Canadian Money Demand Functions: Cointegration–Rank Stability

Alfred A. Haug [0]

This paper applies conventional tests (Johansen, 1995) and new tests (Chao and Phillips, 1999) for cointegration to long–run money demand functions using historical Canadian data back to 1872. If cointegration is found, recently proposed tests by Quintos (1998a) for stability of the cointegration rank are carried out. The paper focuses on two spans of data: one span starting in 1872, the other in 1957 or 1968. Annual data are used for the former span, and annual and quarterly data for the latter. The preferred money demand specification involves M1.

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[0]Department of Economics, York University, 4700 Keele Street, Toronto, Ontario, Canada M3J 1P3. E-mail: haug@econ.yorku.ca. Carmela Quintos kindly provided the GAUSS code for the stability tests and Pierre Siklos made available data on institutional change and long–term interest rates. The author thanks participants at seminars at Simon Fraser University and the University of Canterbury, and at various conferences for helpful comments on an earlier version of this paper. The usual caveat applies. Financial support from the University of Canterbury and from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

1. Introduction

The long–run money demand function has been playing an important, though different, role in macroeconomic models of the various schools of thought. Friedman's (1956) goal has been to find a stable function for money demand that depends on only a very limited number of variables. Meltzer (1963), Laidler (1966), Lucas (1988), and many others, have followed the same line of research. The empirical stability of money demand functions has been a concern for some time.[1] For the last decade, empirical researchers have applied mostly cointegration techniques to uncover a stable money demand relation in the long–run.[2] However, the finding of cointegration does not imply that the relation is stable over time.

Stock and Watson (1993) have applied several methods of estimating cointegrating vectors to U.S. money demand functions over the period 1900 to 1988 and tested for parameter stability. They have considered a semi–logarithmic M1 money demand function with real GNP as the scale variable and various short– and long–term interest rates, in turn, to measure opportunity costs of holding money. They have concluded that a long span of data is necessary in order to estimate long–run money demand functions precisely.[3]

This paper applies the stability tests with unknown change point of Quintos (1998a) to Canadian money demand functions within the framework of a vec-

[1] See the reviews by Judd and Scadding (1982) and Goldfeld and Sichel (1990) of studies focussing on short–run stability.

[2] A few examples of early applications are Johansen and Juselius (1990), Hafer and Jensen (1991), Hendry and Ericsson (1991) with a reply by Friedman and Schwartz (1991), and Hoffman and Rasche (1991). See also Laidler (1993, Chapter 4) and Ericsson (1998) for surveys of other empirical research, and Lütkepohl et al. (1999) for an alternative to linear cointegration.

[3] Miyao (1996) has questioned a stable cointegrating relationship for M2. See also Friedman and Kuttner (1992), and Estrella and Mishkin (1997). However, Ball (2001) has revisited Stock and Watson's study with a data set extended to 1996 and found precise estimates and a stable cointegrating relation for M1 for the U.S. postwar period. Similarly, Haug and Tam (2001) have confirmed a stable relation for the U.S. postwar period, however, have found that M0 is preferable to M1.

tor error–correction model (VECM). The short–run dynamics are not held constant for these tests.[4] Quintos (1998a, 1998b) has developed a framework for testing for cointegration–rank stability. Quintos (1998a) has derived the asymptotic properties of the tests and has shown that a necessary first step is to test for rank stability in order to avoid biased results in subsequent tests within the VECM framework. Quintos (1998a) also has demonstrated with a Monte Carlo experiment that her tests have good size and power properties in finite samples.

In addition, this paper compares the performance of Johansen's (1995) tests for cointegration to a recently proposed alternative method. Johansen's sequential tests for cointegration rank may lead to overestimation of the number of cointegrating vectors even in the limit. The tests also require to first specify the number of lags in the vector error–correction model. I therefore apply as an alternative a new information criterion proposed by Chao and Phillips (1999). This criterion avoids the problem of overestimating the rank and allows to determine the number of lags in the model and the cointegration rank simultaneously. Chao and Phillips have demonstrated in a Monte Carlo study that their criterion performs well in small samples.

For the empirical analysis in this paper, I employ a new data set with a long span back to 1872. The new data are from Metcalf et al. (1998). They have constructed measures of money that take the current Bank of Canada definitions of the monetary base, M1, and M2 back in time to mid–1871. The basic semi–logarithmic money demand specification that I use linearly relates the natural logarithm of real money balances to the natural logarithm of real GNP and to the level of a long–term interest rate. Various alternative specifications are explored, including some specifications with recently developed new measures of money.

Section 2 outlines the econometric methods used. Section 3 describes the data and Section 4 reports the empirical results. Section 5 concludes.

[4]See Ball (2001, p. 37, fn. 5) on this issue.

2. Econometric Methodology

2.1 Unit Roots and Cointegration

As a first step, every time–series that enters the money demand function is tested for one and two unit roots. The augmented Dickey–Fuller and Phillips–Perron tests are applied. Next, Johansen's (1995) maximum likelihood based method is used to test for cointegration and to estimate the cointegrating vectors. Following Hoffman et al. (1995), among others, I allow for linear deterministic time trends in the levels vector moving-average representation of the model, which in turn implies an unrestricted constant and no deterministic time trends in the VECM specification. Allowing for deterministic trends in the data in levels is appropriate given growth and technological change.

Chao and Phillips (1999) have drawn attention to a potential problem with Johansen's method of performing sequential tests to determine the cointegration rank. Johansen's (1992) Theorem 2 shows that the probability of overestimating the rank remains positive in the limit and therefore the cointegration rank is not estimated consistently with the sequential procedure. Furthermore, the VECM of Johansen requires in general to choose an appropriate lag order and results can be sensitive to lag misspecification.[5]

Chao and Phillips have proposed to apply the Posterior Information Criterion (PIC) of Phillips and Ploberger (1996) to VECMs as an alternative to Johansen's method. This criterion allows to determine the VECM lag order and the cointegration rank jointly and leads to consistent estimation of both. The cointegrating vectors can then be consistently estimated by Johansen's method after imposing the lag order and cointegration rank obtained with the PIC. I briefly outline the framework in which the PIC is applied, following Chao and Phillips.

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[5]See Toda and Phillips (1994).

The VECM is given by

$$\Delta Y_t = J^*(L)\Delta Y_{t-1} + J_* Y_{t-1} + \varepsilon_t. \tag{1}$$

Y_t is a vector of dimension m and contains in the empirical application real money balances, real GNP and an interest rate, and

$$J(L) = \sum_{i=1}^{p+1} J_i L^{i-1}$$

so that

$$J_* = J(1) - I_m$$

and

$$J^*(L) = \sum_{i=1}^{p} J_i^* L^{i-1}$$

with

$$J_i^* = -\sum_{l=i+1}^{p+1} J_l, \quad i = 1, \ldots, p.$$

Further, Δ is the first difference operator, and $J_* = \Gamma_r A_r'$ with loading vectors Γ_r and the cointegrating vectors A_r, each a matrix of dimension $m \times r$ with full column rank r and $0 \leq r \leq m$. The cointegration rank is given by r and the columns of A_r contain the r cointegrating vectors. When $r = 0$, then $\Gamma_0 = A_0 = 0$. When $r = m$, then $\Gamma_m = J_*$ and $A_m = I_m$. These special cases deliver a vector autoregression in first differences and in levels, respectively. The interest here is with the cases in between. For ease of exposition, the VECM is here specified without any constant terms.

The lag order p and the cointegration rank r are selected by $(\widehat{p}, \widehat{r})$ = arg min PIC (p, r) and an approximation is given by

$$\mathrm{PIC}(p,r) \approx \ln\left|\widehat{\Omega}_{p,r}\right| + \left[m^2 p + 2r(m-r) + mr\right] T^{-1} \ln T,$$

where T is sample size. The PIC attaches to the parameters $r(m-r)$ of the cointegrating matrix twice the penalty than it does to parameters of stationary regressors. In contrast to Johansen's method, the PIC imposes a penalty on overparameterization in order to correct for upward bias of r. The residual covariance matrix (see

Chao and Phillips on notation)

$$\widehat{\Omega}_{p,r} = \left[\left(\Delta Y - Y_{-1} \widehat{J}_*(p,r)' \right)' M_{W(p)} \left(\Delta Y - Y_{-1} \widehat{J}_*(p,r)' \right) \right] T^{-1}$$

with

$$\widehat{J}_*(p,r) = \left(\widehat{\Gamma}(p,r), \;\; \widehat{\Gamma}(p,r) \widehat{\overline{A}}(p,r)' \right),$$

where $\widehat{\Gamma}(p,r)$ and $\widehat{\overline{A}}(p,r)$ are the maximum likelihood estimators of Γ and $\overline{A}$ when the cointegration rank is assumed to be r and the lag order is assumed to be p, assuming that ε_t is iid N(0, Σ). Further,

$$M_{W(p)} = I_T - W(p)(W(p)'W(p))^{-1}W(p)'$$

with

$$W(p) = [W_1(p), \ldots, W_T(p)]'$$

and

$$W_t(p) = [\Delta Y_{t-1}', \ldots, \Delta Y_{t-p}']'.$$

Chao and Phillips have shown how the PIC is based on Bayesian as well as classical principles. They have also proved weak consistency of the PIC.

2.2 Structural Change: Rank Stability Tests

I first test for the stability of the cointegration rank r of $J_* = \Gamma_r A_r'$, following Quintos (1998a). The null hypothesis is that the rank r stays constant over the full sample:

$$H_0^r : \;\; \mathrm{rank}(J_*^{(t)}) = r.$$

That means that the number of cointegrating vectors does not increase or decrease from some point in time on.

The test statistic depends on the form that the alternative hypothesis takes. I first consider the case of more cointegrating vectors:

$$H_1^r : \;\; \mathrm{rank}(J_*^{(t)}) > r.$$

Quintos has suggested a likelihood ratio test based on the fully–modified vector autoregressive estimation procedure of Phillips (1995). However, she has shown in a subsequent paper (Quintos, 1998b) that the likelihood ratio test for this estimator is degenerate when there are no cointegrating vectors in the system. I therefore apply instead Johansen's method to estimate the eigenvalues for the rank stability test. The test is defined by the sup of the following likelihood ratio statistic:

$$\sup_{\kappa \in \Phi} Q_T^{+}(\kappa) = \sup_{\kappa \in \Phi} [\kappa T] \sum_{i=r+1}^{m} \widehat{\lambda}_i^{([\kappa T])},$$

with $\Phi = [.15, \;\; .85]$ as suggested by Andrews (1993).[6] The asymptotic distribution depends on whether or not a constant is included in the VECM and on what restrictions, if any, are placed on that constant. The critical values from MacKinnon et al. (1999) apply to the above test statistic.

The $\widehat{\lambda}_i^{(\cdot)}$ are the roots of the matrix (see Johansen, 1995, p. 95 on notation)

$$S_{\Delta Z \Delta Z}^{(\cdot)-\frac{1}{2}} S_{\Delta Z Z}^{(\cdot)} S_{ZZ}^{(\cdot)-1} S_{Z \Delta Z}^{(\cdot)} S_{\Delta Z \Delta Z}^{(\cdot)-\frac{1}{2}}. \tag{2}$$

The r largest eigenvectors of this reduced rank regression are the estimators of $\widehat{A}^{(\cdot)}$. Reduced rank estimation is equivalent to maximum likelihood. Further, for example for T,

$$S_{\Delta Z \Delta Z}^{(T)} = T^{-1} \Delta Z^{(T)\prime} \Delta Z^{(T)}$$

$$S_{\Delta Z Z}^{(T)} = T^{-1} \Delta Z^{(T)\prime} Z_{-1}^{(T)}$$

$$S_{ZZ}^{(T)} = T^{-1} Z_{-1}^{(T)\prime} Z_{-1}^{(T)}.$$

$Z_{-1}^{(t)} = (z_0, \ldots, z_t)'$ and $\Delta Z^{(t)}$ are similarly defined. Δz_t and z_{t-1} are the residuals of a regression of ΔY_t on $(\Delta Y_{t-1}, \ldots, \Delta Y_{t-p})$ and of Y_{t-1} on the same set of regressors.

Next, I consider the other alternative hypothesis of less cointegrating vectors than r:

$$\tilde{H}_1^r : \;\; \mathrm{rank}(J_*^{(t)}) < r.$$

[6] For a few small samples in the post–WWII period, I used $\Phi = [.2, \;\; .8]$ and $[.25, \;\; .75]$ instead in order to achieve convergence.

At time t the estimated residuals are

$$\widehat{U}^{(t)\prime} = \Delta Z^{(t)\prime} - J_* Z^{(t)}_{-1}$$

and they are standardized to

$$\widehat{E}^{(t)\prime} = \widehat{\Omega}_{00}^{-\frac{1}{2}} \widehat{U}^{(t)\prime},$$

where Ω_{00} is a function of the long–run covariance matric Ω. I use a quadratic kernel along with an automatic data–based bandwidth selection method as suggested by Andrews (1991) to estimate Ω_{00}. Also, I pre–whiten following Andrews and Monahan (1992).[7]

Define

$$G_t = \widehat{A}_r^{(T)\prime} \widehat{E}^{(t)\prime},$$

where $\widehat{A}_r^{(T)} = S_{\Delta Z \Delta Z}^{(T) - \frac{1}{2}} V_r$ and V_r contains the eigenvectors of equation (2) ordered by the size of the r largest eigenvectors of (2). The Lagrange multiplier test statistic is given by

$$\overline{Q}_T = T^{-2} tr \left\{ \sum_{t=1}^{T} G_t G t' \right\}.$$

The asymptotic distribution is non-standard and depends on r. Critical values were simulated by Quintos (19978a) and results are given in her Table 1.

3. The Data

The annual data cover the period from 1872 to 1997. This was the largest data span available at the time this research was started. The money measures are M0 (the monetary base), M1, and M2. M1 includes currency in the hands of the public and demand deposits held by the public and provincial governments, net of float. M2 includes M1 net plus personal savings deposits and non–personal (chequing and non–chequing) notice deposits. These three measures of money have been constructed by Metcalf et al. (1998) and are historical extensions of the current Bank of Canada

[7] Using instead a Parzen window with four lags does not affect the results significantly. See also Phillips (1995).

definitions. Details on approximations and other problems in data construction are discussed in Metcalf et al. Their monthly data cover the years 1872 to 1967. I averaged the monthly data to arrive at annual figures. Data on these money measures from 1968 to 1997 are from Statistics Canada's (March 1999) CANSIM data base, with B2055, B2033, and B2031 corresponding to M0, M1, and M2, which are the current Bank of Canada definitions.

The annual data for gross national product (GNP) and the GNP deflator are from Urquhart (1986) for the years 1872 to 1925. These series are from Statistics Canada (1975) for the years 1926 to 1960, Catalogue 13-531. For the years 1961 to 1997, the GNP series is from CANSIM, D16441, and the GDP deflator series is also from CANSIM, D205566 up to 1985 with base year 1981, and D19296 from 1986 to 1997 with base year 1992. These GDP–deflators are then linked to the GNP deflator constructed by Urquhart.

The long–term interest rate was kindly supplied by Pierre Siklos to cover the period 1872 to 1985. Data for the period 1986 to 1997 are from CANSIM, B14013 and are consistent with Siklos' series. The series is the annual average of Government of Canada long–term bond yields of over ten years. In addition, I use data from Siklos (1993) to measure institutional change over the period 1900 to 1986 that will be explained in Section 4.

The corresponding quarterly data are from the same sources. The quarterly data cover the period from 1957 to 1997. In addition to the long–term interest rate, the 3 month T–bill rate, CANSIM series B14007, is also considered, following Stock and Watson (1993) and others in considering a short– and a long–term interest rate.

The Bank of Canada has recently calculated new measures of the money stock to include similar deposits outside chartered banks and other relatively liquid funds not captured by M1 and M2: M1++ and M2++. These are more comprehensive measures of money from the 1970s on than M1 and M2 and are therefore considered in the empirical tests because results might be sensitive to the choice of the money measure.[8] These series are available from 1968 on (quarterly and seasonally adjusted) and are from CANSIM (June 1999), series B1652 and B1650, respectively. They are analyzed below in addition to the other money measures. M1++ consists of M1 plus all notice deposits. M2++ is M2 plus the sum of deposits at Trust and Mortgage Loan Companies, at Credit Unions, at Caisses Populaires, Canada Savings Bonds, and all mutual funds.[9]

Seasonal adjustment, when necessary, is carried out with the weighted average (multiplicative) method in EViews 3.1. All unit root and cointegration tests, including PIC, are performed with EViews. The Quintos procedures are performed in GAUSS for Windows.

4. Empirical Results

First, I test each single annual (1872-1997) and quarterly (1957-1997 and 1968-1997) time–series for one and two unit roots with the augmented Dickey Fuller test using Akaike's criterion to select the appropriate lag lengths. I also apply the Phillips–Perron unit root test with the Newey–West correction as implemented in EViews 3.1. I allow in turn for a constant and for a constant plus deterministic time trend in the test regressions. Results are available from the author on request. All money measures and GNP are in real terms and natural logarithms: ln(rM0), ln(rM1), ln(rM2), ln(rM1++), ln(rM2++), and ln(rGNP). The nominal long–term interest rate is specified in natural logarithms, ln(ltir), and alternatively in levels, ltir. The same applies for the short–term interest rate, which is represented by the 3 month T–bill rate. For all variables, the null hypothesis of a unit root cannot be rejected, whereas the null hypothesis of two unit roots is rejected, using a 5% level of significance.

[8] See, for example, Haug and Lucas (1996).

[9] See the Bank of Canada Review, January 1998, *Notes to the Tables.*

The basic money demand relation takes the following form:

$$\ln(\text{real money measure}) - \alpha - \beta \ln(\text{rGNP}) - \gamma \ln(\text{ltir}) = u_t. \tag{3}$$

I explore various money demand specifications that have been used by previous researchers. In contrast to Stock and Watson (1993) and to Ball (2001), Hoffman et al. (1995) and Bordo et al. (1997) have not assumed a known break date when testing stability of the money demand function.[10] Hoffman et al. and Bordo et al. have tested for the stability of the number of cointegrating vectors, which is the cointegration rank, and in addition for stability of the cointegrating parameters.[11] However, they held constant the short–run dynamics of the money demand function for the rank stability tests, despite the well established instability of the short–run money demand function in the literature.[12]

I set up the VECM for the variables in equation (3), using the Schwarz criterion for lag order selection considering up to six lags in the annual data and up to eight lags in the quarterly post–WWII data. The VECM is specified with an unrestricted constant. Johansen's (1995) maximum likelihood based method is used as a first step for testing for the number of cointegrating vectors with the trace test. The P–values are calculated with a program available from MacKinnon et al. (1999).

I consider in the VECM specifications the real money measures ln(rM0), ln(rM1), and ln(rM2) in turn, with the annual data for the period 1872 to 1997. The interest rate is specified in logarithms, however, all test results remain unchanged when the long–term interest rate in levels, ltir, is used instead of ln(ltir). The only

[10] They have used the vector error–correction model of Johansen (1995). Furthermore, Bordo et al. have included additional variables for the long–run money demand function to capture institutional change.

[11] Also, Haug and Lucas (1996) have tested for parameter stability of money demand when the change point is unknown but did not test for rank stability.

[12] See Ball (2001, p. 37, fn. 5). Further, Hoffman et al. and Bordo et al. have not applied the Quintos framework and used a somewhat ad hoc approach instead. In addition, Hoffman et al. included break dummies in the error–correction model so that the standard asymptotic critical values they used are not quite appropriate.

other variable included in the VECM is ln(rGNP). The VECM models with M0 and alternatively with M2 as the measure of money do not lead to a rejection of the null hypothesis of no cointegration. I therefore find no evidence for cointegration when M0 or M2 are measuring the money stock and the Johansen method is applied.

In contrast, the VECM model with M1 as the measure of money leads to the finding that there is one cointegrating vector in the system, even at a quite low level of significance. Results are reported in Table 1. The cointegrating vector estimate gives an income (GNP) elasticity of 1.04 and an interest elasticity of -.96. I tested the hypothesis that the income elasticity is not significantly different from 1 and could not reject it, using a Wald test.[13] This finding has important implication because it allows for a specification in terms of velocity, ln(GNP/M1).

The Johansen method may possibly overestimate the cointegration rank. This point was discussed in the section on econometric methodology. To explore this possibility, I apply the PIC of Chao and Phillips (1999) in order to choose the lag order of the VECM and the cointegration rank simultaneously, and to assure consistent estimation of both. The PIC rejects cointegration for the models with M0 and M2 as measures of money and therefore confirms the above Johansen results. For the model with M1, the PIC takes on a value of -16.6 for $p = 0$ and $r = 1$, and the same value for $p = 0$ and $r = 0$. According to this criterion, there may or may not be cointegration. I will count in the evidence from the Johansen test and side with $r = 1$, i.e., that there is one cointegrating vector in the VECM.

The next step is to test for stability of the rank $r = 1$ of the cointegrating relationship for M1 over the period 1872 to 1997. I first apply the $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ test with the null hypothesis that the rank r is constant at 1 over the whole sample period. The alternative hypothesis is that the rank is greater than 1 for some $\kappa \in \Phi$. The test statistic takes on a value of 54.3. The 1% critical value from MacKinnon et al. is 19.93 and the P value for this statistic is less than .0001. The null hypothesis of a constant cointegration rank over the full sample is therefore decisively rejected in

[13]See Johansen (1995).

favor of the alternative hypothesis that there is more than one cointegrating vector for some κ.

The test for stability of the rank $r = 1$ for M1 against the alternative hypothesis that $r < 1$ leads to a test statistic $\overline{Q}_T$ of .82. According to the critical values presented in Quintos (1998a, p.302) in her Table 1, the null hypothesis of rank constancy against the alternative of a rank less than 1 cannot be rejected even at a 10% level of significance.

The long span of data does not lead to a relationship with constant rank in both directions for the models as specified so far. I therefore carry out a sensitivity analysis. First, I restrict the coefficient of income to one and reapply the above tests for M1. Qualitative results are unchanged. Second, I start the sample in 1914 instead of in 1872. Again, I find with the PIC (and the Johansen trace test) one cointegrating vector for M1. However, the rank is not stable. The $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ equals 38.3. On the other hand, $\overline{Q}_T$ equals 1.16 and therefore indicates no instability at usual significance levels.

Third, I include in the money demand specification a variable to measure the difference between the own yield of money and the market interest rate, as suggested by Friedman and Schwartz (1982, pp. 270–271). This variable is defined as ln[ltir×(high powered money/total money)]. Tests detect for M1 one cointegrating vector but the rank is again unstable in the same direction: $\sup_{\kappa\in\Phi} Q_T^+(\kappa) = 57.9$, whereas $\overline{Q}_T = .34$.

Fourth, Siklos (1993) has argued for a money demand specification that includes variables to capture institutional and technological change in order to account for instabilities in the money demand function.[14] In particular, he has found two variables useful in addition to income and interest rates. One is the ratio of nonbank financial assets to total financial assets. It is supposed to capture financial sophistication. The other variable is the ratio of currency to money which is supposed to mirror the spread of commercial banking. I use M2 velocity and real per capita permanent

[14] See also Mulligan (1997).

income in addition to these two variables so that the specification is identical to that of Siklos. All these data are from Siklos and cover the period 1900 to 1986. The results with the Johansen tests and the PIC support the finding of one cointegrating vector in this model, as found by Siklos. However, the Quintos test rejects stability for both rank stability tests ($\sup_{\kappa\in\Phi} Q_T^+(\kappa) = 183.4$ and $\overline{Q}_T = 398.8$).

The results for the period from 1872 to 1997 suggest that M0 and M2 do not lead to a cointegrating relation for a standard money demand function. The results for M1 are more promising. The Johansen test and the PIC indicate both a strong cointegrating relation with $r = 1$ for M1. This relation is robust to changing the starting date from 1872 to 1914, to using a modified long–term interest rate that accounts for the own yield of money, and to using a velocity specification.

The cointegration–rank stability for M1 gives mixed results. The support for a cointegration rank of at least $r = 1$ is very solid. On the other hand, the null hypothesis that $r = 1$ is rejected against the alternative that $r > 1$. This implies that at some point in time an additional cointegrating vectors appears in the VECM system. This is not detrimental for the money demand relation with M1. It does not negate the cointegrating vector for money demand.[15] What this finding does call into question is further inference based on the VECM. A properly specified VECM requires a stable rank. Otherwise, tests for restrictions on cointegrating vectors or tests for weak and super exogeneity will be biased. Single equation methods for analyzing cointegrating relationships potentially avoid this problem.

The overall finding with the long span of data is that the cointegration rank is not stable in at least one direction. I therefore analyze the post–WWII data for stability. I take first annual data from 1957 to 1997 and repeat all the above tests for this shorter span. M0 and M2 lead to cointegrating relations ($r = 2$ and $r = 1$, respectively), using the PIC. However, the rank is not stable either in either direction ($\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ is equal to 66.5 and 42.4, and $\overline{Q}_T$ is equal to 7.0 and 41.1). M1 leads

[15]In contrast, the specification with M2 and institutional variables in the model is unstable in both directions and cointegration vanishes at times, based on the $\overline{Q}_T$ test.

to a relation with two cointegrating vectors. The Schwarz criterion picks one lag, however the PIC chooses no lags instead. The PIC is minimized for $r = 2$ and $p = 0$. The $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ is equal to 51.5 and $\overline{Q}_T$ to 15.7 and the null hypotheses are rejected for both tests, however, this finding does not exclude the possibility that $r = 1$ at a minimum.

Quarterly data are available for the period 1957 to 1997. The results with quarterly data are likely to improve the power of the cointegration tests compared to the results with annual data over the same time span.[16] I repeat the above analysis with quarterly data using the long–term interest rate, and additionally the T–bill rate. For M0 and M2, the PIC rejects cointegration when the long–term interest rate is used, even though the Johansen trace test detects one cointegrating vector in each model. M1 with the long–term interest rate leads to one cointegrating vector, using the PIC. The Johansen results are given in Table 2 and confirm $r = 1$. But, the rank is not stable: $\sup_{\kappa\in\Phi} Q_T^+(\kappa) = 76.5$. On the other hand, $\overline{Q}_T = 1.0$, so that the rank is not less than 1.

The results with respect to stability change somewhat once the long–term interest rate is replaced by the short–term T–bill rate. The PIC detects one cointegrating vector for M0, M1, and M2 each. Though, rank stability is not achieved. The $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ statistic takes on values of 79.1, 110.1, and 64.1, and $\overline{Q}_T$ =.81, 11.8, and 17.8, respectively. Therefore, the result with M0 and the short rate provides some qualified support for a cointegrated money demand function because $r = 1$ is not rejected against the alternative $r < 1$.

It is of interest to study whether the recently published M1++ and M2++ measures of money improve results. These series are available from 1968 on. The quarterly models with these money measures, using the long–term interest rate, do not lead to cointegration according to the PIC. Johansen's trace test would have detected one cointegrating vector in each model. Using instead the T–bill rate leads to no cointegration with the PIC for M1++. Johansen's trace test would have suggested

[16]See Haug (2002).

$r = 1$ and produced a wrong sign for the coefficient on the T–bill rate. The PIC supports one cointegrating vector for M2++ using the T–bill rate. However, the $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ and $\overline{Q}_T$ statistics produce highly significant values of 62.6 and 217.0. Therefore, the new measures of money do not lead to a more stable specification of a cointegrated money demand function.

Several empirical studies of money demand have included inflation as an additional explanatory variable.[17] I therefore include inflation in addition to income and the long–term interest rate. For quarterly data, I try again the short–term T–bill rate as an alternative to the long–term rate. Inflation is supposed to measure the yield on non–financial assets (goods) as an alternative to holding money. I test for a unit root in the inflation rate. The augmented Dickey Fuller and the Phillips Perron tests reject a unit root at the 5% level for the long span of data from 1872 or 1914 to 1997. The same holds true for annual and quarterly post–WWII data as far as the Phillips Perron test is concerned. The augmented Dickey Fuller test does not reject a unit root for post–WWII data. Even though the unit root evidence is not conclusive, I carry out the other tests for the post–WWII period. The PIC detects cointegration in the annual data from 1957 to 1997 only for M2. Johansen's sequential tests instead detect cointegration in addition for M0 and M1. The cointegrating relationship for M2 is not stable in either direction (the $\sup_{\kappa\in\Phi} Q_T^+(\kappa) = 93.5$ and $\overline{Q}_T = 62.5$). The PIC detects cointegration for M0, M1, and M2 in the quarterly data, regardless of whether the long– (r=1, 3, and 1, respectively) or short–term interest rate (r=2, 3, and 3, respectively) is used. However, rank stability is again rejected in all cases.[18] Using M1++ or M2++ instead for the period of availability (1968:1-1997:4) does not lead to stability either. The PIC detects one cointegrating vector in each case, regardless of the interest rate used. Rank stability is rejected in all cases in both directions.

[17] See, e.g., Ericsson (1998).

[18] $\sup_{\kappa\in\Phi} Q_T^+(\kappa)$ is equal to 137.5, 158.0, and 95.5 for the long rate, and to 121.1, 46.3, and 76.1 for the short rate. $\overline{Q}_T$ is equal to 5.6, 221.8, and 10.3 for the long rate, and to 12.1, 46.3, and 76.1 for the short rate.

Furthermore, all quarterly data produce an incorrect coefficient sign for either the interest rate or the (annualized) inflation rate, with the exception of M2++.

The rank stability results for the short span of data from 1957 on are consistent with the results for the long span of data from 1872 on. Both data sets support a cointegration rank for at least $r = 1$ when M1 and the long—term interest rate are used. The short–term interest rate leads to different results. Only M0 exhibits a cointegration rank of at least $r = 1$ when the T-bill measures the opportunity costs of holding money.

The empirical findings in this Section show that the PIC of Chao and Phillips gives results that are in line with theoretical predictions that Johansen's sequential tests possibly overestimate the cointegration rank. This finding is only prevalent in shorter spans of data like the post–WWII period. Longer spans of data do not produce differences between the PIC and Johansen's sequential method as far as the cointegration rank is concerned. In addition, the PIC chooses a lag order for the vector error–correction model below that of the Schwarz criterion for many short as well as for a few long spans of data.

5. Conclusion

This paper examined the Canadian money demand relationship with historical data starting in 1872. It focused on the reliability of the trace test for the cointegration rank in vector error-correction models (VECMs) and on the stability of the cointegration rank. Chao and Phillips' (1999) posterior information criterion (PIC) and Quintos' (1998a) cointegration–rank stability tests were applied.

The cointegration tests results for the long span of data are not sensitive to the method used in order to determine the cointegration rank, however, the postwar period reveals some significant differences between the trace test and the PIC. The results for the stability of the cointegration rank in the VECM suggest that at least one cointegrating vector is present when a money demand specification with M1 and the long–term interest rate is used, regardless of the data spans considered. For the postwar period, a specification with M0 and the short–term interest rate also supports at least one cointegrating vector.

This paper supplements numerous other studies that have considered parameter stability for a money demand relationship but have not tested for cointegration–rank stability.[19] A stable cointegration rank is essential for further inference within a VECM in order to avoid biased results. Due to the uncovered instabilities in the VECM setting, it seems preferable on this account to employ single–equation based methods for cointegration analysis instead of full–system based methods as the VECM.

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[19]See for example Haug and Lucas (1996).

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TABLE 1

Johansen Trace Test for Cointegration and Cointegrating Vector Estimate
1872 –1997: ln(rM1) on ln(rGNP) and ln(ltir)

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	P-Value	Hypothesized No. of Vectors
0.21	37.18	29.80	0.006	None
0.06	7.39	15.49	0.53	At most 1
0.001	0.16	3.84	0.69	At most 2

Normalized Vector:

ln(rM1)	ln(rGNP)	ln(ltir)	Constant
1	-1.04	0.96	5.40

TABLE 2

Johansen Trace Test and Cointegrating Vector Estimate
1957:1-1997:4: ln(rM1) on ln(rGNP) and ln(ltir)

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	P-Value	Hypothesized No. of Vectors
0.11	32.12	29.80	0.03	None
0.07	12.44	15.49	0.14	At most 1
0.0009	0.14	3.84	0.71	At most 2

Normalized Vector:

ln(rM1)	ln(rGNP)	ln(ltir)	Constant
1	-1.24	1.25	2.90