Environmental Labeling

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Abstract

This paper studies how information disclosed by voluntary environmental labels creates incentives for firms to invest in environmentally-friendly production technologies. I develop a model with differentiated products and imperfectly-informed consumers. Consumers care about the environmental characteristics of goods (for example, how they were produced), but cannot directly observe these product characteristics. Firms differ in their abilities to develop "clean" technologies, but have no incentive to do so absent government regulation or a policy that provides information to consumers. A scheme of voluntary labels, awarded to firms that achieve some chosen level of environmental friendliness, gives some firms enough incentive to develop clean technologies, while others choose to produce "dirty" goods. Each consumer is individually ineffective in reducing aggregate environmental damage but consumers purchase products according to how they privately value environmental quality. I parameterize the relationship between the environmental quality consumers experience privately from their own consumption of a product and the intensity of its environmental damage. I use the model to explain how voluntary labels improve consumer welfare and characterize the welfare maximizing labeling standard. I also contrast the effects of a labeling program on consumer welfare with those of compulsory environmental regulation.

Keywords: credence goods, disclosure, environmental policy, firm heterogeneity and product labeling.

JEL Classification: L15 and Q58.

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1 Introduction

Environmental labeling is a market-based environmental policy measure that relies on consumer preferences for environmental quality to create incentives for firms to improve their process and production methods (PPMs). Environmental labeling programs provide information to consumers about a product’s unseen means of production that would not otherwise be recognized prior to its purchase, or even after its consumption.\(^1\)\(^2\) They are typically voluntary and are administered by credible third parties such as government agencies or private non-profit entities.\(^3\) The first ecolabeling program, Germany’s Blue Angel, was established in 1977. Today there are more than twenty-five national ecolabeling programs that certify products if they satisfy numerous criteria such as acceptable levels of air emissions, water effluents, energy usage or content rules for recycled materials. There are also hundreds of environmental product labels with a more specific focus that, for instance, certify products if they satisfy standards for sustainable forestry and fishing practices, a reduced "carbon footprint," farming practices deemed "organic," or ethical standards such as "dolphin-safe" tuna.\(^4\) This paper addresses fundamental questions about how to best utilize labels as an environmental policy instrument and compares voluntary labeling to compulsory regulation in terms of their effects on environmental quality and welfare.

While environmental quality is usually hidden to the consumer, it is often known to the producer. The underlying problem corresponds to one of hidden information, or adverse

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\(^1\)In the economics literature, products with such attributes are referred to as credence goods. See Tirole (1988) for a detailed description.

\(^2\)The ISO differentiates among environmental product claims by classifying them into "Type I" claims, or ecolabels, which are defined by fixed, multi-issue, third party criteria; "Type II" claims, which are based on self-declarations by manufacturers; and "Type III" claims, which present quantified product information. In addition, labels granted by a third party certification agency that refer to a specific environmental or ethical characteristic of a product are referred to as "single issue" labels.

\(^3\)For instance, Germany’s ecolabel, the Blue Angel, is managed in a public-private partnership however the maintenance and ambition of the program, by defining targets, thresholds and levels of standards is a function of the public authorities. See Muller (2005).

selection (see Akerlof (1970)), since the environmental quality of a firm’s PPMs typically depends on irreversible investments that have been undertaken by the firm. Since environmental quality is unobservable, producers have an incentive to cheat and market harmful products as environmentally friendly. Consequently, consumers reduce their willingness to pay for the products, which drives environmental quality out of the market. Labeling is a means to resolve this market failure since a third party’s claim may be more credible to consumers than a producer’s own assertion. In principle, this market failure would be entirely resolved if information about a product’s environmental quality could be fully disclosed. The precision of information concerning a product’s PPMs, however, is severely restricted by the costs of disclosing more detailed information.\footnote{Labels that provide quantified information concerning environmental quality typically concern product usage. For example, the EU Energy Label rates products from 'A' to 'G', with 'A' being the most efficient.} For instance, ecolabeling programs assess a product’s entire life-cycle and are complex and costly to generate.\footnote{The life-cycle of a product begins with the extraction of raw materials, and then encompasses its production, distribution, use and disposal.} Consequently, in practice, environmental labels are characterized by fixed, third-party criteria and there is a necessary role for an authority to choose a standard of environmental quality to be upheld by the program.\footnote{Although international bodies such as the International Organization for Standardization (ISO) and the Global Ecolabeling Network oversee and promote a set of guiding principles for environmental labeling programs (see ISO 14024 (1999)), they are chiefly a code of good practice and no specific guidelines for how to set PPM criteria have been established.}

If individuals acting in the market place do not take into account the impact of their consumption on aggregate environmental damage, it is unclear, a priori, the degree to which ecolabeling programs can enhance welfare or produce environmental benefits. Unless consumers receive a private benefit from purchasing environmentally friendly products,\footnote{Private consumption benefits may arise from personal health benefits, such as the avoidance of exposure to pesticides, or may be due to a desire to receive social acclaim, to avoid the scorn of others, or for 'warm glow' (see Becker (1974); Andreoni (1990)).} environmental labeling programs will have no effect on consumption. Some empirical work has been undertaken to investigate whether consumer behavior is affected by information con-
cerning the environmental or ethical aspects of goods. Most notable are Nimon and Beghin (1999), who, in their study of the US apparel industry, find a premium for organic fibers in apparel goods of 37% of the price for adult items and 90% of the price for baby items. Teisl et al. (2002) find that the introduction of the dolphin-safe label in the US eventually resulted in a market share of canned tuna that was roughly 1% higher than without the label. Due to a lack of cross sectional variation in the data (virtually all tuna sold in the US after 1990 was dolphin-safe), however, their result could have been due to market trends that were unaccounted for. Bjorner et al. (2004) utilize data on purchases of paper products and detergents to estimate the marginal willingness to pay for the Scandinavian ecolabel, the Nordic Swan. They find that the marginal willingness to pay for the ecolabel ranges from 13%-29% of the labeled good's price. Their study provides convincing evidence that consumers experience a private benefit from consuming goods that have a lessened environmental impact, since the types of products analyzed by Bjorner et al. (2004) are unlikely to confer any direct health benefits (in contrast with the organic apparel items analyzed by Nimon and Beghin (1999)).

Hence while it is likely that consumers are individually ineffective in reducing aggregate environmental damage, the empirical evidence suggests that they do respond in the marketplace to information that is provided about a product’s environmental quality.

Several theoretical papers study certification or environmental labeling and share a context in which there is a severe information asymmetry that precludes product warranties (see Grossman (1981)), or the seller’s reputation (see Allen (1984); Shapiro (1982)) from providing a remedy. Lizzeri (1999) studies the extent to which private, for-profit certification intermediaries reveal information to uninformed parties. Since information revelation

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9 According to Nimon and Beghin (1999), catalogues included in their database claim that organic cotton clothes are “healthy for babies because there are no pesticide or chemical residues to be absorbed through the skin” and some imply potential toxicity from wearing conventional cotton and raise fears of contamination by dioxins resulting from the bleaching and dying processes.
is a strategic decision in this framework, the disclosure rule used by the certifier is endoge-
nous. Lizzeri (1999) finds that for a monopoly intermediary, if the product is unsafe with
some probability, the profit maximizing disclosure rule takes the form of a minimum quality
standard. Jansen and Lince de Faria (2002) analyze why governments may pursue different
labeling policies and assess how international trade affects welfare if tastes for environmental
quality and production costs differ across countries. They apply the framework by Rosen
(1974) to study labeling under perfect competition. Since firms have zero profits, they are
indifferent to selling labeled or unlabeled products. Once a labeling program is established,
since consumer preferences for environmental quality are heterogeneous, both labeled and
unlabeled products are provided in equilibrium to satisfy consumer demand. Bottega and
De Freitas (2009) consider numerous institutions that regulate environmental quality and
study the welfare implications of their coexistence. They apply the framework by Mussa
and Rosen (1978) to study labeling in the presence of a monopoly goods producer. A reg-
ulator sets a mandatory minimum quality standard with the objective of maximizing social
welfare; a nongovernmental organization (NGO) and a private certifier each administer a
labeling program with the objective of maximizing environmental quality and the monopo-
list’s profits (which are extracted), respectively. Bottega and De Freitas (2009) find that in
the presence of a labeling program administered by an NGO or a private certifier, the scope
for public intervention by the regulator is decreased since either labeling program increases
the environmental quality of products on the market. Another article related to this paper
is by Petrakis et al. (2005), who compare the use of information provision with taxation as
an environmental policy measure for "dirty" products that impose private damages directly
onto consumers and also generate an environmental externality. They assume that some
consumers are initially uninformed about the individual damages, but can be informed by
way of a costly advertisement undertaken by the government. They consider two firms in
Bertrand competition, one of which produces a clean good that does not generate any dam-

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ages, and assume the quality produced by both firms is exogenous. Despite that taxation can never be fully efficient since the regulator cannot recognize each consumer’s type and must levy a uniform tax, Petrakis et al. (2005) find that taxation usually dominates a policy of information provision. Even though informed consumers do not purchase the good that generates the externality, taxes directly change relative prices while information can only change prices indirectly, by changing consumer behavior.

This paper is also related to the wider literature on corporate social responsibility (CSR) and voluntary environmental programs (VEPs) that attributes firms’ over-compliance with minimum environmental standards to consumer preferences for environmental quality. VEPs include environmental labeling programs but also encompass voluntary agreements between regulators and polluters as an alternative to mandatory approaches. Arora and Cason (1995) evaluate the factors leading to participation in the US Environmental Protection Agency’s 33/50 program\(^\text{10}\) to assess the potential ability of voluntary programs to augment command and control regulation. They find that large firms with substantial chemical releases are the most likely to participate, since they have the greatest reduction potential. Also, since the program offers public recognition to participants, public awareness plays an important role in the success of information based voluntary programs, in addition to competition in environmental quality among firms. Antweiler and Harrison (2003) utilize the National Pollutant Release Inventory (NPRI)\(^\text{11}\) to empirically investigate the effectiveness of green consumerism on a national level. If consumers use the NPRI to identify facilities with high levels of pollution and to identify the companies that own them, then multi-sector firms can experience a spillover effect, which they refer to as "environmental leverage": a low-revenue high-emission sector may negatively impact sales in a high-revenue low-emission

\(^{10}\)The 33/50 Program was implemented in 1991 with the objective of reducing the releases and transfers of 17 high-priority toxic chemicals.

\(^{11}\)The NPRI currently tracks over 300 substances and was created by Environment Canada in 1992 to provide information on pollutants released to the environment and transferred for disposal.
sector. Antweiler and Harrison (2003) find that emissions are lower for companies that are simultaneously environmentally leveraged and exposed to consumer markets. They also find, however, that green consumerism is a weak force and question whether it can be a substitute for other emission-reducing incentive systems such as taxes or regulation. Besley and Ghatak (2007) identify CSR as the creation of public goods (or curtailment of public bads), such as the provision of environmental quality, jointly with the production of private goods. They utilize a perfectly competitive framework to compare CSR with private voluntary contributions to the public good from consumers, provision by non-profit organizations, and government regulation. Firms undertake CSR by choosing a level of public goods provision in response to consumer preferences for public goods, and hence is part of a profit-maximizing strategy by firms whose businesses have external effects. They assume firms can make credible promises to provide public goods (and also consider the case where firms may renege on their promises but can be monitored) so that certification programs are not necessary. Given these assumptions, Besley and Ghatak (2007) derive numerous striking equivalence results. For instance, they show that a small uniform regulation on the level of public goods provision leaves the total contribution to the public good unchanged since the level of CSR is crowded out one-for-one by the regulation.

This paper develops a theoretical model to study the incentive for firms to undertake investment to improve the environmental quality of their PPMs in response to information that is disclosed by a voluntary labeling program. I assume that the "quality" produced by any firm is inversely related to the environmental damage caused by its production process and is not discernible in the final characteristics of its product. Hence, unlike Besley and Ghatak (2007), consumers cannot ascertain a product’s environmental quality even after it has been consumed. Also, due to the costs of disclosing more detailed information about

12 Note that in Besley and Ghatak (2007), products are experience goods rather than credence goods, since each firm’s contribution to the public good can be ascertained by consumers upon consumption.
the way a product has been produced, the label discloses a single standard of quality that is chosen by a credible third party. Consequently, firms either produce a product with the level of environmental quality established by the label, or sell an unlabeled product that has a minimal level of environmental quality. In order for a firm to certify its product, I assume that it is necessary for it to undertake a (subsequently irreversible) fixed cost investment. The labeling program resolves a problem of adverse selection in the market, since firms have no incentive to invest unless there is a labeling program or mandatory standards exist within the industry. I assume that the labeling authority chooses its standard of quality to maximize social welfare. Since the regulator and the labeling authority share the same objective function, this permits a clear understanding of how labeling, as a voluntary policy measure undertaken by government, differs from mandatory regulation. The model builds on the Dixit-Stiglitz model of monopolistic competition with firm heterogeneity as developed by Melitz (2003). I assume, however, that firms differ in terms of their (voluntary) investment cost to produce a given standard of environmental quality. As a consequence, for a given labeling standard, it is optimal for only a proportion of firms to label their products. This is consistent with empirical evidence since ecolabeling criteria are set to reward only the top environmental performers in a product category. Firms choose to label their products according to their individual competitive advantage and firms that label profit from undertaking investment. Ex post profits are necessary to ensure that some firms have an incentive to undertake costly investment in environmentally friendly technology. Monopolistic competition is an appropriate characterization of the market structure since environmental labeling programs are designed with the intent of rewarding particular brands

\[13\] Germany’s ecolabel, the Blue Angel, has successfully promoted innovation in a number of product categories such as varnishes and burners. Also, according to a survey in Germany in 1998, 76% of companies believe that the ecolabel has increased competition for environmental innovation in their branch. See Muller (2005) and Global Ecolabelling Network (1998).

\[14\] In Melitz (2003), firms are heterogeneous in terms of their productivity, which permits an analysis of how exposure to international trade reallocates market shares and affects aggregate productivity.
by differentiating them from competitors in a given product category, thereby encouraging competition in environmental quality among firms.\textsuperscript{15}

To enter the industry, I assume that a firm must make an initial investment to establish a baseline production process that has only a minimal level of environmental quality. Prior to entering the industry, firms cannot accurately observe how difficult it will be to improve the quality of their PPMs to satisfy a given environmental standard. For simplicity, I assume that information concerning the magnitude of these outlays arrives only subsequent to a firm’s entry, once it is able to acquire an understanding of its ability to adopt the necessary technology, and will determine the desirability of an additional investment. For instance, organic farming methods often replace herbicides with mechanical cultivation practices such as tillage to provide weed control. The cost of adopting the technology will depend on the firm’s understanding of how to improve soil quality in conjunction with factors that are specific to the farm’s location such as climate or geography, which can be understood with precision upon making an initial investment in a parcel of land. I also assume that identical consumers are individually ineffective in reducing aggregate environmental damage so that, when choosing how much of a given product to consume, they disregard the effects of their consumption on others. They are environmentally conscious, however, in the sense that they enjoy a private benefit from consuming environmentally friendly products. For the case where there is a negative externality, I parameterize the relationship between a consumer’s private valuation of a product’s environmental quality and the intensity of its environmental damage to determine the sensitivity of expenditure to changes in environmental damage. Unlike Petrakis et al. (2005), the private benefit that consumers obtain from purchasing environmentally friendly products does not necessarily arise from the avoidance of individual damages and even products that are revealed to have low environmental quality are consumed in the absence of a regulation that restricts their production. While

\textsuperscript{15}See Chapter 1 in Zarrilli et al. (1997).
efficiency could be restored in this framework if the government were to levy a tax per unit of environmental damage, the informational requirements would exceed those necessary to implement a labeling program or mandatory government regulation. Hence such a tax is not directly comparable with these regimes in the absence of an explicit way to measure the costs of acquiring the additional information.

The model provides several key insights. First, a labeling program can never decrease consumer welfare. Since a labeling program is voluntary, a firm will invest to improve the environmental quality of its PPMs if and only if it will obtain a greater profit from labeling its product. The rents earned from investing in environmentally friendly production are passed on to consumers in the form of greater variety since, for a given labeling standard, the number of varieties produced increases until expected profits are competed away through entry by firms to the industry. The model provides a clear description of the welfare maximizing labeling standard and a simple rule for a labeling authority to follow in practice. If there is no externality, for plausible choices of the investment cost function and the distribution of firm ability, the welfare maximizing labeling standard maximizes the environmental quality experienced privately by consumers, averaged over all available product varieties. In the presence of a negative externality, the optimal labeling standard should be set higher if and only if expenditure is sufficiently sensitive to changes in the intensity of a product’s environmental damage. The model also guides policy makers to choose the best instrument. If there is a negative externality, regulation provides greater welfare than labeling unless expenditure is not very sensitive to changes in the intensity of a product’s environmental damage, or if an outside good that imposes no environmental damage is a poor substitute for the product under consideration and consumers place sufficient value on minimal quality products. Accounting for the sensitivity of expenditure to changes in the intensity of environmental damage reveals circumstances in which regulation provides greater welfare than labeling, even if consumers derive a large private benefit from products that impose
environmental damage. In practice, this relationship can be estimated from data obtained from operative labeling programs, consumer surveys or experiments, and environmental impact information. In contrast with Besley and Ghatak (2007), in this framework voluntary labeling is not equivalent to government regulation. Also, while Petrakis et al. (2005) find that taxation usually dominates information provision since it is a more direct instrument, this model identifies instances in which information provision provides greater welfare than regulation since regulation can be too blunt of an instrument.

In Section 2, I present the basic model. I characterize the environmental quality that is experienced privately by each consumer in Section 3 and the optimal labeling standard in Section 4. In Section 5, I extend the model to include a negative environmental externality and in Section 6 I compare labeling with mandatory regulation. I conclude the paper in Section 7.

2 The Model

I model the interaction between consumers, producers and a labeling authority. One sector of the economy produces varieties of a horizontally differentiated good and environmental damage results from the production of each variety. The other sector produces a homogeneous consumption good with no environmental impact and hence the labeling program pertains only to the differentiated goods sector. The homogeneous good is the numeraire. I assume that it is produced under constant returns to scale in a competitive market and that one unit of labor produces one unit of output. These assumptions imply a unit wage rate, since workers are mobile between sectors. Each consumer owns one unit of labor, which is the only factor of production.

First, the labeling authority chooses the standard \( \tilde{q} \), the level of environmental quality specified by the label, to maximize social welfare \( W \). Second, firms decide whether to enter
the differentiated goods sector. There are an infinite number of potential entrants, and nature draws each entrant’s type $\theta$ independently from the common distribution $H$. Each firm pays an identical fixed cost $F$, thereafter sunk, to enter the industry and each firm learns its type upon entry. Given the standard set by the labeling authority $\tilde{q}$, a firm’s type determines its fixed investment cost $\delta(\tilde{q}, \theta)$ to improve the environmental quality of its variety from $q$ to $\tilde{q}$. In the absence of investment, any variety possesses minimal quality $q$. It is costless for firms to enter the homogeneous goods sector, and homogeneous goods have no environmental impact. Third, upon learning its type, each firm that produces a differentiated good decides whether to invest to label its product. A firm will invest if and only if the additional profit from labeling is not less than its voluntary investment cost. Fourth, firms produce. All firms that invest produce a product variety with quality $\tilde{q}$, while other firms in the differentiated goods sector produce a variety with minimal quality $q$. Fifth, consumers observe which product varieties are labeled and decide how much to consume of each.

### 2.1 Consumers

There is a continuous set of identical consumers $I$ indexed by $i$. Consumers love variety but they also value environmental quality. They infer the level of quality of any differentiated good by observing whether it is labeled. Any firm that has invested to improve its quality to the level $\tilde{q}$ will obtain the label to signal its quality to consumers. Since there is no incentive for any firm to over-invest, consumers must infer that no firm will produce a variety with quality greater than $\tilde{q}$. Also, since there is no incentive for a firm to invest a positive amount less than what is needed for the label, consumers must also infer that unlabeled varieties possess the minimal level of quality $q$.

Specifically, let $V$ be a continuous set of horizontally differentiated varieties indexed by $v$. 


For each variety \( v \), consumer \( i \) weights the utility he receives from consuming the quantity \( c_i(v) \) with the subjective function \( \lambda(q(v)) \) that serves to characterize how he privately values the environmental quality of variety \( v \). I assume that \( \lambda \) is continuously differentiable and increasing at a rate that is diminishing in quality so that \( \lambda'(q) > 0, \lambda''(q) < 0 \) and, as \( q \to \infty \), \( \lambda'(q) \to 0 \). Also, consumers value varieties that possess a minimal level of quality so that \( \lambda(\tilde{q}) > 0 \). The preferences of consumer \( i \) are given by

\[
U_i = z_i + \frac{1}{\beta}C_i^\beta
\]

(1)

where \( z_i \) is consumption of the homogeneous good, \( 0 < \beta < 1 \), and

\[
C_i = \left[ \int_{v \in V} \lambda(q(v))^{\alpha}c_i(v)^{\rho}dv \right]^\frac{1}{\beta}
\]

(2)

is a C.E.S. subutility function over a continuum of goods indexed by \( v \), where \( \alpha > 0 \). Also, the elasticity of substitution between any two varieties is given by \( \sigma = \frac{1}{1-\rho} \) and since the varieties are substitutes, \( 0 < \rho < 1 \).\(^{16}\) For a given \( \tilde{q} \),

\[
q(v) = \begin{cases} 
\tilde{q}, & \text{if variety } v \text{ is labeled} \\
q, & \text{if variety } v \text{ is unlabeled}
\end{cases}
\]

and hence, once some firms label, varieties are both horizontally and vertically differentiated.

As first shown by Dixit and Stiglitz (1977), \( C_i \) can be thought of as a composite good with an aggregate price \( P \).\(^{17}\) Maximizing (1) subject to the budget constraint

\[
z_i + PC_i \leq Y_i
\]

\(^{16}\)The C.E.S. utility function is often referred to as ‘love of variety’ preferences since the same level of expenditure spread out over more varieties increases utility.

\(^{17}\)Also see Krugman (1980) for an application of the monopolistic competition framework with homogeneous firms to international trade.
where \( Y_i \) is the income of consumer \( i \), yields the aggregate demand function \( C_i = P^{-\epsilon} \) with constant price elasticity \( \epsilon = \frac{1}{1-\beta} \). It follows that the share of a consumer’s income spent on differentiated goods is \( \xi_i = \frac{P^{1-\epsilon}}{Y_i} \) and the optimal consumption of the outside good is \( z_i = Y_i - P^{1-\epsilon} \).

Next, maximizing the subutility function (2) subject to the budget constraint

\[
\int_{v \in V} p(q(v))c_i(v)dv \leq \xi_i Y_i
\]

yields the optimal consumption of each variety \( v \)

\[
c_i(q(v)) = \frac{p(q(v))^{-1} \lambda_p(q(v))}{P^{1-\sigma}} \xi_i Y_i
\]

where the real price index for differentiated goods is given by

\[
P = \left[ \int_{v \in V} \lambda_p(q(v))dv \right]^{\frac{1}{1-\sigma}}
\]

and \( \lambda_p(q(v)) = \frac{\lambda(q(v))^{x+\sigma}}{p(q(v))^{\rho+\sigma}} \). The function \( \lambda_p(q(v)) \) depends on the consumer’s private valuation of variety \( v \)’s environmental quality, normalized by its price. If \( \alpha > \rho \), then for any product variety \( v \), the consumer’s private valuation of its environmental quality is relatively more important as a determinant of expenditure than its price.

### 2.2 Firms

Potential entrants to the differentiated goods sector are identical and know that there is a labeling program in the industry that upholds a given standard \( \bar{q} > q \).

To enter the industry, each firm must make an initial irreversible investment \( F > 0 \) (measured in units of labor), which enables it to produce a product with a minimal level of environmental quality.

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\(^{18}\)I assume that consumers have positive demand for the numeraire good so that \( z_i > 0 \).

\(^{19}\)Eco-labeling schemes are typically based upon multiple criteria. These could be represented by a single dimension, however, if each criterion is weighted according to its relative importance.
Upon entry, each firm learns its type $\theta$, which is drawn independently according to the common distribution $H$ with support on $\Theta$. Since firms do not know their individual investment cost with certainty prior to entry, this models a firm’s initial uncertainty about its ability to learn the environment-friendly technology needed to obtain the label. The investment cost (measured in units of labor) for a firm with type $\theta$ to be able to produce a variety with a level of environmental quality $q \geq \bar{q}$ is given by

$$\delta(q, \theta) = \frac{q - q}{\theta^\eta}$$

for some $\eta \geq 1$ that measures the sensitivity of the investment cost function with respect to firm type. The investment cost is optional, as it is incurred only by firms that label their products. From (5) it follows that, given a firm’s type, its investment cost is increasing in the standard of environmental quality and, for a given standard, decreasing in its type. Also, $\delta_{\theta\theta}(q, \theta) < 0$ so that the marginal investment cost of a higher standard is strictly decreasing in firm type.

Production with a higher standard of environmental quality also necessitates that firms incur a greater marginal cost. Each firm’s marginal cost $\gamma(q)$ (measured in units of labor) is increasing at a rate that is increasing in quality $q$ so that $\gamma'(q) > 0$, $\gamma''(q) > 0$, where $\gamma(q) \geq 1$, but, for simplicity, is independent of its type. Hence to produce a differentiated good with a standard of environmental quality $\tilde{q} > q$, a firm with type $\theta$ must incur an investment cost of $\delta(\tilde{q}, \theta)$ and a cost of $\gamma(\tilde{q})$ per unit of output, while minimal quality products only require a cost of $\gamma(q)$ per unit of output. I assume that $\lambda(q)$ is increasing sufficiently rapidly in $q$ relative to $\gamma(q)$, so that $\lambda'_p(q) > 0$. Also, I assume that $\alpha < \frac{1}{\sigma}$ and $\sigma > 2$, which, as shown in the appendix, is sufficient to ensure $\lambda_p(q)$ is increasing at a rate that is diminishing in quality $q$ so that $\lambda''_p(q) < 0$. These assumptions ensure that a firm’s
operating profits are increasing in environmental quality \( q \),
however at a diminishing rate.

Each differentiated product \( v \) is produced by a single monopolistically competitive firm
that is small relative to the economy. From (3) it follows that it will perceive itself as facing
a downward sloping demand curve with constant elasticity \( \sigma \).

Each firm maximizes its profits by charging a markup over marginal cost equal to \( \frac{\sigma}{\sigma - 1} \) and hence \( p(q) = \frac{\gamma(q)}{\rho} w \), where \( w \) is the wage rate. Upon entry, it follows that, gross of investment costs, the operating
profits for a firm with type \( \theta \) are given by

\[
\pi(q(\theta)) = [p(q(\theta)) - \gamma(q(\theta)) w] c(q(\theta)) = \frac{\lambda_p(q(\theta))}{\sigma P^{1-\sigma}} \xi Y \tag{6}
\]

where \( c(q(\theta)) = \int_{i \in I} c_i(q(\theta)) di \) is the total consumption of the variety with environmental
quality \( q \) produced by a firm with type \( \theta \), and \( Y = \int_{i \in I} Y_i di \) is aggregate income. The share
of aggregate income spent on differentiated goods is

\[
\xi = \frac{P^{1-\epsilon} L}{Y} \tag{7}
\]

where \( L \) is the total measure of workers and hence aggregate consumption of the outside
good is

\[
z = Y - P^{1-\epsilon} L. \tag{8}
\]

A firm that has entered the differentiated goods sector will undertake the investment
needed to acquire the label if and only if the additional profit from doing so is not less than
its investment cost. The firm with type \( \theta^* \) that is indifferent between investing to obtain

\[\text{We'll see from (9) that this condition is necessary for a labeling program to be effective.}\]

\[\text{From (3), the aggregate demand for any variety } v \text{ is given by } c(q(v)) = \frac{\nu(q(v))^{\sigma} \lambda(q(v))^{\sigma}}{\rho} \xi Y \text{ and since each firm is small, it is unable to influence } P \text{ or } \xi.\]

\[\text{Note that since } Y = wL, \xi = \xi_i.\]
the label that certifies quality \( \tilde{q} \) and selling its product unlabeled is defined by

\[
\pi (\tilde{q}) - w\delta (\tilde{q}, \theta^*) = \pi (q) .
\] (9)

Firms that draw a high type \( \theta > \theta^* \) have a sufficiently low cost of attaining the label and will optimally choose to invest to acquire the label, while those that draw a low type \( \theta < \theta^* \) will optimally choose not to invest and sell their products unlabeled. Low type firms that do not label make positive operating profits since consumers attribute positive value to unlabeled products, and they do not incur investment costs. In equilibrium, each firm that does not label produces a positive level of output equal to the total quantity of an unlabeled product demanded by consumers \( c (\tilde{q}) \). Also, because investment is voluntary, it must be that high type firms also make positive operating profits. In equilibrium, each firm that acquires the label produces a positive level of output equal to the total quantity of a labeled product demanded by consumers \( c (\tilde{q}) \). It follows that the market share for a labeled good relative to an unlabeled good is

\[
\frac{p (\tilde{q}) c (\tilde{q})}{p (q) c (q)} = \frac{\lambda_p (\tilde{q})}{\lambda_p (q)} .
\] (10)

Given the threshold firm type \( \theta^* \) in (9), from (4) the real price index \( P \) can be expressed as

\[
P^{1-\sigma} = QM
\] (11)

where

\[
Q = H(\theta^*)\lambda_p (\tilde{q}) + (1 - H(\theta^*)) \lambda_p (\tilde{q})
\] (12)

is an index of the average quality of available product varieties and \( M \) is the total mass of entrants to the industry. From (7) and (11) it follows that the share of income spent on differentiated goods \( \xi \) is increasing in the total environmental quality \( QM \) of differentiated goods, since expenditure on differentiated goods increases in response to their lower price.
I assume that each firm’s type $\theta$ is drawn independently from a Pareto distribution with
shape parameter $h > 1$ and scale parameter 1, so that

$$H(\theta) = 1 - \theta^{-h}$$ \hspace{1cm} (13)

for $\theta \in [1, \infty)$. With this distribution, there are relatively few high type firms in the population
with the greatest potential to improve the environmental quality of their product, since
the probability that a firm draws $\theta > \theta^*$ is decreasing in $\theta^*$. The Pareto distribution has
been used by economists to model firm productivity and firm size (see Axtell (2001), Melitz
(2003) and Luttmer (2006)). It is a plausible characterization of the firm type distribution
in this framework since, from (3), a firm’s type $\theta$ is the underlying determinant of its market
share.

From (5) and (13), for a given standard $\bar{q}$, the expected investment cost $I$ (measured in
units of labor) is

$$I = (1 - H(\theta^*)) E [\delta(\bar{q}, \theta) | \theta > \theta^*]$$ \hspace{1cm} (14)

$$= \theta^{*-h} \frac{h}{h + \eta} \frac{\bar{q} - q}{\theta^{*\eta}}.$$ 

Prior to learning its type, a firm’s expected profits are given by

$$E[\Pi] = H(\theta^*) \pi(q) + (1 - H(\theta^*)) \pi(\bar{q}) - wI$$

We’ll see in Section 4 that $\sigma > \epsilon$ is a necessary condition for the existence of a welfare maximizing
labeling standard.
which, using (6) and (11), can be expressed simply as

\[ E[\Pi] = \frac{\xi Y}{\sigma M} - wI. \]

Free entry ensures zero expected profits net of the entry fee so that

\[ \frac{\xi L}{\sigma M} - I = F \]  \hspace{1cm} (15)

and hence a bounded number of firms enters the industry.

Recalling that \( w = 1 \), an equilibrium is the solution to the system of equations (7), (8), (9), (11), (12), (14) and (15), which implicitly defines the endogenous variables \( \xi, z, \theta^*, P, Q, I \) and \( M \) as functions of \( \tilde{q} \). As shown in the appendix, for a given \( \tilde{q} \), the equilibrium exists and is unique.

3 Private Quality

Average quality \( Q \) is the relevant notion of quality experienced privately by each individual since consumers purchase every available variety, and even those varieties that do not satisfy the standard \( \tilde{q} \) can be legally produced under a labeling program. As shown in Lemma 1, the magnitude of the labeling standard \( \tilde{q} \) is not adequate as a measure of overall quality because as \( \tilde{q} \) increases, a smaller proportion of firms choose to acquire the label. Since a labeling standard is voluntary, there is a natural trade-off between the magnitude of the labeling standard and the proportion of firms that invest to label their products.

**Lemma 1** The threshold firm type \( \theta^* \) is increasing in \( \tilde{q} \).

**Proof.** See the appendix. \( \blacksquare \)

\(^{24}\)Note that the labor market clears according to Walras' law.
As a consequence of diminishing returns to quality, the additional operating profit from acquiring the label relative to the investment cost is decreasing in $\tilde{q}$ for any given firm type $\theta$. It follows from the definition of $\theta^*$ in (9) that if the standard is increased, then a higher type of firm is indifferent to labeling. Due to this trade-off between the severity of the standard and the proportion of firms that choose to comply, $Q$ is quasi-concave in the labeling standard $\tilde{q}$. As $\tilde{q}$ is increased from its minimal level $\underline{q}$, average quality $Q$ begins to increase but once a sufficiently small proportion of firms invest to label their products, it begins to fall. Hence an excessively high standard can render a labeling program ineffective and there is an optimal standard at which the overall quality that is enjoyed privately by consumers is maximal.

4 The Optimal Labeling Standard

Welfare is given by

$$W = L + \frac{1}{\epsilon - 1} P^{1-\epsilon}$$

(16)

which, from (11), is increasing in total environmental quality $QM$. In this section we’ll ascertain that greater average quality $Q$ induced by a higher labeling standard $\tilde{q}$ increases incentives for investment, and hence the expected investment cost $I$. Due to free entry (15), if the share of income spent on differentiated goods $\xi$ does not rise sufficiently, the mass of entrants $M$ will then fall. High type firms that label earn a rent from investment, which attracts a greater mass of entrants $M$ to the industry than if investment rents did not arise. Consequently, the increase in average quality $Q$ due to a greater labeling standard $\tilde{q}$ is not offset by a decrease in the mass of entrants $M$ and total environmental quality $QM$, and hence welfare $W$, increases. Investment rents are created by a labeling program since

\footnote{Recall that $\lambda'_{\theta^*}(q) < 0$.}

\footnote{Note that since expected profits are zero, social welfare in (16) is equivalent to consumer welfare.}
it utilizes the technological ability of firms, which is an unemployed resource prior to the program’s introduction.

Firms that choose to label realize a net profit \( \pi (\tilde{q}) - \delta (\tilde{q}, \theta) \) that must exceed \( \pi (q) \), since each firm has the option to sell its variety unlabeled. From (9) it follows that the rent from investment to a firm of type \( \theta \) is

\[
s (\tilde{q}, \theta) = \pi (\tilde{q}) - \pi (q) - \delta (\tilde{q}, \theta)
\]

which is increasing in \( \theta \) and positive for all \( \theta > \theta^* \). Hence expected rents from investment \( S \) are given by the expected additional operating profit from labeling less the expected investment cost

\[
S (\tilde{q}) = \int_{\theta^*}^{\infty} s (\tilde{q}, \theta) dH (\theta) = \frac{Q - \lambda_p (q)}{Q} \frac{\xi L}{\sigma M} - I.
\]

**Proposition 2** For any continuous firm type distribution \( H \) and investment cost function \( \delta(q, \theta) \) that satisfies \( \delta_q > 0, \delta_\theta < 0 \) and \( \delta_{q\theta} < 0, \) if \( \sigma > \epsilon \), then \( \tilde{q}^* \) maximizes welfare if and only if it maximizes expected rents from investment \( S \).

**Proof.** See the appendix.

**Corollary 3** If \( H \) is given by (13) and \( \delta \) is given by (5), then \( \tilde{q}^* \) maximizes welfare if and only if it maximizes average quality \( Q \).

**Proof.** See the appendix.

As shown in the appendix, expected rents \( S \) are quasi-concave in the labeling standard \( \tilde{q} \). Since a labeling program is voluntary, expected rents \( S \) are greater for any finite \( \tilde{q} > q \) so that welfare must be improved by the program. Due to free entry (15), if \( S \) is increasing.
in the labeling standard, then, for a given share of income spent on differentiated goods $\xi$, the mass of entrants $M$ falls less than proportionately in response to a rise in $Q$ induced by a higher standard $\tilde{q}$. Consequently total environmental quality $QM$ rises. Analogously, for a given $\xi$, if $S$ is decreasing, $M$ falls more than proportionately in response to a rise in $Q$ and $QM$ falls. While, from (7) and (11), the share of income spent on differentiated goods $\xi$ increases in response to greater total quality $QM$, this effect of a higher labeling standard $\tilde{q}$ is secondary since $\sigma > \epsilon$ and hence $QM$ is quasi-concave in $\tilde{q}$.\footnote{Also note that if $\xi$ is not held constant and $\epsilon$ is sufficiently large, an increase in $Q$ will result in an increase in $M$. This is discussed later in this section of the paper.} Hence, under general conditions, $S$ is maximal whenever $QM$ is maximal and from (16), maximizing welfare $W$ is equivalent to maximizing expected investment rents $S$. At the optimal standard $\tilde{q}^*$, since expected investment rents are maximal, the technological ability of firms is best utilized and investment is efficient since no other standard yields greater expected investment rents.

If $H$ and $\delta$ are given by (13) and (5), then using the definition of $\theta^*$ in (9), the definition of $P$ in (11), and the expression for $I$ in (14), the expected additional operating profit from labeling is given by

$$\frac{Q - \lambda_p(q)}{Q} \frac{\xi L}{\sigma M} = \frac{h + \eta}{h} I.$$ \hfill (18)

Equation (18) describes the industry-wide incentive compatibility constraint for investment since it specifies the expected additional operating profit that must be anticipated by firms to elicit a given level of expected investment expenditure.\footnote{Note that assumptions on the choice of $H$ and $\delta$ are necessary to obtain an analytical expression for (18).} Since investment is voluntary, a greater expected investment cost $I$ is consistent with greater profitability from investment. The share of income spent on differentiated goods $\xi$ expressed in (7), the incentive compatibility constraint (18) and the zero profit condition (15) together determine the equilibrium share of income spent on differentiated goods $\xi$, the equilibrium mass of entrants $M$ and the equilibrium expected investment cost $I$, for an equilibrium level of $Q$ that is induced by
some labeling standard $\tilde{q}$.

Holding $\xi$ and $M$ fixed, from (18) it follows that an increase in $Q$ that is induced by an increase in $\tilde{q}$ reallocates the share of expected operating profits earned by firms that do not invest to those that undertake investment. Since an increase in average quality $Q$ increases the expected additional operating profits from labeling, the incentive for firms to acquire the label increases. Hence, given $\xi$ and $M$, $I$ increases in response to greater average quality $Q$. From the incentive compatibility constraint (18) and the definition of $S$ in (17) it follows that expected rents from investment $S$ are a constant proportion of $I$ and hence $S$ also increases in response to greater average quality $Q$.

Entry to the industry, however, depends on the share of income spent on differentiated goods $\xi$, which, from (7), increases in response to a decrease in the real price index $P$ according to the size of the price elasticity of demand for the aggregate differentiated good $\epsilon$. For a given mass of entrants $M$, from the definition of $P$ in (11), $P$ falls in response to greater average quality $Q$ and hence if $\epsilon$ is sufficiently small, it follows from free entry (15) that $M$ falls whenever $Q$ rises and there is a trade-off between average quality $Q$ and variety $M$. If $\epsilon$ is sufficiently large, $M$ increases. In either case, however, since expected investment rents $S$ are increasing in $Q$, from Proposition 2 it follows that total environmental quality $QM$ is greater.

So far we have ascertained that in response to an increase in average quality $Q$ induced by a higher labeling standard $\tilde{q}$, the expected investment cost $I$ increases, expected investment rents $S$ increase and hence, from Proposition 2, total environmental quality $QM$ increases, while the mass of entrants $M$ may increase or decrease, depending on the elasticity of demand for the aggregate differentiated good $\epsilon$. Also, from (7), the share of income spent on differentiated goods $\xi$ increases so that, from (8), consumption of the homogeneous good $z$ decreases. Analogously, a decrease in $Q$ results in the opposite effects. It follows that

\[ S = \frac{Q}{I}. \]

\[ \text{From (18) and (17) it follows that } S = \frac{Q}{I}. \]
average quality $Q$ is maximal if and only if expected investment rents $S$ are maximal or, equivalently, welfare $W$ is maximal. It is sufficient for the regulator, then, to choose the labeling standard to maximize the average environmental quality $Q$ of available product varieties.$^{30}$ Furthermore, if entry to the industry $M$ is fixed in the short run, then since the optimal labeling standard $\tilde{q}^*$ maximizes total environmental quality $QM$, the regulator should choose the labeling standard to maximize average quality $Q$ in the short run, even under very general conditions for $H$ and $\delta$.

5 The Environment

In the previous sections consumers were unaffected by the environment and purchasing labeled products simply provided each consumer with a greater private benefit derived from their own consumption. I now allow for the possibility that consumers are affected by the consumption decisions of others. Consumers still value environmental quality that is experienced from their consumption but they are now additionally concerned for the environment itself. For instance, in addition to enjoying certified organic food for health benefits that are experienced privately, consumers now care about the beneficial effects of organic farming on biodiversity, and soil and water quality,$^{31}$ which depend on aggregate consumption.$^{32}$ I assume that consumers are individually ineffective in reducing aggregate environmental damage so that consumer behavior continues to be motivated by how environmental quality is privately valued. Since consumers do not take into account the impact of their consumption on others (including themselves), even with perfect information the efficient level of

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$^{30}$Or, equivalently, the regulator can choose $\tilde{q}$ to maximize $\frac{Q}{\lambda_p(\tilde{q})} = (1 - \theta^{s-k}) + \theta^{s-k} \frac{\lambda_p(\tilde{q})}{\lambda_p(q)}$ where, from (10), $\frac{\lambda_p(\tilde{q})}{\lambda_p(q)}$ is given by the relative market share for labeled goods and, from (13), $\theta^{s-k}$ is the proportion of firms that label their products.

$^{31}$Organic farming prevents water contamination due to pesticide runoff.

$^{32}$While consumers are affected by environmentally unfriendly production indirectly through aggregate consumption, it must be equal to aggregate production in equilibrium.
environmental damage cannot be obtained without government intervention.

Specifically, I assume that there is a negative environmental externality given by

\[ E = \int_{v \in V} c(v)\phi(q(v))dv \]  \hspace{1cm} (19)

where the externality per unit of consumption of variety \( v \) (the intensity of environmental damage) is given by \( \phi(q(v)) > 0 \). I assume that firms producing higher quality varieties pose a smaller environmental hazard and that \( \phi \) is decreasing at a rate that is increasing in further improvements to quality so that \( \phi'(q) < 0 \) and \( \phi''(q) > 0 \), for all \( q \in [q_1, \infty) \). Furthermore, I assume that the preferences of consumer \( i \) are now given by

\[ U_i = z_i + \frac{1}{\beta} C_i^\beta - E. \]

A consumer’s utility, then, is affected by the environmental quality of each product variety \( v \) by way of a private component that depends on their own consumption, as well as a nonrival and nonexcludable component \( E \) that depends on aggregate consumption. It follows that welfare is given by

\[ W = W^P - E^* \]  \hspace{1cm} (20)

where, from (7), the private component of welfare \( W^P = L + \frac{1}{\epsilon - 1} \xi \) corresponds to (16) and \( E^* \) is the externality resulting from optimal consumption decisions. From (19) and (3) it follows that

\[ E^* = \Phi \xi L \]  \hspace{1cm} (21)

where the externality per unit of expenditure on differentiated goods

\[ \Phi = (1 - \mu)\phi_p(q) + \mu \phi_p(\tilde{q}) \]  \hspace{1cm} (22)
is a weighted average of the externality per unit of expenditure on labeled differentiated goods $\phi_p(q) = \frac{\phi(q)}{\mu(q)}$ and the externality per unit of expenditure on unlabeled differentiated goods $\phi_p(q) = \frac{\phi(q)}{\mu(q)}$, where the weight $\mu = \frac{(1-H(\theta^*))\lambda_p(q)}{Q}$ is the share of expenditure on differentiated goods that is allocated to labeled varieties. The externality per unit of expenditure on differentiated goods $\Phi$ is quasi-convex in the labeling standard $\tilde{q}$ and hence there is a standard $\tilde{q}_E$ at which $\Phi$ is minimal. A higher labeling standard works directly to decrease $\Phi$ since the externality per unit of expenditure on labeled goods $\phi_p(q)$ is decreasing in $\tilde{q}$. It also works indirectly to increase $\Phi$, since the share of expenditure allocated to labeled varieties $\mu$ is decreasing in $\tilde{q}$. As $\tilde{q}$ is first increased from $q$, the former effect dominates and $\Phi$ decreases but once a sufficiently small number of firms invest to label their products, $\Phi$ begins to increase.

For a given $\Phi$, it follows from (21) that an increase in the share of income spent on differentiated goods $\xi$ leads to a greater externality $E^*$. Since $\xi$ is increasing in total environmental quality $QM$ (or, equivalently, decreasing in the real price index $P$), a higher standard $\tilde{q}$ that results in greater $QM$ can result in both a smaller externality per unit of expenditure on differentiated goods $\Phi$ and greater expenditure on differentiated goods $\xi L$. If the price elasticity of demand for the aggregate differentiated good $\epsilon$ is sufficiently large, a decrease in $P$ has a large impact on $\xi$ and a higher labeling standard can perversely lead to greater aggregate environmental damage $E^*$. It follows from (20) and (21), however, that if $\frac{1}{e-1} > \Phi(\tilde{q}) L$, then an increase in the externality due to greater expenditure on differentiated goods is offset by an increase in private welfare $W_p$.

I assume an explicit relationship between how consumers privately value the environment-
tal quality of each variety \( v \) and the intensity of its environmental damage given by

\[
\lambda_p(q(v)) = \phi_p(q(v))^{-k}
\]

(23)

for some finite \( k > 0 \). The elasticity \( k \) denotes the percentage decrease in expenditure on each variety \( v \) relative to a percentage increase in its environmental damage per unit of expenditure. If \( k > 1 \) \((k < 1)\), then consumers care more (less) about the environment than their consumption, since their expenditure on each variety \( v \) decreases more (less) than proportionately in response to an increase in its environmental damage per unit of expenditure. For instance, if \( \phi_p(q(v)) \) specifies the emissions of a pollutant per unit of expenditure on variety \( v \) and if labeled varieties emit half the pollution of unlabeled varieties per unit of expenditure, then whenever \( k > 1 \) \((k < 1)\) consumer spending on labeled varieties is more (less) than twice the amount for unlabeled varieties. Concern for the environment is increasing in \( k \) since a consumer’s expenditure for any variety becomes more sensitive to its environmental damage per unit of expenditure.

The following proposition extends Corollary 3 and demonstrates that in the presence of a negative externality, whenever consumers care more (less) about the environment than their consumption, then the welfare maximizing labeling standard \( \tilde{q}^* \) is greater (smaller).

**Proposition 4** If \( \lambda_p(q) \) is given by (23) and \( \epsilon < \frac{1}{\Phi(q)L} + 1 \) for all \( q > \hat{q} \), then (i) welfare \( W \) is quasi-concave in \( \tilde{q} \) and (ii) if we define \( \tilde{q}_P = \arg \max W_P \), then \( \tilde{q}_P < \tilde{q}^* \) if and only if \( k > 1 \).

**Proof.** See the appendix.

If \( \frac{1}{\epsilon - 1} > \Phi(\tilde{q}) L \), then, due to a higher labeling standard \( \tilde{q} \), the increase in the externality due to greater expenditure on differentiated goods is offset by an increase in private welfare \( W_P \). Consequently, as shown in the appendix, there is a unique standard \( \tilde{q}^* > \hat{q} \) that
maximizes welfare. If \( k = 1 \), then private quality is inversely proportional to the intensity of environmental damage and there is no tension in using a single policy instrument to deal with both market failures, imperfect information and the externality, simultaneously. In this case, the standard \( \tilde{q}_P \) that maximizes private welfare \( W_P \) is equivalent to the standard \( \tilde{q}_E \) that minimizes the externality per unit of expenditure on differentiated goods \( \Phi \). Consequently, the optimal labeling standard is unchanged in the presence of the externality and, from (20), the objective of maximizing welfare \( W \) is equivalent to minimizing environmental damage \( E^* \).\(^{34}\) If \( k > 1 \), however, since the percentage increase in \( \lambda_p (\tilde{q}) \) due to a higher labeling standard \( \tilde{q} \) is greater than the percentage decrease in \( \phi_p (\tilde{q}) \), the share of expenditure on differentiated goods allocated to labeled varieties \( \mu \) decreases less rapidly in \( \tilde{q} \) than if \( k = 1 \), and \( \Phi \) is minimized at a standard \( \tilde{q}_E \) that exceeds \( \tilde{q}_P \). For this case, the optimal labeling standard \( \tilde{q}^* \) is greater than \( \tilde{q}_P \) since it is optimal for consumers to forgo some of the quality they experience privately in order to experience less environmental damage. Similarly, if \( k < 1 \), then since \( \lambda_p (\tilde{q}) \) is less responsive to changes in \( \phi_p (\tilde{q}) \), the share of expenditure on differentiated goods allocated to labeled varieties \( \mu \) is decreasing more rapidly in the labeling standard \( \tilde{q} \). Hence \( \tilde{q}_P \) exceeds \( \tilde{q}_E \) and the optimal labeling standard \( \tilde{q}^* \) is smaller in the presence of the externality.

6 Regulation

Labels provide consumers with information so they may recognize high quality products but a government authority could alternatively forbid the production of products produced with low environmental quality. Under such regulation, the chosen mandatory standard \( \tilde{q} > q \) determines the requisite level of investment with cost \( \delta (\tilde{q}, \theta) \) in order for a firm with type

\(^{34}\)Recall from Corollary 3 that \( Q \) is maximized at \( \tilde{q}_P \), and from (22) and (23) we have that if \( k = 1 \), \( \Phi = \frac{1}{\tilde{q}} \) for all \( \tilde{q} \).
to produce. After learning its type, a firm that finds it unprofitable to invest will choose to exit the industry. The following proposition demonstrates that in the absence of an externality, despite that average quality $Q$ may be lower under a labeling program, labeling provides greater welfare than regulation.

**Proposition 5** The optimal labeling regime yields greater private welfare $W_P$ than the optimal regulatory regime for any $\lambda_p(q) > 0$.

**Proof.** See the appendix. ■

For any given standard of quality $\tilde{q} = \bar{q}$, since minimal quality products can be produced under a labeling program, incentives for investment are weaker than under regulation. Consequently, the expected investment cost $I$ is smaller and, due to free entry (15), the mass of entrants $M$ and hence variety is greater under a labeling program than under regulation. From (9) it follows that a smaller proportion of firms invest under a labeling regime. Hence average quality $Q$ is lower than under regulation whenever the private valuation for minimal quality goods $\lambda_p(q)$ is sufficiently small. The effects of a higher threshold firm type $\theta^*$ on $Q$ and $M$ are precisely offsetting, however, so that the distinguishing impact of a labeling program on welfare is due to the availability of minimal quality varieties. Consequently, total environmental quality $QM$ and private welfare $W_P$ are greater under a labeling program than under regulation for any $\lambda_p(q) > 0$, and the difference is increasing in $\lambda_p(q)$. Indeed, since consumers are not affected by the consumption decisions of others, a regulation that forbids the consumption of minimal quality varieties, which have value for consumers, must result in lower welfare. Since welfare under labeling is greatest when the standard is chosen optimally, it follows that the optimal labeling regime yields greater welfare than the optimal regulatory regime.

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35 Since firm type $\theta^*$ does not earn a rent from investment, from (17) it follows that $\frac{dS}{d\theta^*} = 0$ and hence, from Proposition 2, we have $\frac{dQM}{d\theta^*} = 0$. 

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If consumers are affected by the consumption decisions of others, in principle the government could levy a tax per unit of environmental damage so that consumers fully bear the costs that result from their consumption.\(^{36}\) Since a tax is a precise instrument that can internalize the damages for each variety, if chosen optimally, it is the first-best policy measure. The optimal labeling regime and the optimal regulatory regime result in lower welfare since regulation over-corrects for the damages imposed by minimal quality varieties, since they are not produced at all, while labeling under-corrects for their damages, since minimal quality varieties are produced but their price does not reflect the true cost of their consumption. Implementing the optimal tax requires the government to have full information, however, regarding each potential level of environmental quality that can be produced by each firm. In response to incentives created by the tax, each firm will invest to produce the level of environmental quality that maximizes its profit (net of tax payments), which depends upon its type. Implementing a labeling program that discloses a single standard of quality, or regulation that imposes a minimum standard of quality, however, necessitates only that an authority verify whether the environmental quality of a firm’s product is at least the level established by the labeling or regulatory standard.\(^{37}\) Hence a tax is not directly comparable with these regimes without an unequivocal measure of the costs of acquiring the additional information, which could be prohibitive or may vary widely depending on the context of the problem.

Proposition 6 extends Proposition 5 to the instance where consumers are affected by the consumption decisions of others and determines whether labeling or regulation is the optimal policy measure for different values of the model’s parameters. It demonstrates that

\(^{36}\)The tax that implements the welfare maximizing level of environmental damage is commonly referred to as a Pigovian tax. See Kolstad (2000).

\(^{37}\)For instance, it may be prohibitive to quantify the environmental damage caused by a farm that applies pesticides to its fields, or to determine the number of dolphins killed by a firm that produces tuna. To implement a voluntary standard or a regulation that prohibits farmers from using inorganic pesticides, or fishermen from using purse seine nets, however, could be as simple as checking whether a specific piece of equipment is in place.
if differentiated goods and the outside good $z$ are not poor substitutes, even if the private valuation for minimal quality goods $\lambda_p(q)$ is large, regulation provides greater welfare than labeling if consumption is sufficiently sensitive to environmental damage. If differentiated goods and the outside good $z$ are poor substitutes, however, then labeling provides greater welfare than regulation if the private valuation for minimal quality goods $\lambda_p(q)$ is sufficiently large. It also demonstrates that Proposition 4 provides a thorough characterization of the optimal labeling standard in the presence of the externality. We’ll see that if $\frac{1}{\epsilon - 1} \leq \Phi(\tilde{q})L$, then regulation is the optimal policy measure. Hence Proposition 4 provides a general characterization of the optimal labeling standard whenever it is possible for labeling to be the optimal policy measure for any $\tilde{q} > q$.

**Proposition 6** If $\lambda_p(q)$ is given by (23) then i) for any given $k$, $\lambda_p(q)$ and $\tilde{q} = \overline{q}$, if $\epsilon \geq \frac{1}{\Phi(q)L} + 1$, then regulation provides greater welfare than labeling. ii) If $\epsilon > \frac{1}{L} + 1$, then for any $\lambda_p(q) > 1$ there exists a unique $k^*$ such that the optimal regulatory regime provides greater welfare than the optimal labeling regime if and only if $k > k^*$. iii) If $\epsilon \leq \frac{1}{L} + 1$, then there exists a unique $\lambda_p(q) > 1$ such that if $\lambda_p(q) \geq \lambda_p(q)'$ then, for any $k$, the optimal labeling regime provides greater welfare than the optimal regulatory regime.

**Proof.** See the appendix. ■

From Proposition 5 it follows that the expenditure on differentiated goods $\xi L$ is greater under a labeling program than under regulation.\(^{38}\) If $\frac{1}{\epsilon - 1} \leq \Phi(\tilde{q})L$, so that differentiated goods and the outside good $z$ are close substitutes, from (20) it follows that the externality due to the production of differentiated goods exceeds their contribution to private welfare $W_P$. Hence greater expenditure on differentiated goods results in lower welfare and regulation is the optimal policy measure. Also, if $\frac{1}{\epsilon - 1} < L$, so that differentiated goods and the outside good $z$ are substitutes, then for a given $\lambda_p(q) > 1$ there is a threshold value of

\(^{38}\)Recall from (7) that $\xi$ is increasing in $QM$. 31
environmental concern $k^*$ such that regulation provides greater welfare than labeling if and only if $k > k^*$.\textsuperscript{39} As $k$ increases, since consumption becomes more sensitive to the intensity of environmental damage, consumers are hurt more by the environmental damage that results from the aggregate consumption of minimal quality varieties. As shown in Figure 1, where $W^L$ denotes welfare under labeling and $W^R$ denotes welfare under regulation, welfare is decreasing more rapidly in $k$ under labeling than under regulation, since minimal quality varieties are not consumed under regulation. If consumers place less value on minimal quality varieties, welfare under labeling decreases while welfare under regulation is unchanged. Hence, as shown in Figure 1, $k^*$ decreases and regulation is optimal for a larger range of environmental concern $k$. If $\epsilon \leq \frac{1}{k} + 1$, so that differentiated goods and the outside good $z$ are poor substitutes, then labeling provides greater welfare than regulation if consumers place sufficient value on minimal quality varieties for any degree of environmental concern $k$. Labeling provides greater welfare than regulation since, in particular, the outside good $z$ is a poor substitute for minimal quality varieties that are unavailable to consumers under regulation.

\textsuperscript{39}Note that since $\lambda_p (q) > 1$, it follows from (23) that $\Phi (q) < 1$ so that (i) is a special case of (ii). Hence for a given $\lambda_p (q) > 1$ and $k$, if $\frac{1}{\epsilon - 1} \leq \Phi (q) L < L$, then $k > k^*$. 

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Figure 1. Welfare as a function of $k; \epsilon > \frac{1}{L} + 1$ and $1 < \lambda_p(q)_2 < \lambda_p(q)_1$.

7 Conclusion

This paper examines how to best utilize product labeling as an environmental policy measure. Consumers are individually ineffective in reducing aggregate environmental damage and consumption depends on how they privately value environmental quality. Due to imperfect information concerning the environmental quality of products, there is no incentive for a firm to undertake investment unless there is a labeling program or compulsory regulation in the industry. Firms differ in their ability to produce environmental quality so that, for a given standard of quality established by the label, more able firms earn a greater rent from investment.

If consumers are not affected by the consumption decisions of others, the optimal labeling standard maximizes expected investment rents. Consequently, the technological ability of firms is best utilized and investment is efficient. Furthermore, for plausible choices of the investment cost function and the distribution of firm ability, the optimal labeling standard also maximizes the average environmental quality experienced privately by consumers. If
there is a negative externality, the optimal labeling standard is higher if and only if consumers care more about the environment than their consumption, which is determined by the elasticity $k$. This provides a clear rule for a labeling authority to follow in practice. The model also guides regulators to choose the best policy instrument. If there is a negative externality, labeling provides greater welfare than regulation if consumption is not very sensitive to environmental damage, or if an outside good that imposes no environmental damage is a poor substitute for the product under consideration and consumers place sufficient value on minimal quality products. Despite that labeling can never be fully efficient, in these instances regulation is too blunt of an instrument since forbidding the production of minimal quality goods creates a greater welfare loss than if they are available to consumers without imposing an additional cost for their consumption.

The model demonstrates that in the presence of a negative externality, the optimal labeling standard and policy instrument depend critically on $k$, which parameterizes the relationship between the environmental quality that consumers experience privately from their consumption of a product and the intensity of its environmental damage. In principle, this parameter could be estimated from data obtained from operative labeling programs, consumer surveys or experiments, and environmental impact information. A more thorough analysis of this empirical prediction awaits future research.
Appendix

Preliminaries

Concavity of $\lambda_p$:

If $\alpha \sigma < 1$ and $\sigma > 2$, then $\lambda_p(q)$ is concave in $q$. We have that $\lambda_p(q) = \frac{\lambda(q)^{\alpha \sigma}}{\rho(q)^{\alpha \sigma}} = \rho^{\alpha \sigma} \frac{\lambda(q)^{\alpha \sigma}}{\gamma(q)^{\alpha \sigma}}$. Since, by assumption, $\lambda'(q) > 0$ and $\lambda''(q) < 0$, $\lambda(q)^{\alpha \sigma}$ is concave if $\alpha \sigma < 1$. Also, since, by assumption $\lambda'_p(q) > 0$, it follows that $\lambda''_p(q) < 0$.

Existence and uniqueness of equilibrium:

Existence of the equilibrium follows if $F$ is not too large. Equations (9), (12), (14) and (15) yield the following polynomial in $\theta^*$

$$\frac{\eta (\tilde{q} - q)}{h + \eta} \theta^{-(h+\eta)} + \frac{\lambda_p(q)}{\lambda_p(q) - \lambda_p(q)} \theta^{\sigma - \eta} = F \quad (A.1)$$

If $\tilde{q} > q > 0$, the left hand side of (A.1) exceeds $F$ at $\theta^* = 1$. If $\tilde{q} < q$, since $\frac{\lambda_p(q) - \lambda_p(q)}{\tilde{q} - q} \rightarrow \lambda'_p(q)$ as $\tilde{q} \rightarrow q$, if we require $F < \frac{\lambda_p(q)}{\lambda'_p(q)}$, the left hand side of (A.1) continues to exceed $F$ at $\theta^* = 1$. Hence if $F < \min \left\{ \frac{\eta}{h+\eta}, \frac{\lambda_p(q)}{\lambda'_p(q)} \right\}$, the left hand side of (A.1) exceeds $F$ at $\theta^* = 1$ for all $\tilde{q} > q$. Furthermore, as $\theta^* \rightarrow \infty$, the left hand side of (A.1) approaches zero, giving us existence. Uniqueness follows by noting that the left hand side of (A.1) is monotone decreasing in $\theta^*$.

Comparative statics for $\tilde{q}$:

Differentiating the system of equations (7), (8), (9), (11), (12), (14) and (15) with respect to $\tilde{q}$ yields:

$$\frac{d \log \xi}{d \tilde{q}} = \frac{\epsilon - 1}{\sigma - \epsilon} \Omega \quad (A.2)$$

$$\frac{d \log z}{d \tilde{q}} = - \frac{\xi}{1 - \xi} \frac{\epsilon - 1}{\sigma - \epsilon} \Omega \quad (A.3)$$

$$\frac{d \log \theta^*}{d \tilde{q}} = \frac{1}{\eta} (\Omega - \Delta) \quad (A.4)$$

$$\frac{d \log (QM)}{d \tilde{q}} = \frac{\sigma - 1}{\sigma - \epsilon} \Omega \quad (A.5)$$

$$\frac{d \log Q}{d \tilde{q}} = \tau \Omega \quad (A.6)$$

$$\frac{d \log I}{d \tilde{q}} = h + \eta \frac{\lambda_p(q)}{Q - \lambda_p(q)} \Omega$$

$$\frac{d \log J}{d \tilde{q}} = h + \eta \frac{\lambda_p(q)}{Q - \lambda_p(q)} \Omega$$
\[
\frac{d \log M}{d q} = \left( \frac{\sigma - 1}{\sigma - \epsilon} - \tau \right) \Omega \quad (A.7)
\]

where
\[
\Omega = \frac{\theta^{*-h}}{Q} \left[ \lambda'_p (\tilde{q}) - \frac{h}{h + \eta} \lambda_p (\tilde{q}) - \frac{1}{\tilde{q} - q} \lambda_p (q) \right] \quad (A.8)
\]
\[
\Delta = \frac{\lambda'_p (\tilde{q})}{\lambda_p (\tilde{q}) - \lambda_p (q)} - \frac{1}{\tilde{q} - q} \quad (A.9)
\]

and \( \tau = \frac{\eta Q + h \lambda_p (q)}{\eta Q} \).

### Comparative statics for \( \lambda_p (q) \):

Differentiating the system of equations (7), (8), (9), (11), (12), (14) and (15) with respect to \( \lambda_p (q) \) yields:

\[
\frac{d \log \xi}{d \lambda_p (q)} = \frac{\varepsilon}{1 - \varepsilon} \frac{1 - \theta^{*-h}}{Q} > 0 \quad (A.10)
\]
\[
\frac{d \log z}{d \lambda_p (q)} = \frac{- \xi}{1 - \xi} \frac{\varepsilon}{1 - \varepsilon} \frac{1 - \theta^{*-h}}{Q} < 0 \quad (A.11)
\]
\[
\frac{d \log \theta^*}{d \lambda_p (q)} = \frac{1}{\eta} \left[ \frac{1}{\lambda_p (\tilde{q}) - \lambda_p (q)} + \frac{1 - \theta^{*-h}}{Q} \right] > 0 \quad (A.12)
\]

so that, as \( \lambda_p (q) \rightarrow \lambda_p (\tilde{q}) \), \( \frac{d \theta^*}{d \lambda_p (q)} \rightarrow \infty \). Also,

\[
\frac{d \log (QM)}{d \lambda_p (q)} = \frac{1}{Q} \frac{1 - \theta^{*-h}}{1 - \varepsilon} > 0 \quad (A.13)
\]
\[
\frac{d \log Q}{d \lambda_p (q)} = \frac{1}{Q} \left[ (1 - \theta^{*-h}) - \frac{h \theta^{*-h} \lambda_p (\tilde{q})}{\eta Q} \right] \quad (A.14)
\]

which is positive if \( \theta^{*-h} \) is close to 0 and negative if \( \theta^{*-h} \) is close to 1 and hence from (A.1), \( \frac{d \log Q}{d \lambda_p (q)} \) is negative if \( \lambda_p (q) \) is sufficiently small and positive if it is sufficiently large. Furthermore,

\[
\frac{d \log I}{d \lambda_p (q)} = - (h + \eta) \frac{1}{\eta} \left[ \frac{1}{\lambda_p (\tilde{q}) - \lambda_p (q)} + \frac{1 - \theta^{*-h}}{Q} \right] < 0 \quad (A.15)
\]
\[
\frac{d \log M}{d \lambda_p (q)} = \frac{\varepsilon}{1 - \varepsilon} \frac{1 - \theta^{*-h}}{Q} + \frac{I (h + \eta)}{(I + F) \eta} \left[ \frac{1}{\lambda_p (\tilde{q}) - \lambda_p (q)} + \frac{1 - \theta^{*-h}}{Q} \right] > 0. \quad (A.16)
\]

### Second order conditions:
From \((A.4)\) we have that

\[
\frac{d \log (QM)}{d \tilde{q}} = \frac{\sigma - 1}{\sigma - \epsilon} \Omega
\]

where

\[
\Omega = \frac{\theta^{\epsilon-h}}{Q} \left[ \lambda_p' (\tilde{q}) - \frac{h}{h + \eta} \lambda_p (\tilde{q}) - \lambda_p(q) \right].
\]

Also, from \((16)\) it follows that in the absence of an externality, maximizing welfare \(W\) is equivalent to maximizing \(QM\). Since by assumption \(\sigma > \epsilon\), it remains to show that there exists a unique \(\tilde{q}^*\) such that \(\Omega > 0\) if and only if \(\tilde{q} < \tilde{q}^*\). As \(\tilde{q} \to \infty\), since \(\lambda_p\) is concave in \(\tilde{q}\), \(\lambda_p' (\tilde{q}) \to 0\) faster than the secant line \(\frac{\lambda_p (\tilde{q}) - \lambda_p (q)}{\tilde{q} - q}\) and hence \(\frac{\lambda_p' (\tilde{q}) - \lambda_p' (q)}{\tilde{q} - q} \to 0\). Also, since \(\lambda_p\) is concave in \(\tilde{q}\), we have that \(\frac{\lambda_p' (\tilde{q})}{\tilde{q} - q}\) is decreasing in \(\tilde{q}\). Hence, since \(0 < \frac{h}{h + \eta} < 1\), it follows that there exists a unique \(\tilde{q}^*\) such that \(\Omega > 0\) if and only if \(\tilde{q} < \tilde{q}^*\).

**Proof of Lemma 1:**

Since \(\lambda_p\) is concave in \(q\), we have \(\Delta < 0\) for every \(\tilde{q} > q\). From \((A.8)\) we can express \(\Omega\) as

\[
\Omega = \frac{Q - \lambda_p (q)}{(h + \eta) Q} \left[ h \Delta + \frac{\eta \lambda_p' (q)}{\lambda_p (\tilde{q}) - \lambda_p (q)} \right]
\]

so that

\[
\Omega - \Delta = - \left[ \frac{\eta Q + h \lambda_p (q)}{(h + \eta) Q} \right] \Delta + \frac{Q - \lambda_p (q)}{(h + \eta) Q} \frac{\eta \lambda_p' (q)}{\lambda_p (\tilde{q}) - \lambda_p (q)} > 0
\]

and hence, from \((A.3)\), \(\frac{d \log \theta^*}{d \tilde{q}} > 0\) for all \(\tilde{q} > q\).

**Proof of Proposition 2:**

Note that \((7)\), \((11)\), \((15)\) and \((17)\) are independent of the specific choice of cost function \(\delta\) and firm type distribution \(H\). A continuous firm type distribution \(H\) is necessary to ensure that all variables are continuous in \(\tilde{q}\) and the conditions for \(\delta\) \((\delta_q > 0, \delta_q < 0\) and \(\delta_{q\theta}(\tilde{q}, \theta) < 0\)) are necessary to ensure that if it is optimal for a given firm type to invest, then it is optimal for all higher firm types to also invest. From \((7)\), \((11)\), \((15)\) and \((17)\) we have

\[
S = F - \frac{\lambda_p (q) L}{\sigma \ (QM)^{1-\epsilon}}
\]

where, since \(\sigma > \epsilon\), \(\epsilon = \frac{\sigma - 1}{\sigma - 1} < 1\). We have

\[
\frac{dS}{d\tilde{q}} = (1 - \varepsilon) \frac{\lambda_p (q) L (QM)^{-2}}{\sigma} \frac{d (QM)}{d \tilde{q}}.
\]
From (11), (17) and (A.17),
\[
\frac{dS}{dq} = (\sigma - \epsilon) \frac{\lambda_p(q) L dW}{\sigma Q M \frac{d\bar{q}}{dq}}
\]
and hence maximizing \(S\) is equivalent to maximizing \(W\). Furthermore, quasi-concavity of \(S\) follows from the second order conditions.

**Proof of Corollary 3:**
From (16) and (A.4) we have that \(\frac{dW}{dq} = 0\) if and only if \(\frac{dQM}{dq} = \frac{\sigma - 1}{\sigma - \epsilon} QM \Omega = 0\). Hence \(\Omega(\bar{q}^*) = 0\) and it follows from (A.5) and the second order conditions established in the preliminaries above that at \(\bar{q}^*\), \(Q\) is maximal. Moreover, at \(\bar{q}^*\), from (A.7), if \(\tau > \frac{\sigma - 1}{\sigma - \epsilon}\), \(M\) is minimal while if \(\tau < \frac{\sigma - 1}{\sigma - \epsilon}\), \(M\) is maximal.

**Proof of Proposition 4:**
From (20) and (21) we have
\[
W = L + \left( \frac{1}{\epsilon - 1} - \Phi(\bar{q}) \right) \xi. \hspace{1cm} (A.18)
\]
Recall that with the externality \(E^*\), \(W\) is maximized at \(\bar{q}^*\), \(W_P\) is maximized at \(\bar{q}_P\), and \(\Phi\) is minimized at \(\bar{q}_E\). Also, from (A.2) and the second order conditions, we have that \(\xi\) is quasi-concave in \(\bar{q}\). Also, from (22), \(\Phi\) is quasi-convex in \(\bar{q}\) and from (A.18), that \(W\) is quasi-concave in \(\bar{q}\). From the definition of \(\Phi\) in (22) and \(\lambda_p\) in (23), we can express \(\Phi = \frac{\Lambda}{\bar{q}}\), where \(\Lambda = (1 - \theta^{* - h}) \lambda_p(q)^{1 - \frac{1}{\tau}} + \theta^{* - h} \lambda_p(q)^{1 - \frac{1}{\tau}}\), so that from (A.2), (A.5) and (A.18)
\[
\frac{dW}{dq} = \Phi \xi L \left[ \frac{\frac{1}{\epsilon - 1} - \Phi \frac{L d\log \xi}{dq} - \frac{d \log \Phi}{dq}}{\Phi L \frac{\frac{1}{\epsilon - 1} - \Phi \frac{L d\log \xi}{dq} - \frac{d \log \Phi}{dq}}{\Phi L} \right], \hspace{1cm} (A.19)
\]
Also, from (A.5) and Corollary 3 it follows that \(\frac{d \log \theta^*}{dq} |_{\bar{q} = \bar{q}_P} = \frac{\lambda_p'(\bar{q})}{h(\lambda_p(q) - \lambda_p'(q))}\) and hence at \(\bar{q} = \bar{q}_P\)
\[
\frac{d \log \Lambda}{dq} = \frac{\theta^{* - h} \lambda_p'(\bar{q})}{\Lambda} \left[ \frac{d \lambda_p'(\bar{q})^{1 - \frac{1}{\tau}}}{d \lambda_p'(\bar{q})} - \frac{\lambda_p'(\bar{q})^{1 - \frac{1}{\tau}} - \lambda_p(q)^{1 - \frac{1}{\tau}}}{\lambda_p'(\bar{q}) - \lambda_p(q)} \right]. \hspace{1cm} (A.20)
\]
It follows from (A.20) that at \(\bar{q} = \bar{q}_P\), \(\frac{d \log \Lambda}{dq} > 0\) if and only if \(k < 1\), since \(\lambda_p(q)^{1 - \frac{1}{\tau}}\) is concave (convex) in \(\lambda_p(q)\) if \(k > 1\) (\(k < 1\)) so that the slope of \(\lambda_p(q)^{1 - \frac{1}{\tau}}\) at \(\lambda_p(q)\) is greater than the secant line through \(\lambda_p'(\bar{q})^{1 - \frac{1}{\tau}}\) and \(\lambda_p(q)^{1 - \frac{1}{\tau}}\) if and only if \(k < 1\). Finally, since \(\Omega(\bar{q}_P) = 0\), it follows from (A.19) that \(\frac{dW}{dq} |_{\bar{q} = \bar{q}_P} > 0\) if and only if \(k > 1\). Hence, since \(W\) is quasi-concave in \(\bar{q}\), it follows that \(\bar{q}_P < \bar{q}\) if and only if \(k > 1\).

It remains to show that \(W\) is quasi-concave in \(\bar{q}\). If \(k > 1\), from the argument above, \(\bar{q}_P < \bar{q}_E\). From the first line of (A.19) it follows that since \(\xi\) and \(-\Phi\) are quasi-concave on \([\bar{q}, \infty)\) and \(\frac{1}{\epsilon - 1} - \Phi(\bar{q}) L > 0\), \(\frac{dW}{dq} > 0\) on \([\bar{q}, \bar{q}_P]\) and \(\frac{dW}{dq} \leq 0\) on \((\bar{q}_E, \infty)\). Over the interval
there exists a unique \( q^* \) such that at \( q^* \), 
\[
\frac{1}{\Phi L} \frac{d \log \xi}{dq} = 0 > \frac{d \log \Phi}{dq} = 0 > \frac{1}{\Phi L} \frac{d \log \xi}{dq}.
\]
It follows that there exists a unique \( q^* \in (\tilde{q}_P, \tilde{q}_E) \) such that at \( q^* \), 
\[
\frac{1}{\Phi L} \frac{d \log \xi}{dq} = 0 = \frac{d \log \Phi}{dq} = 0 = \frac{1}{\Phi L} \frac{d \log \xi}{dq}.
\]
An analogous argument holds for the case where \( k \leq 1 \). If \( k = 1 \), since \( \tilde{q}_P = \tilde{q}_E \), \( \tilde{q}_P = \tilde{q}_E = q^* \).

**Proof of Proposition 5:**

i) First note that the decision problem for consumers under regulation is equivalent to their decision problem under a labeling program with the additional constraint \( \lambda_p (q) = 0 \), since \( c(q) = 0 \) if and only if \( \lambda_p (q) = 0 \). Hence it is possible to compare the effects of a mandatory standard with those of a labeling standard by comparing the solution to the decision problem under a labeling program while imposing \( \lambda_p (q) = 0 \), with the solution to the decision problem under a labeling program where \( \lambda_p (q) \) is determined by consumer preferences. From the comparative statics for \( \lambda_p (q) \), it follows from (A.15), (A.11), (A.12), (A.16) and (A.10), respectively, that for a given standard \( \tilde{q} = \tilde{q}_I \), \( I \) is smaller, \( z \) is smaller, \( \theta^* \) is greater, \( M \) is greater, and \( \xi \) is greater under a labeling program than under regulation for any \( \lambda_p (q) > 0 \). Also, from (A.14), \( Q \) is greater under labeling if and only if \( \lambda_p (q) \) is sufficiently large. Finally, from (16) and (A.13), we have that, in the absence of an externality, welfare \( W \) is greater under labeling than regulation for any \( \lambda_p (q) > 0 \) since
\[
\frac{d \log (W)}{d \lambda_p (q)} = \frac{1}{\sigma - \epsilon} (QM)^{\epsilon} \frac{1 - \theta^{s-h}}{Q} > 0.
\]
Since welfare is greater under labeling for any given standard of quality, it follows that \( W^L_P (q^*) \geq W^R_P (q^*) > W^R_P (\tilde{q}) \), where \( W^L_P \) is private welfare under labeling and \( W^R_P \) is private welfare under regulation, and asterisks denote optimal standards.

**Proof of Proposition 6:**

We can express the entire system of equations as: (7), (8), (14), (15) and
\[
\frac{\lambda_p (q) - \Upsilon \lambda_p (q)}{Q} \frac{\xi L}{\sigma M} = \frac{\tilde{q} - q}{\theta^s - \theta^s-h}
\]
\[
Q = \Upsilon \lambda_p (q) \left( 1 - \theta^s-h \right) + \lambda_p (q) \theta^s-h
\]
and, from (21) and (22)
\[
E^* = \frac{(1 - \theta^s-h) \phi_p (q) \Upsilon \lambda_p (q) + \theta^s-h \phi_p (q) \lambda_p (q)}{(1 - \theta^s-h) \Upsilon \lambda_p (q) + \theta^s-h \lambda_p (q)} \xi L
\]
for all \( q > \bar{q} \), where the indicator variable \( \Upsilon = 1 \) under labeling and \( \Upsilon = 0 \) under regulation,
and $\varepsilon = \frac{\ell - 1}{\sigma - 1}$. We can solve the system to yield

$$
\zeta = \frac{\xi_R}{\xi} = \left[ \frac{\eta Q_R}{h \lambda_p(\bar{q}) + \eta Q} \right]^{\frac{\zeta - 1}{\zeta}}
$$

(A.22)

where the subscript $R$ is used to distinguish variables that pertain to a regulatory regime. From (7) and Proposition 5 it follows that $\zeta < 1$, for all $\tilde{q} > q$ and $\lambda_p(\bar{q}) > 0$. Welfare under labeling is given by $W^L = W^P - E^*$ and welfare under regulation is given by $W^R = W^P - E^*_R$. Hence $W^L > W^R$ if and only if $\Delta W_P > \Delta E$ where, from (17),

$$
\Delta W^P = W^P - W^P_R = \frac{1}{\varepsilon - 1} (\xi - \xi_R)
$$

(A.23)

and, from (A.21) it follows that

$$
\Delta E = E^* - E^*_R
= \frac{(1 - \theta^* - h) \lambda_p(q) \phi_p(q) + \theta^{* - h} \lambda_p(q) \phi_p(q)}{(1 - \theta^* - h) \lambda_p(q) + \theta^{* - h} \lambda_p(q)} \xi L - \phi_p(q) \xi_R L
= \Phi(\bar{q}) \xi L - \phi_p(q) \xi_R L
$$

where the share of expenditure on differentiated goods allocated to labeled varieties is $\mu = \frac{\theta^{* - h} \lambda_p(q)}{Q}$.  

i) For any given $k$ and $\tilde{q}$, if $\varepsilon \geq \frac{1}{\Phi(\bar{q}) L} + 1$, then $W^R > W^L$.  

From (A.24) we have $\Phi(\bar{q}) = \zeta \phi_p(q) + (1 - \zeta) \frac{\Delta E}{(\xi - \xi_R) L}$. Since $0 < \zeta < 1$ for any $\lambda_p(q) > 0$, it follows that $\frac{\Delta E}{(\xi - \xi_R)} > \Phi(\bar{q}) L > \phi_p(q) L$ and hence $\Phi(\bar{q}) L \geq \frac{1}{\varepsilon - 1}$ implies

$$
\Delta E > \frac{1}{\varepsilon - 1} (\xi - \xi_R)
\iff \Delta E > \Delta W^P.
$$

ii) If $\varepsilon \leq 1 + \frac{1}{L}$, then there exists a unique $\lambda_p(q') > 1$ such that if $\lambda_p(q) \geq \lambda_p(q')$ then $W^L > W^R$ for all $k$. If $\varepsilon > 1 + \frac{1}{L}$, then for any $\lambda_p(q) > 1$ there exists a unique $k^*$ such that $W^L > W^R$ if and only if $k < k^*$.

The proof of ii) will proceed in 3 steps:

1) If $\varepsilon = \frac{\ell - 1}{\sigma - 1} \leq \frac{1}{L}$, then for any $\bar{q} > q$, there exists a unique $\lambda_p(q') \in (1, \lambda_p(q))$ such that if $\lambda_p(q) \geq \lambda_p(q')$, $\frac{\Delta E}{dk} > 0$ for all $k$. If $1 < \lambda_p(q) < \lambda_p(q')$, then $\frac{\Delta E}{dk} > 0$ if and only if $k < k'$, where $k' = \frac{\log(\lambda_p(q'))}{\log(\lambda_p(q)\lambda_p(q') - \lambda_p(q)\lambda_p(q'))}$. If $\zeta - \mu < 0$, then $\frac{\Delta E}{dk} > 0$ for all $k$ and $\lambda_p(q') > 1$.
From (A.24) and (23) it follows that \( \frac{d\Delta E}{dk} > 0 \) if and only if
\[
\left( \frac{\lambda_p(q)}{\lambda_p(q')} \right)^{\frac{1}{\xi}} \frac{\log(\lambda_p(q))}{\log(\lambda_p(q'))} > \frac{\zeta - \mu}{1 - \mu}.
\]
Since \( \lambda_p(q) > \lambda_p(q') \), it follows that if \( \frac{\zeta - \mu}{1 - \mu} \frac{\log(\lambda_p(q))}{\log(\lambda_p(q'))} \leq 1 \), then \( \frac{d\Delta E}{dk} > 0 \) for all \( k > 0 \). Also, it can be shown that if \( \varepsilon \leq \frac{1}{2} \), then \( \zeta - \mu > 0 \) for all \( \lambda_p(q) > 0 \). Hence if \( \lambda_p(q) > 1 \) and \( \varepsilon \leq \frac{1}{2} \), then \( \frac{d\Delta E}{dk} > 0 \) if and only if \( \frac{1}{k} > \frac{\log(\lambda_p(q))}{\log(\lambda_p(q'))} \).

Also, from (A.22) it follows that as \( k \to 0, \Delta W^P > \Delta E \) if and only if \( k < k^* \), (b) there exist a \( k^* > k' \) such that \( \Delta W^P > \Delta E \) whenever \( k < k^* \) or \( k > k^* \), and \( \Delta W^P < \Delta E \) whenever \( k^* < k < k^* \), or (c) \( \Delta W^P \geq \Delta E \) for all \( k \).

Taking limits of (A.24), we have that if \( \lambda_p(q) > 1 \), as \( k \to 0, \Delta E \to 0 \). As \( k \to \infty, \Delta E \to (\xi - \xi_R) L > 0 \). Also, from (A.23) we have that \( \Delta W^P = \frac{1}{1 - \frac{\xi - \xi_R}{\xi - \xi_R}} \frac{1}{L} \), which is independent of \( k \). If \( \lambda_p(q) \geq \lambda_p(q') \) and \( \epsilon > 1 + \frac{1}{L} \), as shown in Figure A.1, then since \( \frac{d\Delta E}{dk} > 0 \) for all \( k > 0 \) there exists a unique \( k^* \) such that \( \Delta W^P > \Delta E \) if and only if \( k < k^* \). If \( \epsilon \leq 1 + \frac{1}{L} \), then \( \Delta W^P > \Delta E \) for all \( k \). If \( 1 < \lambda_p(q) \), and \( \epsilon \leq 1 + \frac{1}{L} \), then, since \( \sigma \geq 3 \), we have that \( \epsilon \leq \frac{1}{2} \) if \( L > 1 \). From step (1) it follows that \( \frac{d\Delta E}{dk} > 0 \) if and only if \( k < k' \). Hence either (a) there exists a unique \( k^* \) such that \( \Delta W^P > \Delta E \) if and only if \( k < k^* \), (b) there exist a \( k^* < k' \) and a \( k^* > k' \) such that \( \Delta W^P > \Delta E \) whenever \( k < k^* \) or \( k > k^* \), and \( \Delta W^P < \Delta E \) whenever \( k^* < k < k^* \), or (c) \( \Delta W^P \geq \Delta E \) for all \( k \). Note that (a) follows if \( \epsilon = 1 + \frac{1}{L} \), (b) follows if \( L < \frac{1}{\epsilon - 1} \leq \frac{\Delta E(k')}{\xi - \xi_R} \), and (c) follows if \( \frac{1}{\epsilon - 1} > \frac{\Delta E(k')}{\xi - \xi_R} \).

Note that if \( 1 < \lambda_p(q) \) and \( \epsilon \leq 1 + \frac{1}{L} \) but \( \zeta - \mu < 0 \), then since \( \frac{d\Delta E}{dk} > 0 \) for all \( k > 0 \) we have an additional case where \( \Delta W^P \geq \Delta E \) for all \( k \).
however, $\epsilon > 1 + \frac{1}{L}$ and $\epsilon \leq \frac{1}{2}$, then since $\zeta - \mu > 0$, $\frac{d\Delta E}{dk} > 0$ if and only if $k < k'$. It follows that there exists a unique $k^*$ such that $\Delta W^P > \Delta E$ if and only if $k < k^*$. If $\zeta - \mu < 0$, then since $\frac{d\Delta E}{dk} > 0$ for all $k > 0$ there exists a unique $k^*$ such that $\Delta W^P > \Delta E$ if and only if $k < k^*$. Hence if $1 < \lambda_p(q) < \lambda_p(q)'$ and $\epsilon > 1 + \frac{1}{L}$, regardless of the sign of $\zeta - \mu$, there exists a unique $k^*$ such that $\Delta W^P > \Delta E$ if and only if $k < k^*$.

(3) Derivation of Figure 1 and Figure A.1, and comparative statics.

From (16) and (A.21) it follows that $W^L$ and $W^R$ are decreasing in $k$. Also, if $\lambda_p(q) > 1$, we have that as $k \to 0$, $W^L \to L + \frac{1}{\epsilon - 1} \xi = W^P$ and $W^R \to L + \frac{1}{\epsilon - 1} \xi_R = W^P_R$. Hence, from Proposition 5 it follows that at $k = 0$, $W^L > W^R$. As $k \to \infty$, $W^L \to L + \left(\frac{1}{\epsilon - 1} - L\right) \xi$ and $W^R \to L + \left(\frac{1}{\epsilon - 1} - L\right) \xi_R$. If $\epsilon > 1 + \frac{1}{L}$, then as $k \to \infty$, $W^L < W^R$. Hence, from step (2) above, and as shown in Figure 1, for any $\lambda_p(q)_1 > 1$, if $\epsilon > 1 + \frac{1}{L}$, then there exists a unique $k^*$ such that $W^L > W^R$ if and only if $k < k^*$. Now consider $1 < \lambda_p(q)_2 < \lambda_p(q)_1$. Since $\xi$ is increasing in $\lambda_p(q)$, $W^L$ shifts downward. Hence $k^*$ decreases and labeling is optimal for a smaller range of $k$. From step (2) it’s clear that $k^*$ is unique for any $\lambda_p(q)_2 > 1$. If $\epsilon \leq 1 + \frac{1}{L}$, then as $k \to \infty$, $W^L \geq W^R$, where equality holds only for the case $\epsilon = 1 + \frac{1}{L}$. Hence, from step (2) above, for any $\lambda_p(q)_1 \geq \lambda_p(q)'$, $W^L > W^R$ for all $k$.

Now consider $\lambda_p(q)_2 < \lambda_p(q)_1$. Since $\xi$ is increasing in $\lambda_p(q)$, $W^L$ shifts downward. If $\lambda_p(q)_2 > \lambda_p(q)'$, then as shown in step (2), it continues to be the case that $W^L > W^R$ for all $k$. If $\lambda_p(q)_2 \leq \lambda_p(q)'$ and $\sigma \geq 3$ then, as shown in step (2), either (a) there exists a unique $k^*$ such that $\Delta W^P > \Delta E$ if and only if $k < k^*$, (b) there exist a $k^* < k'$ and a $k^{**} > k'$ such that $\Delta W^P > \Delta E$ whenever $k < k^*$ or $k > k^{**}$, and $\Delta W^P < \Delta E$ whenever $k^* < k < k^{**}$, or (c) $\Delta W^P \geq \Delta E$ for all $k$. Case (b) is shown in Figure A.1.

![Figure A.1. Welfare as a function of k; $\epsilon \leq \frac{1}{L} + 1$ and $\lambda_p(q) < \lambda_p(q)'$.](image-url)
References


