Strategic Extremism: Bargaining with Endogenous Breakdown Probabilities and Order of Play

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Abstract

The purpose of this paper is to provide a simple example of an alternating offers bargaining model with breakdowns, in which both the breakdown probabilites and the order of play are determined endogenously. We consider two countries in a conflict over a contested asset. We assume that one of the countries can use extremism as a strategic tool in the conflict. Extremism is useful because it provides a credible threat. On the other hand, it involves a risk since it can cause damage, with probabilities that depend on the outcome of the game. We consider a three-stage game in which the use of extremism, the order of play and the division of the contested asset are determined in the three stages, respectively. We show that the game has unique subgame perfect equilibrium in which: extremism is indeed used strategically, the probability of a breakdown is strictly positive and, most importantly, the country that uses extremism moves first in the asset division bargaining stage. The order of play is determined endogenously in this model, because the expected size of the pie is affected by the order of play. Specifically, we show that the equilibrium order of play yields the largest expected pie size.

KEYWORDS: Alternating Offers, Bargaining, Extremism, Endogenous Breakdowns, Order of Play.

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1 Introduction

Following Rubinstein's (1982) path breaking paper, the alternating offers bargaining model has become an important tool in addressing bargaining and negotiations problems. Since it was first introduced, the model has been generalized to take into account multiple players, incomplete information, deadlines, outside options, risk of breakdowns, strategic delays, etc.[1] In addition to its game theoretic importance, the model and its various generalizations have proven useful in applications in a wide range of fields, such as economics, politics, international conflicts, litigations, organizational behaviour, management, etc.

The standard Rubinstein alternating offers model examines the allocation of a *fixed* pie between parties.[2] Given a fixed pie and without any pre-bargaining, or post-bargaining actions by the parties, the model cannot explain the order of the play. Within its framework, the inability to determine the order of play is not a major issue since, with a fixed pie size, the order of play affects only the distribution of the pie, but not its size.

In many bargaining situations, however, the size of the pie is not fixed because the parties may be able to either make strategic pre-bargaining commitments, or take additional post-bargaining steps, whose implications and choices, respectively, depend (credibly) on the outcome of the bargaining game.[3] In both cases, since the order of play affects the outcome of the bargaining game, it will also affect the size of the pie. Consequently, this suggests that a bargaining protocol which gives rise to an *equilibrium* order of play may emerge. Presumably, in equilibrium, the order of play will maximize the expected size of the pie.

The paper provides an example in which a strategic *pre-bargaining* commitment[4] introduces probabilistic breakdowns which affect the expected size of the pie and, as a consequence, both the order of play and breakdown probabilities are determined endogenously.[5] Specifically, we consider two countries that bargain over the partition of a contested asset. As an example of a pre-bargaining commitment, we assume that one of the countries can use extremism as a credible threat.[6] We characterize extremism by the fact that its mere existence can, with some probability, cause damage to the contested asset. Thus, the use of extremism is risky, but this is what makes it's threast credible.[7] This idea is, clearly, related to what Schelling (1960) calls "the threat that leaves something to chance" (see chapter 8). We can think of the probability that damage will occur as, what Binmore et al. (1986)

[1] For a discussion of these models, see Fudenberg and Tirole (1983), Binmore et al. (1986), Sutton (1986), Admati and Perry (1987), Fudenberg et al. (1987), Cramton (1992), Perry and Reny (1993) and Ma and Manove (1993). For good, comprehensive, general discussions of bargaining models see Sutton (1986), Binmore (1987) and Muthoo (1999).

[2] A fixed pie is, indeed, the common assumption in bargaining models discussed in the literature (see Muthoo (1999)).

[3] An example of a paper in which the size of the pie is affected by the players' actions is Frenkel (1998) where "creative ideas" can affect the size of the pie (but not its distribution).

[4] An example of post-bargaining action is given by the standard right-to-manage model in labour economics, where the firm makes its employment decision after bargaining over the wage is completed. Since the order of play affects the outcome of the bargaining game (the wage) and since the wage affects employment decisions and hence output, it follows that the order of play affects the size of the pie.

[5] The question of the order of play is discussed, for example, in the IO literature, where the order of entry, commitments, or other actions are determined endogenously (see, for example, Appelbaum and Weber (1993), Hamilton and Slutsky (1993), Caruana and Einav (2005)). For an example in the context of litigational negotiations see, Daughety and Reinganum (1993).

[6] The strategic use of extremism has been discussed extensively in the literature. See for example, Glazer et al. (1998), Glazer (2002), Glazer and Konrad (2003), Glaeser, et al. (2004), Wintrobe (2006a, 2006b), Appelbaum (2008) and Appelbaum and Katz (2007). Atkinson et al. (1987), is an early example of an application of the bargaining framework to the problem of terrorism.

[7] This paper is, therefore, also related to Avery and Zemsky (1994), Manzini (1997) and Busch, Shi and Wen (1998), where damage may arise due to destructive bargaining. But, this paper is different in several ways. First, here there is a *strategic precommitment* to probabilistic damage. Second, the probability of damage being caused, is endogenously determined by the outcome of each round of bargaining. Finally, given this endogeneity of the probability, the order of play itself is also determined endogenously.

call, the probability of a bargaining "breakdown".[8]

We examine a three-stage bargaining game with the following time line: (i) in the first stage, one of the countries chooses whether to use extremism strategically, (ii) in the second stage, the two countries engage in an alternating offers bargaining game, in which transfer payments are used to determine who will make the first offer in the forthcoming bargaining over the division of the contested asset, (iii) in the third stage, the two countries engage in an alternating offers bargaining game in which the contested asset is divided, in accordance with the agreed upon order of play.

We show that, in general, there exists a unique subgame perfect equilibrium in which the probability of a breakdown is a strictly positive and moreover, the country that makes the strategic pre-bargaining commitment (to use extremism) always moves first in the asset division bargaining stage. The order of play is determined endogenously in this model, because the expected size of the pie is affected by the order of play. The reason that this happens is that the expected size of the pie depends on the breakdown probability, which in turn depends on the outcome of the game, and that depends on the order of play. Consequently, the expected size of the pie depends on the order of play. Specifically, we show that the equilibrium order of play yields the largest expected size pie. In this case, the expected size of the pie is greater when the country that uses extremism moves first. In equilibrium, therefore, this country will indeed move first.[9]

2 The Model

2.1 Strategic Extremism

In many bargaining situations negotiating parties can engage in preemptive activities whose aim is to strengthen bargaining positions. To the extent that such actions are credible, they can confer a bargaining advantage in the forthcoming negotiations. The strategic use of capital structure, or capacity precommitments are two examples of such preemptive moves on the part of a firm. Similarly, the use of extremism as a strategic tool is another example of a preemptive strategy in political conflicts. Extremists are useful because they provide a credible threat, thus enhancing one's bargaining position in a conflict. At the same time, they also involve a risk because they can cause damage to assets. The use of extremism as a strategic tool is, therefore, a double-edged sword and consequently, the usefulness of extremism as a strategic tool is determined by the balance of these two considerations.

Indeed, the use of extremism as a strategic tool in political conflicts that has been common throughout history. Here are two two historical examples, from the Middle East in the Middle Ages, in which "militants" were used, but could not be controlled and, subsequently, the policy actually backfired. In the aftermath of the Mongol-Turkish invasions, there were numerous cases of masters who were overthrown by their "creations". For example,

[8] A related paper is Merlo and Wilson (1995), where both the size of the pie and the order of play follow a Markov process. The size of the pie is, however, exogenously determined.

[9] A very interesting recent example of a model that determines the 'first mover' is Yildirim (2006), in which parties with different cost functions compete *directly* for the right to be the first mover in a multilateral contest over a *fixed pie*. While this can yield an equilibrium 'first mover', the competition, or rent seeking, in that model is directly for the 'proposer position'. In this paper, on the other hand, there is no direct competition for the first mover position. Given that extremism affects the expected size of the pie, the first mover position is determined as part of the bargaining protocol, even without rent seeking, by the maximization of the expected pie size. For examples, in the context of coalitional games, that are related to the Yildirim (2006) paper, see Perez-Castrillo and Wettstein (2002), (2006).

in Iran, the Samanid dynasty was eventually supplanted by the Ghaznavids (962 AD), who were brought in as mercenary soldiers. In Egypt, in 1250 AD, the Ayyubids were deposed by their "imported" Mamluk soldiers, who went on to form their own dynasty (lasting almost three hundred years, although it was, curiously, itself based on mercenary soldiers. See Hourani (1991)). More recently, and perhaps more closely related to an militancy story: the US encouraged and supported the Mujahadin in Afghanistan in their fight against the Russian, and the Palestinian Authority used Hamas in its conflict with Israel (Interesingly, Israel itself encouranged Hamas, in its early days, as a strategic tool agaist the PLO). As we know, in both cases, the policy backfired.

When we examine extremism, it is useful to separate between its supply and demand. A credible threat, regardless of who provides it, or why it is provided, is beneficial because it improves one's bargaining position. This means that, in general, there will be demand for credibility, hence extremism. The existence of extremists and the explanation for their behaviour, on the other hand, is a question of supply. Clearly, to be able to use extremism as a strategic tool, someone has to be able and willing to supply it. The question of why extremists behave the way they do, is most interesting and has, indeed, been discussed extensively in the economic, political and psychological literature.[10] Moreover, regardless of why extremists behave the way they do, their existence is a fact of life. In this paper we, therefore, focus on the demand for, rather than the supply of, extremism. Instead of modeling the precise behaviour of extremists (the supply question), we focus on the promotion and use of extremism as a strategic tool in bargaining (the demand question).

As an example of a bargaining problem in which both breakdown probabilities and the order of play are determined as part of the equilibrium of the game, we consider a simple model in which two countries are in conflict over a contested asset whose value is 1. For simplicity, we assume that only one of the countries, say Country 2, uses extremism strategically.[11] We characterize extremism by the fact that its mere existence can, with some probability, p, result in what we call an "extremist episode", whose consequence is that the contested asset is destroyed.[12] This event is similar to the breakdown in the Binmore et al. (1986) model. In the following, we refer to such an event as an "explosion".

Although we do not try to explain the behavioural aspects of extremism precisely, we capture some of its underlying determinants. We do so in two ways. First, we assume that the outcome in the bargaining process affects the level of "nationalistic anger", in Country 2. Specifically, if Country 1's share is given by x, then nationalistic anger (in Country 2) is an increasing function of x. Second, we assume that Country 2 recognizes the strategic value of extremism and uses it to improve its bargaining position. It does so, by using various policy instruments, denoted by g, that can harness the anger and convert it into "extremism", which in turn provides an effective threat of an explosion. This can be done, for example, by incitement, indoctrination, inflammation of

[10] For discussions of the behaviour of extremists see, for example, Rubbelke (2005), Victoroff (2005), Blomberg et al. (2004a,b) and Wintrobe (2006a), (2006b).

[11] While it is possible to generalize this and consider a case where both countries use strategic extremism, this will not affect the main message of the paper. Namely, that when the size of the pie is affected by precommitments (which, here, make breakdown probabilities endogenous), the order of play in the alternating offers bargaining is determined (optimally) in the equilibrium of the game. Appelbaum (2008) is an example of a model where both countries use extremism strategically. Both the focus and the structure of that model, however, are different. Specifically, it uses a Nash Bargaining framework, focusing on the determinants of extremism and its root causes. The question of the order of play is thus irrelevant.

[12] Alternatively, we can assume that only a fraction b of the contested asset is destroyed. This can be added to the model without affecting the results.

nationalistic feelings, etc. In general, we can think of g as a continuous policy choice variable. In order to focus on the bargaining aspects of the model, however, we treat g as a simple binary choice variable. Namely, Country 2 either uses extremism as a strategic tool, or does not (we will show later that even if allow for a continuous choice of g, the solution is always at the corner). Extremism is, therefore, the product of two factors: the underlying anger and the policy instruments that harnesses that anger. The probability of an explosion (breakdown), in turn, depends on the level of extremism. These relationships can be summarized as follows:

$$\begin{aligned} anger &= A(x) \\ extremism &= B(g, anger), \\ p &= C(extremism) = C[B(g, A(x))] \equiv p(g, x) \end{aligned} \tag{1}$$

where all the functions in (1) are increasing in their arguments. As an example, and in order to simplify the analysis, we take all the relationships in (1) to be linear. Thus, we assume that the probability of an explosion is given by:[13]

$$p(x, g) = gx, \quad \text{where} \quad g = \{0, 1\} \tag{2}$$

The probability of an explosion is, therefore, simply proportional to Country 2's level of anger, the factor of proportionality being Country 2's binary policy instrument.

2.2 The Structure of the Conflict

The conflict between the two countries has the following structure:

1. In stage one, Country 2 decides whether to use extremism strategically.

2. In stage 2, the two countries engage in negotiations that determine who will make the first offer in the forthcoming bargaining game in which the asset is divided. We refer to this as the order of play bargaining (OPB) game. The OPB game is modelled as an infinite time, alternating offers game, where the two countries use transfer payments to arrive at a mutually agreeable order of play.[14] We assume both countries can opt out of the OPB game in favour of the forthcoming asset division bargaining game in stage 3 (described below). In order to avoid the need to consider the effects of attitudes toward risk (insurance considerations), we assume that both countries are risk neutral, so that they maximize their expected payoffs. We also assume that each country discounts its payoffs by its discount factor, $\delta_i, \; i = 1, 2$ in the OPB game. As in the standard Rubinstein game, the order of play in the game is not determined.

3. In stage three, the contested asset is divided in an infinite time, alternating offers bargaining game, in line with the agreed upon order of play. We refer to this as the asset division bargaining (ADB) game. The same as above, the countries are risk neutral and each country discounts its payoffs by its discount factor,

[13] Note that, in general, we can think of Country 2's choice of a continuous instrument g, as a choice of a probability *function,* from a class of possible functions: $p(x) \in P$, say, the set of all real single valued, continuous, differentiable and increasing functions, defined over the interval $[0, 1]$.

[14] Although we chose to model the OPB as an infinite horizon alternating offers game, it is possible to consider alternative protocols that allow the parties to arrive at the order of play that yields the largest expected size pie. It is clear that, since the order of play affects the size of the pie, the optimal order will be reached by any other protocol.

$\delta_i,\ i = 1, 2$. The structure of the ADB game is the same as in Rubinstein (1982), except that here, in each round, after an offer has been made, an explosion occurs with probability p. Specifically, the two countries make sequential, alternating offers. An offer may be accepted, or rejected. In either case, after an offer is made in which Country 1's share is, say, x, an explosion occurs with probability $p(x) = gx$, regardless of who made the offer, or whether the offer was accepted or rejected.[15] The game continues until one of the countries accepts its rival's offer (at which point the game terminates). In this case, an explosion may still precede the termination of the game.[16] The game also terminates if an explosion occurs.[17]

3 The Solution of the Game:

When $g = 0$ we have the standard Rubinstein game (without breakdowns/explosions). In this case, the size of the pie is fixed, so that the order of play cannot be negotiated and remains indeterminate. The solutions to stages 2 and 3 of the game, in this case, are therefore, straight forward (the standard Rubinstein solution). On the other hand, when $g = 1$ explosions are possible and, consequently, the size of the pie is not fixed and hence the order of play is determinate. The solution in this case is both more complicated and interesting. In the following, we proceed by first solving the second and third stage games for the case when $g = 1$. Then, in Section 3.3, we will determine whether Country 2 is, in fact, better off choosing $g = 1$, or $g = 0$, by comparing its payoffs in the two cases (that is, we will determine the choice of g in stage 1).

3.1 Last Stage: The Bargaining Game

In the third stage, in the case when $g = 1$, given the equilibrium order of play that was determined in stage 2, the two countries engage in the ADB game. In this infinite horizon alternating offers bargaining problem, a game that has not ended, appears the same at all even and at all odd time periods. The solution is, therefore, stationary, in the sense that both countries' offers and payoffs are the same in all even and all odd time periods. Thus, if we define x_1 as Country 1's offered share when Country 1 makes an offer and x_2 as Country 1's offered share when Country 2 makes an offer, then Country 1 always offers x_1 and Country 2 always offers x_2. Let us also define Country $i's$ expected payoff when it makes an offer as V_i and Country $i's$ expected payoff when the other country makes an offer as W_i. Thus, when Country 1 makes an offer we have:

$$\begin{aligned} V_1 &= x_1(1 - x_1) \\ W_2 &= (1 - x_1)(1 - x_1) \end{aligned} \tag{3}$$

and when Country 2 makes an offer the payoffs are:

[15] This particular structure is just an example. It is, of course, possible to consider alternative scenarios where, for example, an explosion occurs only if an offer (by either country) is rejected, or only if an offer by Country 2 is rejected, etc. While these alternative assumptions may affect the outcome, they will still yield an equilibrium order of play.

[16] Again, this is not a necessary assumption.

[17] It is useful to note that our model contains elements of both the Rubinstein (1982) and the Binmore et al. (1986) models. If g is zero (when the probability of an extremist explosion is zero), our model becomes the same as the former. On the other hand, if the probability of an explosion is exogenous, our model is similar to the latter.

$$\begin{aligned} V_2 &= (1-x_2)(1-x_2) \qquad (4) \\ W_1 &= x_2(1-x_2) \end{aligned}$$

Note that whereas Country 2's payoffs are always increasing in its share (i.e., decreasing in Country 1's share), Country 1's payoffs are not monotonic. Specifically,

$$\frac{\partial V_2}{\partial x_2} < 0, \; \frac{\partial W_2}{\partial x_1} < 0 \qquad (5)$$

$$\frac{\partial V_1}{\partial x_1} \begin{cases} > 0 \text{ for all } x_1 < .5 \\ = 0 \text{ for } x_1 = .5 \\ < 0 \text{ for all } x_1 > .5 \end{cases} \quad \frac{\partial W_1}{\partial x_2} \begin{cases} > 0 \text{ for all } x_2 < .5 \\ = 0 \text{ for } x_2 = .5 \\ < 0 \text{ for all } x_2 > .5 \end{cases} \qquad (6)$$

Consequently, as is shown below, this implies that the bargaining equilibrium cannot occur on the downward sloping part of Country 1's payoff function.

Consider the case when Country 1 makes the offer. If Country 1 offers x_1 and Country 2 accepts the offer, Country 2's payoff, z_2, is random (since an explosion may occur this period with probability x_1) and given by:

$$z_2 = \begin{cases} 1-x_1 & \text{with probability } 1-x_1 \\ 0 & \text{with probability } x_1 \end{cases} \qquad (7)$$

Its expected payoff in this case is, therefore,

$$E(z_2) = (1-x_1)(1-x_1) \qquad (8)$$

On the other hand, if Country 1 offers x_1 and Country 2 rejects the offer, Country 2's payoff in the following period, y_2, is also random (since an explosion may occur this period with probability x_1 and next period with probability x_2) and given by:

$$y_2 = \begin{cases} (1-x_2)(1-x_2) & \text{with probability } 1-x_1 \\ 0 & \text{with probability } x_1 \end{cases} \qquad (9)$$

Its expected payoff in this case is, therefore,

$$E(y_2) = (1-x_2)(1-x_2)(1-x_1) \qquad (10)$$

Let us now examine what Country 1's offer should be, given any value of x_2. To see this, first, define the value of x_1 that would make Country 2 indifferent between accepting the offer and rejecting it (and waiting for next period when it would be its turn to make an offer), to be given by x_1'. In other words, x_1' satisfies the condition:

$$\delta_2 E(y_2) = E(z_2) \qquad (11)$$

or:

$$x_1' = 1 - \delta_2(1-x_2)^2 \qquad (12)$$

where δ_2 is Country 2's discount factor. Note that:

$$\frac{\partial x_1'}{\partial x_2} > 0, \; \frac{\partial x_1'}{\partial \delta_2} < 0 \qquad (13)$$

Whether Country 1 will set its offer to satisfy condition (12) depends on the value of x_2 (and δ_2). If x_2 is "sufficiently" low, Country 1's payoffs are increasing at x_1', but if x_2 is "sufficiently" high, Country 1's payoffs are decreasing at x_1'. Let us, therefore, define $x_2(.5)$ as the value of x_2 that satisfies:

$$.5 = 1 - \delta_2[1 - x_2(.5)]^2 \tag{14}$$

It is easy to see that $x_2(.5)$ is monotonically increasing in δ_2 and, moreover, for all $\delta_2 > .5$, we have $x_2(.5) > 0$. To ensure positivity of $x_2(.5)$ let us, first, focus on the case where $\delta_2 > .5$. From conditions (12) and (14) it follows that:

$$\begin{array}{c} x_1' < .5 \text{ for all } x_2 < x_2(.5) \\ x_1' = .5 \text{ for } x_2 = x_2(.5) \\ x_1' > .5 \text{ for all } x_2 > x_2(.5) \end{array} \tag{15}$$

so that:

$$\begin{array}{c} \frac{\partial V_1}{\partial x_1} > 0 \text{ for all } x_2 < x_2(.5) \\ \frac{\partial V_1}{\partial x_1} = 0 \text{ for } x_2 = x_2(.5) \\ \frac{\partial V_1}{\partial x_1} < 0 \text{ for all } x_2 > x_2(.5) \end{array} \tag{16}$$

Thus, we conclude that for all for all $x_2 < x_2(.5)$ Country 1 will set its offer so that Country 2 is indifferent between accepting and rejecting the offer. On the other hand, since for all $x_2 > x_2(.5)$ Country 1 is on the downward sloping portion of its payoff function, it is better off offering $x_1 = .5$, an offer that will be accepted. For all $\delta_2 < .5$, Country 1's offer is simply given by $x_1 = 1/2$, for all x_2. Country 1's offer is, therefore, defined by the condition:

$$H^1(x_1, x_2; \delta_2) \equiv \begin{cases} \begin{array}{c} (1 - x_1) - \delta_2(1 - x_2)(1 - x_2) = 0 \;\text{ for all } x_2 < x_2(.5) \\ x_1 - 1/2 = 0 \;\text{ for all } x_2 \geq x_2(.5) \end{array} & \text{if } \delta_2 > .5 \\ \\ x_1 - 1/2 = 0 \quad \text{if } \delta_2 \leq .5 \end{cases} \tag{17}$$

The equilibrium condition $H^1(x_1, x_2; \delta_2) = 0$ (referred to as the H^1 function) is shown in Figure 1. As the figure indicates, at $x_1 = 1/2$, the H^1 function becomes vertical, which implies that we cannot have a bargaining solution in which Country 1's share is larger than .5.

Note that, unlike in the Rubinstein model where the sum of the (expected) payoffs is 1, here we have:

$$V_1 + W_2 = (1 - x_1) < 1, \text{ for all } 0 < p(x_1) = \; x_1 \leq 1 \tag{18}$$

Consider, now, the case when Country 2 makes the offer. If Country 2 offers x_2 and Country 1 accepts the offer, Country 1's payoff, z_1, is random (since an explosion may occur this period with probability x_2) and given by:

$$z_1 = \begin{cases} x_2 & \text{with probability } 1 - x_2 \\ 0 & \text{with probability } x_2 \end{cases} \tag{19}$$

Its expected payoff in this case is, therefore,

$$E(z_1) = x_2(1 - x_2) \tag{20}$$

On the other hand, if Country 2 offers x_2 and Country 1 rejects the offer, Country 1's payoffs in the following period, y_1, is also random (since an explosion may occur this period with probability x_2 and next period with probability x_1) and given by:

$$y_1 = \begin{cases} x_1(1-x_1) \text{ with probability } 1-x_2 \\ 0 \quad \text{ with probability } x_2 \end{cases} \tag{21}$$

Its expected payoff in this case is, therefore,

$$E(y_1) = x_1(1-x_1)(1-x_2) \tag{22}$$

Country 2 offers Country 1 a share that would make Country 1 indifferent between accepting the offer and rejecting it and waiting for next period when it would be Country 1's turn to make an offer. In other words, Country 2 sets its offer so that:

$$\delta_1 E(y_1) = E(z_1) \tag{23}$$

where δ_1 is Country 1's discount factor. This condition can be written as:

$$H^2(x_1, x_2; \delta_1) \equiv x_2 - \delta_1 x_1(1-x_1) = 0 \tag{24}$$

and is referred to as the H^2 function. In order to focus on the general case, for the rest of the paper we assume that both discount factors are strictly between 0 and 1 (thus, ignoring the two extreme cases, when the discount factors are either 0, or 1).

Again, the sum of the (expected) payoffs is less than 1:

$$V_2 + W_1 = (1-x_2) < 1, \text{ for all } 0 < p(x_2) = x_2 \leq 1 \tag{25}$$

Equations (18) and (25) capture the fact that while extremism may be beneficial for Country 2, it is "socially" costly. We will come back to this point later.

We now have the following:

Proposition 1 *The asset division bargaining game has a unique subgame perfect equilibrium in which: Country 1 always offers* $x_1^*(\delta_1, \delta_2)$ *when it makes the offer and accepts any offer that provides it with at least* $x_2^*(\delta_1, \delta_2)$*, but rejects any other offer. Country 2 always offers* $x_2^*(\delta_1, \delta_2)$ *when it makes the offer and accepts any offer that provides it with at least* $1 - x_1^*(\delta_1, \delta_2)$*, but rejects any other offer. The equilibrium values of* $x_1^*(\delta_1, \delta_2)$ *and* $x_2^*(\delta_1, \delta_2)$ *are given by the solution to equations (17) and (24) .*

Proof. Equations (17) and (24) can be solved to obtain an explicit unique real solution[18] for $x_1^*(\delta_1, \delta_2)$ and $x_2^*(\delta_1, \delta_2)$ as follows. The solution to the equations $(1-x_1) - \delta_2(1-x_2)(1-x_2) = 0$ and $x_2 - \delta_1 x_1(1-x_1) = 0$ is given by the roots of the following polynomials, respectively:[19]

$$\delta_2\delta_1^2 x_1^4 - 2\delta_2\delta_1^2 x_1^3 + \left(\delta_2\delta_1 + \delta_2\delta_1^2 + \delta_2\delta_1\right) x_1^2 + (1 - \delta_2\delta_1 - \delta_2\delta_1)\, x_1 - 1 + \delta_2 = 0$$

and

$$\delta_1\delta_2^2 x_2^4 + \left(-2\delta_1\delta_2^2 - 2\delta_1\delta_2^2\right) x_2^3 + \left(\delta_1\delta_2 - 2\delta_1\delta_2 + 4\delta_1\delta_2^2 + \delta_1\delta_2^2 + \delta_1\delta_2^2\right) x_2^2 + \tag{26}$$

[18] There are several other solutions, but all are either non-real, or fall outside the unit interval.

[19] Note that solving equations (17) and (24) for, say, x_1, we get: $x_1 = 1 - \delta_2(1 - \delta_1 x_1(1-x_1))(1 - \delta_1 x_1(1-x_1)) \equiv G(x_1)$. But, $x_1 \in [0,1]$ and G is a continuous function from the unit simplex into itself. Therefore, there is a fixed point. For all $0 < \delta_1 < 1,\ 0 < \delta_2 < 1$, this solution is unique.

$$+\left(1-\delta_1\delta_2+\delta_1\delta_2+2\delta_1\delta_2-2\delta_1\delta_2^2-2\delta_1\delta_2^2\right)x_2+\delta_1\delta_2-\delta_1-2\delta_1\delta_2\delta_1+\delta_1\delta_2^2=0$$

Let this solution be defines as: $x_1^+(\delta_1,\delta_2)$ and $x_2^+(\delta_1,\delta_2)$. Let us also define the set $G(\delta_1,\delta_2)$ as:

$$G(\delta_1,\delta_2)\equiv\{\delta_1,\delta_2:x_1^+(\delta_1,\delta_2)\leq 1/2\} \tag{27}$$

It is easy to show that the boundary of the set $G(\delta_1,\delta_2)$ can be written as:

$$\delta_2=8/(\delta_1^2-8\delta_1+16) \tag{28}$$

That is:

$$G(\delta_1,\delta_2)\equiv\{\delta_1,\delta_2:\delta_2\geq 8/(\delta_1^2-8\delta_1+16)\} \tag{29}$$

The bargaining game may have either an interior, or a corner solution. Specifically, $x_1^*(\delta_1,\delta_2)$ and $x_2^*(\delta_1,\delta_2)$ are given by:

$$x_1^*(\delta_1,\delta_2) = x_1^+(\delta_1,\delta_2),\ x_2^*(\delta_1,\delta_2)=x_2^+(\delta_1,\delta_2),\ \text{if } (\delta_1,\delta_2)\in G(\delta_1,\delta_2) \tag{30}$$

$$x_1^*(\delta_1,\delta_2) = 1/2,\ x_2^*(\delta_1,\delta_2)=\delta_1/4,\ \text{if } (\delta_1,\delta_2)\notin G(\delta_1,\delta_2) \tag{31}$$

Moreover, we can define $\overline{V}_i$ and $\underline{V}_i$ as the highest and lowest expected utilities that Country i can obtain when it moves first. Similarly, we can define $\overline{W}_i$ and $\underline{W}_i$, as the highest and lowest expected utilities that Country i can obtain when it moves second. It can be verified that $\overline{V}_i=\underline{V}_i$ and $\overline{W}_i=\underline{W}_i$, $i=1,2$, thus implying uniqueness.

■

An example of a unique (interior) solution is shown in Figure 1. The equilibrium conditions $H^1(x_1,x_2;\delta_2)=0$ and $H^2(x_1,x_2;\delta_1)=0$, are shown for the symmetric case with $\delta_1=\delta_2=.9$ and $g=1$. Note that at $x_1=1/2$ the $H^1(x_1,x_2;\delta_2)=0$ function becomes vertical.

Given the equilibrium values $x_1^*(\delta_1,\delta_2)$ and $x_2^*(\delta_1,\delta_2)$, the two countries' expected payoffs, when Country 1 makes an offer, are given by:

$$\begin{aligned} V_1^* &= x_1^*(\delta_1,\delta_2)[1-x_1^*(\delta_1,\delta_2)]\equiv V_1^*(\delta_1,\delta_2) \\ W_2^* &= [1-x_1^*(\delta_1,\delta_2)][1-x_1^*(\delta_1,\delta_2)]\equiv W_2^*(\delta_1,\delta_2) \end{aligned} \tag{32}$$

Similarly, the two countries' expected payoffs, when Country 2 makes an offer, are given by:

$$\begin{aligned} V_2^* &= [1-x_2^*(\delta_1,\delta_2)][1-x_2^*(\delta_1,\delta_2)]\equiv V_2^*(\delta_1,\delta_2) \\ W_1^* &= x_2^*(\delta_1,\delta_2)[1-x_2^*(\delta_1,\delta_2)]\equiv W_1^*(\delta_1,\delta_2) \end{aligned} \tag{33}$$

Finally, remember that the discussion above is for the case when $g=1$. In the case when $g=0$, the model simply collapses to the standard Rubinstein model and equations (17) and (24) yield the solution:

$$x_1^0(\delta_1,\delta_2) = \frac{(1-\delta_2)}{(1-\delta_1\delta_2)} \tag{34}$$

$$x_2^0(\delta_1,\delta_2) = \frac{\delta_1\,(1-\delta_2)}{(1-\delta_1\delta_2)} \tag{35}$$

as in Rubinstein (1982). For comparison purposes, Figure 1 also shows the H^1, H^2 functions (the thinner linear lines in the diagram) for the (Rubinstein) case, where $g = 0$. As the figure shows, for these parameter values, both x_1^0 and x_2^0 are higher in the Rubinstein case. Now, in the case when $g = 0$, define Country $i's$ payoff when it makes an offer as V_i^0 and Country $i's$ payoff when the other country makes an offer as W_i^0. Then we have:

$$V_1^0 = x_1^0(\delta_1, \delta_2) = \frac{(1-\delta_2)}{(1-\delta_1\delta_2)},\ W_2^0 = 1 - x_1^0(\delta_1, \delta_2) = \frac{\delta_2(1-\delta_1)}{(1-\delta_1\delta_2)} \tag{36}$$

$$W_1^0 = x_2^0(\delta_1, \delta_2) = \frac{\delta_1(1-\delta_2)}{(1-\delta_1\delta_2)},\ V_2^0 = 1 - x_2^0(\delta_1, \delta_2) = \frac{(1-\delta_1)}{(1-\delta_1\delta_2)} \tag{37}$$

3.1.1 The Effects of the Order of Play

From equations (17) and (24) it follows that:[20]

$$x_1^*(\delta_1, \delta_2) > x_2^*(\delta_1, \delta_2),\ \text{for all } 0 < \delta_1 < 1,\ 0 < \delta_2 < 1,\ g = 1 \tag{38}$$

which also implies that:

$$1 - x_2^*(\delta_1, \delta_2) > 1 - x_1^*(\delta_1, \delta_2),\ \text{for all } 0 < \delta_1 < 1,\ 0 < \delta_2 < 1,\ g = 1 \tag{39}$$

In other words, both countries get a higher share when they move first. But, given that the probability of an explosion also depends on the order of play, will they also have higher expected payoffs when they move first? Comparing V_2^* with W_2^*, we get:

$$\begin{aligned} V_2^* - W_2^* &= (1-x_2^*)(1-x_2^*) - (1-x_1^*)(1-x_1^*) > 0 \\ \text{for all } 0 &< \delta_1 < 1,\ 0 < \delta_2 < 1,\ g = 1 \end{aligned} \tag{40}$$

since: $1 - x_2^* > 1 - x_1^*$ and $1 - x_2^* > 1 - x_1^*$, because $x_1^* > x_2^*$.

Comparing V_1^* with W_1^*, we have:

$$V_1^* - W_1^* = x_1^*(1-x_1^*) - x_2^*(1-x_2^*) \tag{41}$$

But now, $x_1^* > x_2^*$, whereas $1-x_1^* < 1-x_2^*$. Nevertheless, using the solution for the equilibrium values of $x_1^*(\delta_1, \delta_2)$, $x_2^*(\delta_1, \delta_2)$, it is easy to verify that for Country 1 it is also true that:

$$\begin{aligned} V_1^* - W_1^* &> 0 \\ \text{for all } 0 &< \delta_1 < 1,\ 0 < \delta_2 < 1,\ g = 1 \end{aligned} \tag{42}$$

Both countries are, therefore, better off when they move first. This is, of course, is not surprising: it is the well-known first mover advantage (which vanishes at the limit without friction).

The order of play, however, also affects "social welfare". To see this note that if Country 1 moves first in the bargaining game, total expected payoffs are given by:

$$S_1(\delta_1, \delta_2) \equiv V_1^* + W_2^* = (1 - x_1^*) \tag{43}$$

[20] From equation (17), when $x_1^* < 1/2$ we have: $x_1^* = 1 - \delta_2(1-x_2^*)(1-x_2^*)$, and therefore, $x_1^* - x_2^* = (1-x_2^*)[1-\delta_2(1-x_2^*)] > 0$. When $x_1^* = 1/2$ we have $x_1^* - x_2^* = 1/2 - x_2^* = 1/2 - \delta_1/4 > 0$.

but, when Country 2 moves first total expected payoffs are given by:

$$S_2(\delta_1, \delta_2) \equiv V_2^* + W_1^* = (1 - x_2^*) \tag{44}$$

In both cases, total expected payoffs are given by the expected size of the pie which, here, is simply the probability of no explosion. Alternatively, given the equivalence between the probability of an explosion and the level of extremism, the reduction in the size of the pie is given by the level of extremism.

Looking at the difference in payoffs, and given that $x_1^* > x_2^*$, it is clear that:

$$S_1(\delta_1, \delta_2) - S_2(\delta_1, \delta_2) = \begin{cases} x_2^* - x_1^* < 0, & \text{for } g = 1 \\ 0, & \text{for } g = 0 \end{cases} \tag{45}$$

Thus, since the probability of an explosion is related to Country 1's share, and since Country 1 gets a smaller share when it moves second, total "welfare" as measured by total expected payoffs is higher when Country 2 moves first.

3.2 Stage 2: Bargaining Over the Order of Play

In stage two, the countries engage in the order of play bargaining (OPB). This bargaining game is modelled as an alternating offers game, where the two countries use transfer payments to arrive at a mutually agreeable order of play.[21] We assume that this bargaining game has the following structure. The two countries make alternating offers of a transfer payment to, say, Country 1, where we define t_i as the transfer to Country 1, when Country i makes an offer. To simplify the analysis, we assume that the transfer payments and explosions do not affect each other. Essentially, this means that, except for the order of play, the outcome of the OPB game does not affect the ADB game.[22] We assume both countries can opt out of the OPB game in favour of the ADB game. To be able to fix the opting out values, we assume that if this happens, there is a probability q that Country 2 will make the first offer in the ADB game. The values of the two countries' outside options are, therefore, given by,

$$\overline{P}_1^*(\delta_1, \delta_2) = (1 - q)V_1^* + qW_1^* \tag{46}$$

$$\overline{P}_2^*(\delta_1, \delta_2) = qV_2^* + (1 - q)W_2^* \tag{47}$$

In response to an offer, a country can either accept it, reject it and make a counter offer, or opt out. The OPB game, therefore, ends either when a transfer offer is accepted, or when one of the countries opts out. If the OPB game ends because an offer was accepted, the ADB game begins immediately, according to the agreed upon order of play. On the other hand, if the OPB game ends because one of the countries opts out, the ADB game begins immediately without an agreed upon order of play.[23]

Let us now examine the outcome of the OPB game. First, since total welfare is higher when Country 2 moves first (as in equation (45)), it follows immediately that:

[21] There may be other possible protocols that explicitly strive to maximize social welfare by adopting more complicated bargaining agendas, but we do not pursue them here. For an example of such a bargaining protocol, see Appelbaum (2007).

[22] It is, of course, possible to assume that the transfer payments and explosions affect each other. Since the focus of the paper is on the endogeneity of breakdown probabilities and order of play, rather than the specific details of the model, we do not pursue this here.

[23] Alternatively, we can consider different types of delays between the two games; depending on whether there was an acceptance, opting out, rejection (see, for example, Muthoo (1999)). Again, since we are simply presenting an example of an endogenously determined order of play, we do not pursue this further.

Proposition 2 *In equilibrium, it is always agreed that Country 2 moves first in the ADB game.*

Proof. The amount that Country 1 needs to offer in order be allowed to move first is, at least, $t_1 = -(\overline{P}_2^* - W_2^*) < 0$ (that is, Country 2 will receive a positive transfer of $(\overline{P}_2^* - W_2^*)$). But, if it makes this transfer, it will have at most: $V_1^* + t_1 = \overline{P}_1^* + q(S_1 - S_2) < \overline{P}_1^*$. In other words, Country 1 cannot offer the minimum transfer required to be allowed to move first and still remain as well off as it would be if it were to enter the ADB game without an agreed upon order of play (with a payoff of $\overline{P}_1^*$). On the other hand, if Country 2 would offer Country 1, for example, $t_2 = \overline{P}_1^* - W_1^* > 0$, Country 1's payoffs will then be: $\overline{P}_1^*$, so that such an offer will be accepted. Can Country 2 make such a transfer and still be better off? Yes, since its receipts will be: $V_2^* - t_2 = V_2^* - [(1-q)V_1^* + qW_1^* - W_1^*]$ $= \overline{P}_2^* + (1-q)(S_2 - S_1) > \overline{P}_2^*$. Hence, in equilibrium, it must be the case that Country 2 moves first. ■

The unique subgame perfect equilibrium of the OPB game is characterized as follows:

Proposition 3 *The order of play bargaining game with outside options has a unique subgame perfect equilibrium in which:*

1. It is always agreed that Country 2 moves first in the ADB game
2. (a) Country 1 always offers a transfer of $t_1^* > 0$ and accepts a transfer t, if and only if $t \geq t_2^*$, (b) Country 1 always opts out when it gets an offer $t < t_2^*$, if and only if $\delta_1(W_1^* + t_1^*) \leq \overline{P}_1^*$.
3. (a) Country 2 always offers a transfer of $t_2^* > 0$ and accepts a transfer t, if and only if $t \leq t_1^*$, (b) Country 2 always opts out when it gets an offer $t > t_1^*$, if and only if $\delta_2(V_2^* - t_2^*) \leq \overline{P}_2^*$.
4. t_1^* and t_2^* are given by:

$$t_1^*(\delta_1, \delta_2) = \begin{cases} t_{1a}^* \equiv [(1-\delta_2)V_2^* - \delta_2(1-\delta_1)W_1^*]/(1-\delta_1\delta_2) & \text{if } \delta_1(W_1^* + t_{1a}^*) \geq \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2a}^*) \geq \overline{P}_2^* \\ t_{1b}^* \equiv V_2^* - \delta_2\overline{P}_2^* & \text{if } \delta_1(W_1^* + t_{1b}^*) \geq \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2b}^*) < \overline{P}_2^* \\ t_{1c}^* \equiv (1-\delta_2)V_2^* + \delta_2\overline{P}_1^* - \delta_2 W_1^* & \text{if } \delta_1(W_1^* + t_{1c}^*) < \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2c}^*) \geq \overline{P}_2^* \\ t_{1d}^* \equiv V_2^* - \overline{P}_2^* & \text{if } \delta_1(W_1^* + t_{1d}^*) < \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2d}^*) < \overline{P}_2^* \end{cases} \tag{48}$$

$$t_2^*(\delta_1, \delta_2) = \begin{cases} t_{2a}^* \equiv [\delta_1(1-\delta_2)V_2^* - (1-\delta_1)W_1^*]/(1-\delta_1\delta_2) & \text{if } \delta_1(W_1^* + t_{1a}^*) \geq \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2a}^*) \geq \overline{P}_2^* \\ t_{2b}^* \equiv \delta_1 V_2^* - (1-\delta_1)W_1^* - \delta_1\overline{P}_2^* & \text{if } \delta_1(W_1^* + t_{1b}^*) \geq \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2b}^*) < \overline{P}_2^* \\ t_{2c}^* \equiv \overline{P}_1^* - W_1^* & \text{if } \delta_1(W_1^* + t_{1c}^*) < \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2c}^*) \geq \overline{P}_2^* \\ t_{2d}^* \equiv \overline{P}_1^* - W_1^* & \text{if } \delta_1(W_1^* + t_{1d}^*) < \overline{P}_1^* \text{ and } \delta_2(V_2^* - t_{2d}^*) < \overline{P}_2^* \end{cases} \tag{49}$$

Proof. The game is a standard alternating offers bargaining game with outside options. The proof that in equilibrium it is always agreed that Country 2 moves first in the ADB game follows from Proposition 2. The rest of the proof is the same as the one provided in Muthoo (1999). The proof follows from the requirement that in a subgame perfect equilibrium the offers are such that a player is always indifferent between accepting, or rejecting the other player's offer. Since in equilibrium Country 2 always moves first in the ADB game, this implies that we need to solve the two conditions (corresponding to the cases when Country 2 and Country 1 make offers, respectively):[24]

$$W_1^* + t_2^* = \max\{\delta_1(W_1^* + t_1^*), \overline{P}_1^*\} \quad (50)$$

$$V_2^* - t_1^* = \max\{\delta_2(V_2^* - t_2^*), \overline{P}_2^*\} \quad (51)$$

The equilibrium values t_1^* and t_2^* are the unique solution to these two equations. ■

There are, therefore, four types of (unique) equilibria (corresponding the four lines in equations (48) and (49): $\{t_{1j}^*, t_{2j}^*\}$, $j = a, b, c, d$. Which of the possible equilibria actually occurs, clearly, depends on the values of δ_1 and δ_2.[25] Let us examine the nature of the equilibrium. First, it is easy to verify that in equilibrium we always have $t_{1j}^* > 0$ and $t_{2j}^* > 0$, $j = a, b, c, d$. In other words, it is indeed the case that Country 2 always provides a transfer to Country 1, so that it would be allowed to move first. Second, as is the case in the Rubinstein model, we cannot determine who moves first in the OPB game. Third, from the definitions of $\{t_{1j}^*, t_{2j}^*\}$ and the boundary requirements of the four equilibria in equations (48) and (49), it follows that there is a first mover advantage (in the OPB game): namely, $t_{1j}^* > t_{2j}^*$, $j = a, b, c, d$. Finally, regardless of who moves first in the OPB game, the order of play in the ADB game is always the same: Country 2 moves first. Thus, regardless of who moves first in the OPB game, the order of play in the ADB game will be the socially optimal order of play (resulting in total welfare of $V_2^* + W_1^*$).

3.3 Stage 1: The Choice of Extremism as a Strategic Tool

In stage one of the game, Country 2 decides whether to use extremism as a strategic tool; that is, whether it should choose $g = 1$, or $g = 0$, taking into account the outcomes of the forthcoming OPB and ADB games in stages two and three, respectively. Since this is a binary choice between $g = 1$, and $g = 0$, we can simply compare Country 2's payoffs in the two cases.

As was shown above, if Country 2 chooses $g = 0$, the model collapses to the standard Rubinstein model. In this case, the size of the pie is fixed (as is shown by equations (34)-(37)), so that social welfare is the same regardless of the order of play ($V_1^0 + W_2^0 = V_2^0 + W_1^0 = 1$). Consequently, as in the Rubinstein model, the order of play cannot be negotiated and remains indeterminate. Thus, for $g = 0$, and assuming that the probability that Country 2 will make the first offer in the ADB game is again given by q, the countries' payoffs are simply the average payoffs, $\overline{P}_1^0(\delta_1, \delta_2)$ and $\overline{P}_2^0(\delta_1, \delta_2)$, given by:

$$\overline{P}_1^0(\delta_1, \delta_2) = (1-q)V_1^0 + qW_1^0 \quad (52)$$

$$\overline{P}_2^0(\delta_1, \delta_2) = qV_2^0 + (1-q)W_2^0 \quad (53)$$

On the other hand, if Country 2 chooses $g = 1$, its payoff is $V_2^* - t_1^*$ if Country 1 moves first (in the OPB game) and $V_2^* - t_2^*$ if Country 1 moves second (in the OPB game). Again, assuming that the probability that

[24] Note that since in equilibrium Country 2 always moves first in the ADB game, on the right side of equation (50) we have W_1^*, rather than V_1^* and on the left hand side of equation (51) we have V_2^*, rather than W_2^*.

[25] For example, equilibrium $\{t_{1b}^*, t_{2b}^*\}$ occurs when δ_1 is 'sufficiently' high for any given value of δ_2, equilibria $\{t_{1a}^*, t_{2a}^*\}$ and $\{t_{1c}^*, t_{2c}^*\}$ can only occur if δ_2 is 'sufficiently' high and equilibrium $\{t_{1d}^*, t_{2d}^*\}$ occurs when δ_1 is 'sufficiently' low.

Country 2 will move first (in the OPB game) is q, Country 2's expected payoff, defined as $\overline{P}_2^{**}$, is given by:[26]

$$\overline{P}_2^{**} \equiv q(V_2^* - t_2^*) + (1-q)(V_2^* - t_1^*) = V_2^* - [qt_2^* + (1-q)t_1^*] \tag{54}$$

Using the solutions for V_2^*, t_1^*, t_2^* and $\overline{P}_2^0$ we can easily calculate $\overline{P}_2^{**}$ and $\overline{P}_2^0$ (the payoffs when $g = 1$ and when $g = 0$). Comparing $\overline{P}_2^{**}$ with $\overline{P}_2^0$ we find that indeed:

$$\overline{P}_2^{**} > \overline{P}_2^0, \text{ for all } \delta_1 \text{and } \delta_2 \tag{55}$$

That is, Country 2's expected payoff is always higher when it chooses $g = 1$. Country 2 will, therefore, always choose to use extremism strategically.

Finally, it should be noted that we always get a corner solution at $g = 1$, for all $\delta_1$and δ_2, because of (i) the discreet choice of g, (ii) the linearities assumed in obtaining the probability function in equation (2). It can be shown that with a more general probability function, $p(x, g)$, we can get an interior solution for g, thus allowing for varying degrees of extremism.

4 Conclusion

This paper, develops an alternating offers bargaining model with breakdowns, in which both the breakdown probabilities and the order of play are determined endogenously. The model shows that if extremism can be used strategically in a conflict, in general, there exists a unique subgame perfect equilibrium in which it will, indeed, be used. As a result, the probability of a breakdown is determined endogenously. The endogeneity of the probability implies that the size of the expected pie is affected by the order of play, which in turn means that the order of play can also be determined endogenously. Specifically, we show that in equilibrium, the order of play will the one that leads to the larger expected pie size. In this context, this means that the country that uses extremism strategically will move first.

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[26] More generally, we can take the probabilities that Country 2 makes the first offer to be different in the OPB and ADB games. For simplicity, we do not pursue this here.

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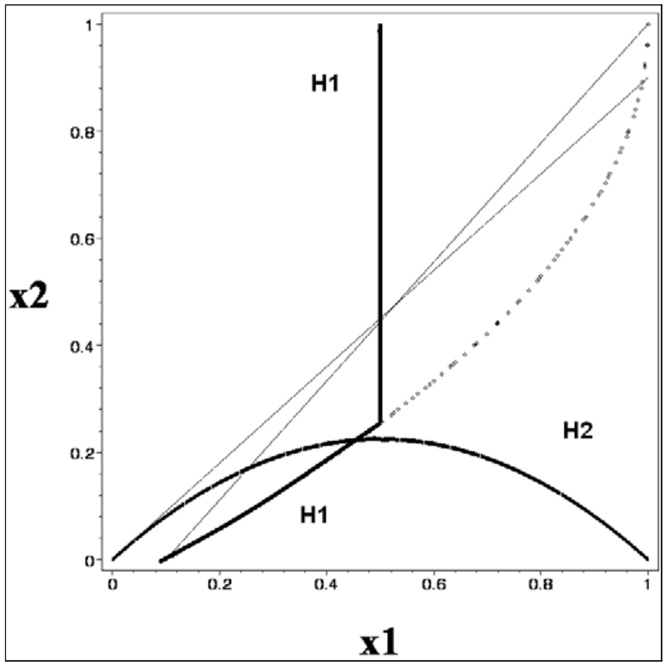

Figure 1: Equilibrium with $g = 1$, $\delta_1 = \delta_2 = .9$. Thin Lines - H^1 and H^2 for the case where $g = 0$.