's marginal product and thereby generate additional rent to finance the full employment output of the capital sector. Investment is stimulated and real interest rates reduced. Since firms (rationally) predict that they will be sales constrained at output levels consistent with full employment, their input choices do not satisfy the classical marginal productivity conditions.

The plan of the paper is as follows. In the next section, the microfoundations of firm behaviour are examined. A full employment macro equilibrium is presented in the third section, where the `Keynesian' and `classical' are the two nested cases. The final section contains concluding remarks.

2 Microfoundations

Consider the neoclassical investment model. A representative firm in discrete time (index t) maximizes its present real value V_t with discount rate ρ_t. It chooses labor N_t with real wage ω_t and the growth rate α_t in its predetermined capital stock K_t. The objective function is

$$V_t = \max_{\alpha_t;\ N_t} (1+\rho_t)^{-1}\{F(N_t;K_t) - \phi(\alpha_t)K_t - \alpha_t K_t - \omega_t N_t + \psi_t[Q_t - (F(N_t;K_t) - \phi(\alpha_t)K_t)] + V(K_{t+1};Z_{t+1})\}. \quad (1)$$

The gross output function $F(\cdot)$ is linearly homogeneous with positive and diminishing marginal products. Adjustment costs $\phi(\alpha_t)K_t$ are gross output lost in the installation of new capital where (letting $'$ denote differentiation) $\phi';\phi'' > 0$ and $\lim_{\alpha \to 0}\phi = \lim_{\alpha \to 0}\phi' = 0$. New capital purchased $\alpha_t K_t$ takes one period to install so that $K_{t+1} = K_t(1+\alpha_t)$. Capital is immobile in the sense that it may only produce the output of the sector (capital or consumption) in which it was first installed. Depreciation, debt and taxes are ignored for simplicity. The upper bound (if any) to sales in period t is Q_t where ψ_t is the shadow price of such a constraint. The expected firm value next period is $V(K_{t+1}; Z_{t+1})$ where Z_{t+1} denotes expected future values of wages, sales constraints, etc.

The relative price of output sold to new capital bought is one in all periods. This is because (1) describes the objective of competitive firms in either the capital or consumption sectors and both outputs are produced by an identical, constant returns to scale technology.[2] This is consistent with the static theory of the firm where relative prices reflect relative marginal costs. A formal derivation of the unit relative price in the present dynamic context is found in the Appendix.

The first order conditions for N_t and α_t are

$$F_{N_t} = \omega_t(1 - \psi_t)^{-1}; \tag{2}$$

$$V(\cdot_{t+1})_{K_{t+1}} + \psi_t \lambda'_t = 1 + \lambda'_t \tag{3}$$

(a subscript denotes a partial derivative and $\cdot_\tau$ means the arguments of a function evaluated in period τ). Differentiation of the maximized value of (1) with respect to K_t and substitution of (2) and (3) yield the discrete time

[2] The two goods are distinguished in use by some costless characteristic. For example, consumption goods are blue marbles produced by labor and installed red marbles. It takes a period to install newly produced red marbles in either sector with installation costs in the form of lost (gross) output of the purchasing firm.

equivalent of the continuous time Euler equation,

$$V(\Delta_{t+1})_{K_{t+1}} - V(\Delta_t)_{K_t} = \rho_t V(\Delta_t)_{K_t} - (F_{K_t} + \alpha_t \lambda'_t - \lambda_t)(1 - \psi_t). \qquad (4)$$

2.1 The Classical Firm

Consider the firm's choices of N_t and α_t if it perceives no sales constraints. With $\psi_t = 0$ for all t, the solution for N_t reflects Keynes' 'first classical postulate': from (2), the marginal product of labor equals the real wage.

The solution method for α_t is more complex. An important initial result is that in the limit; that is, for $\alpha_{t-1} = \alpha_t$, profit in any period is only positive if $\rho_t > \alpha_t$. To see this, substitute (3) and its $t-1$ counterpart into (4) to get the first order difference equation

$$\rho_t - F_{K_t} + \lambda'_{t-1}(1+\rho_t) = \lambda'_t(1+\alpha_t) - \lambda_t \qquad (5)$$

(where given ω_t determines F_{K_t}). Next, eliminate time subscripts in (5) and rearrange it as $F_K = \rho + \lambda + \lambda'(\rho - \alpha)$. From Euler's Theorem and (2), profit is $(F_{K_t} - \lambda_t - \alpha_t)K_t$. Substituting for F_{K_t} from the previous expression, we get $(\rho_t - \alpha_t)(1 + \lambda'_t)K_t$ which is positive only if $\rho_t > \alpha_t$.

Figure 1 about here

Consider the solution for limit α assuming the explicit form $\lambda(\alpha_t) = b\alpha_t^2$. The RHS of (5) becomes $g(\alpha_t) \equiv 2b\alpha_t + b\alpha_t^2$ and its LHS is $h(\alpha_{t-1}, \rho_t, \omega_t) \equiv (\rho - F_K) + (1+\rho)2b\alpha_{t-1}$. As shown in Figure 1, there are two limit solutions:

$\alpha = \rho_t \pm (\rho_t^2 - (F_{K_t} - \rho_t)/b))^{\frac{1}{2}}$. The larger root is stable but greater than ρ_t which implies a loss as we have seen. The smaller root is the limit solution for α. More important, it is a saddle point. The solution[3] for α_t in any t is found from (5) with $\alpha_{t-1} = \alpha_t$, assuming that it is less than or equal to ρ_t. Consistent with Lucas (1967) and Uzawa (1969), it is helpful to arrange (5) under these conditions as: [4]

$$\frac{F_{K_t} + \lambda'_t \alpha_t - \lambda_t}{1 + \lambda'_t} = \rho_t; \qquad (\text{where } \rho_t \geq \alpha_t). \tag{6}$$

2.2 The Keynesian Firm

This firm faces a sales constraint Q_t less than its classical net output and so $\psi_t > 0$ in (2) and (3).[5] Given its intertemporal objective, solutions for N_t and α_t will depend on both current and anticipated future sales constraints. In t, we have $Q_t = [f(n_t) - \lambda(\alpha_t)]K_t$ where $f(\cdot)$ is the intensive form gross production function (with $n_t \equiv N_t/K_t$) and we now specify that the firm expects the same sales constraint intensity, $q_t \equiv Q_t/K_t$, will exist in all

[3] A solution may not exist; if $F_K > b\rho^2 + \rho$, then $h(.)$ lies below $g(.)$ in Figure 1 and all roots in (5) are imaginary. Existence is discussed in the next section.

[4] Our (6) is Uzawa's (50): $(F_K - \gamma)(\rho - \alpha)^{-1} = \gamma'$ where $\gamma \equiv \lambda + \alpha$. Clearly $\rho > \alpha$. Our (2) (with $\psi_t = 0$) and (6) correspond to (8) and (9) in Lucas (1967). His (9) is displayed as Figure 1 in that paper. He specifies the adjustment cost curvature properties as above plus the restriction: $\lambda' \longrightarrow \infty$ as $\alpha \longrightarrow \beta \equiv \rho + \sigma - \phi$, where σ is the depreciation rate (zero here) and ϕ is "some positive number" (Lucas 1967, p. 325). Lucas does not comment on the motive for this restriction. However, bounding the domain of the adjustment cost function from above by ρ implies the 'no loss' condition: $\rho > \alpha$. An explicit form consistent with this specification is $\lambda(\alpha_t) = \alpha_t \ln\{\beta/(\beta - \alpha_t)\}$. If $F_K > \rho$ then $h(.)$ intersects $g(.)$ once from below; i.e., a saddle point solution exists.

[5] To interpret (3) here, note that its LHS is the benefit of an additional unit of installed capital in t + 1 and the second term states that such an increase will reduce net sales in t by λ'_t below the sales constraint which has a shadow price of ψ_t.

future periods τ. Sales constraints are thus expected to grow at the same rate chosen for the capital stock (but no faster; there is still a constraint). While this specification appears arbitrary, it is shown in the next section to be consistent with the expectation of a full employment, Keynesian macro equilibrium. For given q, pairs (n_t, α_t) which satisfy $f(n_t) - \lambda(\alpha_t) = q$ appear in Figure 2 as the function $n_t = l(\alpha_t; q)$.

Figure 2 about here

In the next section's discussion of Keynesian macroeconomic equilibrium, only steady states are considered and so we assume here that ω, n and ρ are identical in adjacent periods. This implies, from (2), that $\psi_{t-1} = \psi_t$. Substitution of this equality as well as (2), (3) and their $t-1$ versions into (4) yields the Keynesian counterpart of (5):

$$\left(\frac{F_{N_t}}{\omega_t} \cdot \rho - F_{K_t}\right) + \lambda'_{t-1}(1+\rho_t) = \lambda'_t(1+\alpha_t) - \lambda_t. \tag{7}$$

For given n_t, (7) resembles (5) displayed in Figure 1 (they are identical if $F_N = \omega$). Considering only the steady-state case of $\alpha_t = \alpha_{t-1}$, inspection of Figure 1 reveals (noting that the first term on the LHS of (7) varies inversely with n_t) that pairs (n_t, α_t) satisfying (7) form the function $n_t = m(\alpha_t; \rho_t; \omega_t)$ concave to the α axis in Figure 2. [6]

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[6] In Figure 2, $\lim_{\alpha \to 0} m(\cdot) < \lim_{\alpha \to 0} l(.)$ since K_t is less than required to produce Q_t and also satisfy the static efficiency requirement $F_N/F_K = \omega/\rho$.

Steady-state Keynesian solutions for n_t and α_t are found in Figure 2 at the first intersection of the $l(\cdot)$ and $m(\cdot)$ loci. The classical solutions are at point C on the positively-sloped range of $m(\cdot)$ where $F_{N_t} = \omega_t$. For given K_t, the classical pair produce Q_t^* and so $q_t^* \equiv Q_t^*/K_t$ positions the $l(\alpha_t, q_t^*)$ locus passing through point C. Since our concern is the sales constrained firm, we have $q < q_t^*$ and so the $l(\alpha_t, q)$ locus lies below point C. Keynesian n_t and α_t are thus both less than their classical counterparts.

In sum, Keynesian solutions for n_t, α_t and ψ_t are found from (2), the constraint equation $f(n_t) - \lambda(\alpha_t) = q$ and (7) where $\alpha_{t-1} = \alpha_t$. From (7) and (2) it is helpful to derive the Keynesian version of (6):

$$\frac{F_{K_t} + \lambda_t' \alpha_t - \lambda_t}{(1 - \psi_t)^{-1} + \lambda_t'} = \rho_t. \tag{8}$$

These solutions are admissable only if losses are avoided. From (2) and the expression for profit, $(F_{K_t} - \alpha_t - \lambda_t)K_t + (F_{N_t} - \omega_t)N_t$, this requires

$$\psi_t \geq -\left[\frac{F_{K_t} - \lambda_t - \alpha_t}{F_{N_t} n_t}\right]. \tag{9}$$

3 Macroeconomic Equilibria

In the macro models below, there are competitive firms in each sector and the representative agent aggregation assumption is invoked. Net outputs are:

$$C_t = F(N_t^C, K_t^C) - \lambda(\alpha_t^C)K_t^C, \tag{10}$$

$$I_t = F(N_t^K, K_t^K) - \lambda(\alpha_t^K)K_t^K. \tag{11}$$

3.1 A Classical Model

This model solves recursively. Assuming an inelastic labor supply N_t, solutions for N_t^i (where $i = C, K$) and ω_t follow from (2) and the labor market equilibrium condition:

$$F_N(N_t^C, K_t^C) = F_N(N_t^K, K_t^K) = \omega_t, \qquad (12),(13)$$

$$N_t^C + N_t^K = N_t. \qquad (14)$$

The solutions for N_t^i in turn imply the gross output and the (identical) marginal products of capital in both sectors. The labor capital ratio in each is $n_t \equiv N_t/K_t$ where $K_t \equiv K_t^K + K_t^C$.

The rates of capital growth α_t^i and the yield on equity ρ_t are found from (6) and the capital market equilibrium condition:

$$\frac{F_{K_t^C} + \lambda_t^{\prime}\alpha_t^C - \lambda_t}{1 + \lambda_t^{\prime}} = \frac{F_{K_t^K} + \lambda_t^{\prime}\alpha_t^K - \lambda_t}{1 + \lambda_t^{\prime}} = \rho_t, \qquad (15),(16)$$

$$F(N_t^K, K_t^K) - \lambda(\alpha_t^K)K_t^K = \alpha_t^C K_t^C + \alpha_t^K K_t^K. \qquad (17)$$

With identical $F_{K_t^i}$ in each sector, (15) implies that $\alpha_t^K = \alpha_t^C$; that is, there is a common capital growth rate, call it α_t. Its solution is found from (17). Equilibrium ρ_t then follows from (16). Given K_t^i, the solutions for N_t^i and α_t yield the net outputs from (10) and (11). Note that net consumption sector output [7] could be expressed as if derived from an aggregate (i.e., one sector) production function, $C_t = F(N_t, K_t) - I_t - \lambda_t K_t$ where $I_t = \alpha_t K_t$.

[7] In order to `close' the model, an equilibrium condition for consumer goods should be

The key observation from (12) - (17) is that ρ_t and α_t are functions of given N_t and predetermined K_t^i. Nothing requires that ρ_t exceed α_t to preclude losses. The conditions under which this requirement is met are next investigated. It is helpful to specify a Cobb Douglas production function $f(n_t) = n_t^{\delta}$ where $\delta \in (0;1)$. Given common α_t and defining $\theta \equiv 1 + K_t^C = K_t^K$ where θ is predetermined, (17) is compressed to

$$n_t^{\delta} = \lambda + \alpha_t \theta: \qquad (17')$$

Next, the no loss condition requires that (for either sector) the LHS of (16) exceed α_t. This is reduced to $F_{K_t} > \alpha_t + \lambda_t$, or

$$(1 - \delta) n_t^{\delta} \geq \alpha + \lambda: \qquad (18)$$

This states that the gross marginal product of capital must be large enough to finance both the purchase and installation of new capital since labor absorbs the remaining output when its real wage equals its marginal product. Combining (17') and (18) and assuming the explicit adjustment cost function $\lambda(\alpha_t) = b\alpha_t^2$, we get

$$\alpha \leq \frac{\theta(1 - \sigma) - 1}{b\sigma}: \qquad (19)$$

This equation specifies the maximum admissable (i.e., no loss) rate of capital growth. Indeed, the RHS of (19) may be negative in which case no positive

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specified. The assumption of an inelastic labor supply could be relaxed so that solutions for ω_t and ρ_t are consistent with household optimization. Alternatively, since the equity market value of firms and its rate of return have been determined, we could introduce a second asset, say money, whose real value and/or rate of return adjust to reconcile households to their output share already determined.

value for such ®$_t$ exists.

Several results now follow. First, since capital growth and the yield on equity are inversely related from (16), maximum ®$_t$ implies a minimum ½$_t$. Moreover, since n_t and ®$_t$ are positively related from (17^0), an upper bound also exists for the labor capital ratio n_t. With K_t predetermined, an employment maximum thus exists in t which can be less than the exogenous labor supply or, for that matter, the labor supply chosen by an optimizing household sector observing the solutions for !$_t$ and ½$_t$ (see fn. 7 above). The `second classical postulate' of Keynes (the marginal utility of real wage income equals the disutility of labor) may not be satis¯ed. Finally, assume that a full employment temporary equlibrium exists in t. If labor supply grows at an exogenous rate ¹ greater than ®$_t$, then both the labor capital ratio and the capital growth rate rise through time. If ¹ is greater than the RHS of (19), however, this economy cannot converge to a full employment classical steady state (where ¹ = ®). In ¯nite time, (19) will not be satis¯ed.

The important conclusion is that labor markets may not clear in the classical model. This outcome will be associated with a downwardly rigid real interest rate and upwardly rigid investment. A labor intensive economy with rapid labor supply growth (high ¹) and a large labor share of gross output (high ±) may not support a full employment classical equilibrium, particularly if its capital is weighted heavily in the capital sector (low µ).

Several adjustments in the model's speci¯cation could remedy the problem. If capital were instead mobile and could produce either output (as in Uzawa 1969), we would effectively have a single sector model and θ could go to infinity if necessary to eliminate any upper bound in (19). Alternatively, under perfect foresight, it might be assumed that initial θ was set correctly to evade the constraint in (19). Or, if unemployment appears in a classical temporary ‘equilibrium’, the competitive markets specification might then be abandoned and the relative price of new capital allowed to fall below unity. In the next section, the solution explored is to abandon the first classical postulate and let the real wage fall below labor's marginal product.

3.2 A Keynesian Model

A full employment macro model is presented below in which the ‘classical’ and ‘Keynesian’ versions appear as two nested cases. The model is specified first and its details examined in notes which follow. We have:

$$F_N(N_t^C, K_t^C) = F_N(N_t^K, K_t^K) = \frac{\omega_t}{1-\psi_t} \qquad (20);(21)$$

$$N_t^C + N_t^K = N_t. \qquad (22)$$

$$\frac{F_{K_t^C} + \lambda_t'\alpha^C - \lambda_t}{(1-\psi_t)^{-1} + \lambda_t'} = \frac{F_{K_t^K} + \lambda_t'\alpha_t^K - \lambda_t}{(1-\psi_t)^{-1} + \lambda_t'} = \rho_t \qquad (23);(24)$$

$$F(N_t^K, K_t^K) - \lambda(\alpha_t^K)K_t^K = \alpha_t^C K_t^C + \alpha_t^K K_t^K \qquad (25)$$

$$\psi_t = \max\left[0,\; -\left\{\frac{F_{K_t} - \lambda_t - \alpha_t}{F_{N_t} n_t}\right\}\right] \qquad (26)$$

A preliminary solution method for this system now follows. Given predetermined capital stocks in each sector, inelastically supplied labor N_t is allocated by (20) and (22) so that the marginal products of labor (and thus of capital) are the same in each sector. Since the shadow prices ψ_t are also the same (this is formally established below), (23) (which reflects (8)) implies that there is a common capital growth rate α_t. Its level is supply side determined from the capital market equilibrium condition (25). The value of ψ_t is next computed from (26) which reflects (9). If $F_{K_t} > \delta_t + \alpha_t$, then $\psi_t = 0$ and the system collapses to become the classical model above. With a reverse inequality, $\psi_t > 0$ and a (zero profit) Keynesian equilibrium exists. With ψ_t so determined, equilibrium ω_t and ρ_t are found from (21) and (24) respectively.

In a Keynesian equilibrium (i.e., with $\psi_t > 0$), aggregate sales constraints are given by (9) and (10). In order to see why their shadow prices are identical in (20) - (26), it is helpful to rearrange (20) and (21) to get the more general form: $(1 - \psi_t^K)F_{N^K} = (1 - \psi_t^C)F_{N^C} = \omega_t$. Next, use (2) to eliminate the ψ_t^i terms from the LHS equality. After some manipulation, $G_t^i \equiv [f(n_t^i) - (\delta_t^i + \alpha_t^i)][n_t^i]^{-1}$ appears on each side of this equality, where $G_{n_t^i} > 0$ and $G_{\alpha_t^i} < 0$. These signs imply that if labor were shifted, for example, from the capital to consumption sectors so that $n_t^K < n_t^C$, it would be necessary that $\alpha_t^K < \alpha_t^C$ to preserve the equality. A symmetrical exercise with respect to (23), however, yields the opposite conclusion: from (2) and

(23), if $n_t^K < n_t^C$ then $\alpha_t^K > \alpha_t^C$ is required to preserve (23). We conclude that $n_t^C = n_t^K$ and so, from (2), a common ψ_t is a necessary condition for a Keynesian equilibrium.

A positive solution for ψ_t is seen in (26) to be a function of n_t. Recall, however, the assumption underlying (7), (8), (9) and thus (26) was that $\psi_t = \psi_{t-1}$ which required a constant labor capital ratio n_t. We conclude that in the Keynesian case, (20) - (26) must be more narrowly interpreted as a temporary equilibrium in the steady state. This alters the recursive solution method from that described above. In the Keynesian steady state, α_t equals the exogenous rate of labor supply growth μ. The solution for the steady state labor capital ratio is then found from (25) given the distribution of installed capital (θ) across sectors. Solutions for n_t and α_t imply the marginal products of both inputs and the values of ψ_t; ω_t and ρ_t.

Figure 3 about here

The Keynesian equilibrium is characterized in Figure 3 which is a variation on Figure 1 in Barro and Grossman (1971). It characterizes here full employment, not an excess supply of labor as in that paper. The locus LBD is the short-run marginal product of labor schedule and conventional labor demand curve under competitive conditions. Maximum employment in the classical model is $\hat{N}$ where $\hat{N} < N$ (the classical real wage would be $F_{\hat{N}}$). The reinforced locus LBC is the Keynesian demand for labor curve. The `kink' at N, with an inelastic range for real wages below F_N, reflects the firm perception that more output from additional employment cannot be sold. For real wage $\omega^* = F_N(1 - \psi)$, the sales constrained output would be produced at zero profit. Labor demand N^*, read off the conventional demand locus LBD at ω^*, would be `notional' in the sense of Clower (1965), not only because firms believe that more output cannot be sold but also because even if it could, it would generate a loss.

While the discussion above assumed a zero profit Keynesian equilibrium, it should be clear that a positive minimum profit target would require an even lower real wage. Given an inelastic labor supply, a solution for real wages (and real profit) appears indeterminate. These might be found, however, if the distribution of income were added to the mechanisms listed above (see fn. 6) to clear the consumer goods market. In the spirit of traditional Keynesianism, the role of flexible real wages would then be to ensure sufficient demand for output, not to clear the labor market.

Finally, observe that unlike its role as the interest rate `floor' in the classical model, the rate of investment becomes an interest rate `ceiling' in the Keynesian model. From (8) and (9) met with equality we get:

$$\rho_t = \alpha - \left[\frac{(1 - \psi_t)\psi_t F_N n + \alpha\psi_t}{1 + \theta'(1 - \psi_t)}\right]: \tag{27}$$

The real interest rate is necessarily less than the rate of capital accumulation. In general, (6) and (8) will piece together a downward sloping marginal efficiency of investment schedule in $(\rho; \alpha)$ space. The classical range of this schedule lies above the 45° ray from the origin and the Keynesian range below it. More generally, we conclude that in the transition from a state of classical unemployment to that of (steady state) Keynesian full employment, a fall in both the real wage and the real interest rate would be observed.

4 Conclusion

Downward inflexibility in real wages and/or real interest rates are the traditional causes of low output and high unemployment. Nothing in this paper changes that general conclusion. What is changed are the roles of the first classical postulate and demand constraints on output sales. The classical equality of the real wage and labor's marginal product is itself a rigidity leading to potential labor market disequilibrium which can be resolved through the perception by firms of constraints on output sales.

Appendix

In this appendix, it is established that the relative price of uninstalled new capital to consumption goods is one. In general, the objective function of the classical firm is:

$$V_t = \max_{\alpha_t, N_t} (1+\rho_t)^{-1}\{P_Q[F(N_t K_t) - \lambda(\alpha_t)K_t] - P_I\alpha_t K_t - \omega_t N_t + V(K_{t+1}, Z_{t+1})\}, \tag{A 1}$$

where P_Q, P_I and ω are the real (i.e., consumption good) prices of output, uninstalled new capital and labor respectively. The equivalents of (2), (3), (4) and (5) in the main text are

$$P_Q F_{N_t} = \omega \tag{A 2}$$

$$V(\cdot_{t+1})_{K_{t+1}} = P_I + \lambda'_t \tag{A 3}$$

$$V(\cdot_{t+1})_{K_{t+1}} - V(\cdot_t)_{K_t} = \rho_t V(\cdot_t)_{K_t} - (P_Q[F_{K_t} - \lambda_t] + \alpha_t \lambda'_t) \tag{A 4}$$

$$P_I \rho_t - P_Q F_{K_t} + \lambda'_{t-1}(1+\rho_t) = \lambda'_t(1+\alpha_t) - P_Q. \tag{A 5}$$

In the limit, real profit is $(\rho - \alpha)(P_I + \lambda' K)$ which requires $\rho > \alpha$ to be positive. Assuming the explicit quadratic form $\lambda(\alpha_t) = b\alpha_t^2$, the solution for the rate of capital growth any period t is

$$\alpha_t = \frac{\rho_t - [\rho_t^2 - (2-P_Q)(P_Q F_K - P_I\rho_t)b^{-1}]^{\frac{1}{2}}}{2-P_Q}. \tag{A 6}$$

In this model, there are two specific cases to consider. For the consumption good sector we have $P_Q \equiv 1$ and for the capital goods sector we have $P_Q \equiv P_I$.

From (A 6), this yields

$$\alpha_t^C = \rho_t - (\rho_t^2 - (F_{K_t^C} - P_I \rho_t) b^{-1})^{\frac{1}{2}}; \qquad \text{(A 7)}$$

$$\alpha_t^K = \frac{\rho_t - (\rho_t^2 - P_I (F_{K_t^K} - \rho_t)(2 - P_I) b^{-1})^{\frac{1}{2}}}{2 - P_I}. \qquad \text{(A 8)}$$

Given K_t^i and ρ_t, note that (A 7) and (A 8) are two equations in three unknowns: α_t^C α_t^K and P_I. The third equation is the (uninstalled) capital market equilibrium condition:

$$\alpha_t^C K_t^C = (f(n_t^K) - \lambda_t - \alpha_t^K) K_t^K; \qquad \text{(A 9)}$$

where n_t^K follows from (A 2) for given ω_t. Since the same relative price P_I is found in the next period, the `horizontal shifts' in both the demand and supply curves for new capital across periods must be the same. In t + 1, therefore, the equilibrium condition can be expressed as

$$\alpha_t^C K_t^C (1 + \alpha_t^C) = (f(n_t^K) - \lambda_t - \alpha_t^K) K_t^K (1 + \alpha_t^K) \qquad \text{(A 10)}$$

From (A 9) and (A 10) we see that $\alpha_t^C = \alpha_t^K$. It follows from inspection of (A 7) and (A 8) that identical α_t^i, in each sector imply that $P_I = 1$.

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Figure 1

Quadratic Adjustment Costs

Figure 2

The Keynesian Steady State

Figure 3

The Keynesian Equilibrium