On the Implications of Different Cash-in-Advance Constraints With Endogenous Labour

by

Arman Mansoorian
Department of Economics
York University
North York, ON
Canada, M3J 1P3

and

Mohammed Mohsin
Department of Economics
University of Tennessee
Knoxville, TN 37996
U.S.A.

Abstract

It is shown that, in contrast to models with fixed labour, a change in monetary policy involving an increase in the inflation rate would have the same qualitative effects on steady state capital, consumption, and employment, regardless of whether only consumption or both consumption and investment are subject to Cash-in-Advance (CIA) constraints. The dynamics of the two models regarding employment and capital are also very similar qualitatively. Only the dynamics of consumption are slightly different. Some numerical analysis is also carried out to gauge the quantitative difference between the two models.

JEL Classification: E22, E52, E58
Keywords: Monetary Policy, Cash-in-Advance, Employment, Capital Accumulation

----

Financial support from the Social Sciences and Humanities Research Council of Canada (grant # 410-97-1212) is gratefully acknowledged.

I. Introduction

One of the long-standing issues in Monetary Economics is the debate on the effects of changes in the rate of growth of money on investment and the capital stock. The two prominent models which have been used to discuss this issue are the money-in-utility model (Sidrauski, 1967) and the cash-in-advance model (CIA) (Stockman, 1981, and Able, 1985). These two alternative models give rise to sharp and contrasting results.

With Sidrauski's money-in-utility formulation an increase in the rate of growth of money will have no steady state effects, because the equality of the rate of time preference and the marginal productivity of capital dictates the level of the capital stock which must be maintained in the steady state. The steady state capital stock is, therefore, unaffected by changes in the rate of growth of money (money is super-neutral in the steady state).[1]

With the CIA formulation the results are sensitive to the assumptions made regarding the CIA constraints. If the CIA constraints are on consumption alone, then changes in the rate of growth of money will have no steady state effects. Higher inflation will increase the cost of current consumption. Higher inflation will also increase the cost of future consumption arising from the dividend payment on titles to capital. On net, the trade-off between current and future consumption is unaffected, and, therefore, there are no effects on steady state capital.[2]

On the other hand, if there are CIA constraints on all transactions, including transactions involving assets, then an increase in the rate of growth of money will reduce the steady state capital stock. The reason is that, then, the higher inflation will act as a tax on the purchases of assets, reducing the real rate of return on assets and the steady state capital.

---

[1] Nevertheless, Fischer (1979) has shown that there are significant real effects along the adjustment path to the steady state.

[2] As with money-in-utility, there will be some off steady state effects.

It is important to establish how robust these results are, by relaxing some of the assumptions made in their derivations. One important assumption is the abstraction from labour/leisure choice. Turnovsky (2000, pp. 264-268) considers the Sidrauski model with labour/leisure choice. His analysis shows that the long run super-neutrality of Sidrauski holds if the instantaneous utility function has the common form of Cobb-Douglas utility, or strong separability between consumption and leisure on the one hand, and real balances on the other.

To date, the effects of labour/leisure choice on the Stockman and Abel results have not been fully worked out. The reason could be that the CIA constraints have traditionally been employed in discrete time frameworks. In that setting, the analysis tends to be very cumbersome, which has given rise to the general view that CIA constraints are relatively hard to work with. Thus, according to Turnovsky (1997, p. 20), "One difficulty with this [CIA] approach is that the introduction of the various constraints, embodying the role played by money in transactions, can very quickly become intractable." (See also Turnovsky (2000, p. 264)). Similarly, Blanchard and Fischer (1989, p. 155) state that "Models based explicitly on (CIA) constraints . . . can quickly become analytically cumbersome. Much of the research on the effects of money has taken a different shortcut, that of . . . putting real money services directly in the utility function."

In this paper we consider the CIA constraints in a continuous time setting. Further, we assume that the central bank targets the inflation rate (not the rate of growth of money *per se*).[3] This precludes complicated, yet no so crucial, off steady state effects, similar to those analyzed by Fisher (1979). It, thus, reduces the dimensions of the dynamic system corresponding to the model.[4] It is

3 This assumption is consistent with the assumptions in the literature concerned with the time consistency of monetary policy (e.g., Kydland and Prescott (1977), Backus and Drifill (1985), and Walsh (1995)), where it is also assumed that the central bank targets the inflation rate (not the rate of growth of money *per se*). Recently, Mishkin (2000) also argued that the central banks of most developed as well as emerging countries do indeed target the inflation rate rather than the rate of growth of money.

4 When the central bank targets the rate of growth of money, instead, the inflation rate will be endogenous,

well known that the steady state policy effects are the same, regardless of whether the central bank fixes the rate of growth of money or the inflation rate. With these two assumptions (continuous time setting and inflation rate targeting) a considerable amount of tractability is attained, enabling us to derive the full dynamics of the model.

We show that the steady state effects of the an increase in the inflation rate on the steady state levels of the important macroeconomic variables are qualitatively the same, regardless of whether the CIA constraint is on consumption alone, or on both consumption and investment. In both cases, the steady state levels of these variables fall. Hence, with endogenous labour supply the discussion of the qualitative effects of policy changes on the steady state equilibrium, which was central in the CIA models with fixed labour, is a rather mute point. The focus now should be on the dynamics, and the quantitative analysis of the steady state effects.

In both models, the increase in inflation will, by increasing the relative price of consumption, will lead to a substitution of leisure for consumption. With CIA constraint on investment as well as on consumption, there is an additional effect, because the higher inflation acts as a tax on investment. This tends to reduce the return on investment, tilting the consumption and leisure profile towards the present. Hence, an increase in the inflation rate leads to a larger fall in the steady state levels of capital, employment and consumption when the CIA constraint is on consumption as well as on investment.

The dynamic adjustments of the important macroeconomic variables are also qualitatively very similar. On impact, there will be a substantial fall in employment in both models, as the representative agent substitutes consumption for leisure. The fall in employment is substantially more with CIA on consumption and investment, as then the increase in the inflation rate directly

---

and variable off the steady state. This then increases the dimension of the dynamic system.

reduces the return on investment, tilting the leisure profile to the present. The fall in employment would reduce the marginal productivity of capital, reducing capital accumulation. Hence, capital will be falling along the adjustment path to the new steady state.

With capital falling along the adjustment path, there will be two competing effects on employment along the adjustment path. First, with capital falling the marginal productivity of labour will be falling, which tends to reduce employment along the adjustment path. Second, with capital falling, the marginal productivity of capital will be rising along the adjustment path. This tends to increase employment over time, through the intertemporal substitution of leisure. In both cases, the second effect dominates, and employment rises over time along the adjustment path.

Consumption is the only important macroeconomic variable whose adjustment along the optimum path is different for the two models. Immediately after the increase in the inflation rate, in both models there is a fall in consumption, because the representative agent substitutes leisure for consumption. Along the adjustment path, with falling capital, there are two forces impinging on the adjustment of consumption. First, with falling capital, there will be falls in the wage rate, inducing the representative agent to substitute consumption for leisure. Second, with falling capital, there will be increases in the marginal productivity of capital, which would tend to tilt the consumption profile towards the future. With CIA constraint on consumption alone the first effect dominates; and consumption falls along the adjustment path. With CIA constraint on investment as well as consumption, the second effect dominates; and consumption rises along the adjustment path.

The paper is organized as follows. The model with CIA on both consumption and investment is presented in Section II. The model with CIA on consumption alone is presented in Section III. The effects of an increase in the inflation rate on the two models are compared and contrasted in Section IV. Some concluding remarks are made in Section V.

II. The Model with CIA on Consumption and Investment

The economy is modeled as a representative worker-entrepreneur with an infinite planning horizon. He chooses his consumption, *c*, labor supply, *l*, and holding of capital, *K*, in order to maximize

$$\int_0^\infty u(c_t, l_t) e^{-\beta t} dt \tag{1}$$

where $u(c_t, l_t) = U(c_t) + V(l_t)$, with $U'(c_t) > 0$, $U''(c_t) < 0$, $V'(l_t) < 0$ and $V''(l_t) < 0$.

Output is produced with a standard neoclassical constant return to scale production function $Y_t = F(K_t, l_t)$ with the following properties: $F_K > 0$, $F_l > 0$, $F_{KK} < 0$, $F_{ll} < 0$, $F_{Kl} > 0$, and $F_{ll}F_{KK} - F_{Kl}^2 = 0$. The agent also receives monetary transfer with real values of τ from the government. There are two kinds of assets in the model, money balances (*m*) and capital (*K*). The total real value of the assets held by the representative agent is a_t:

$$a_t = K_t + m_t. \tag{2}$$

His flow budget constraint is

$$\dot{K}_t + \dot{m}_t + \varepsilon_t m_t = F(K_t, l_t) + \tau_t - c_t,$$

assuming that capital does not depreciate. Using (2), we can re-write this equation as

$$\dot{a}_t = F(K_t, l_t) + \tau_t - \varepsilon_t m_t - c_t. \tag{3}$$

The agent also has the No Ponzi game condition

$$\lim_{t\to\infty} e^{\int_0^t r_v dv} a_t \geq 0. \tag{4}$$

Money is introduced through a cash in advance (CIA) constraint. In this section we assume that the agent will demand money for both his consumption and investment expenditures:

$$m_t \geq c_t + \dot{K}_t \qquad \forall t. \tag{5}$$

As money does not yield utility directly and as the return on bonds completely dominates the return on money, equation (5) will always hold with strict equality:

$$\dot{K}_t = m_t - c_t . \tag{6}$$

Therefore, the representative agent's problem is to maximize (1) subject to (2)–(4), (6) and the initial conditions a_0 and K_0. The Hamiltonian for the representative agent's problem is

$$\mathrm{H} = U(c) + V(l) + \lambda\big[F(K,l) + \tau - c - \varepsilon(a - K)\big] + \mu\big[a - K - c\big].$$

The optimality conditions are[5]

$$U'(c) = \lambda + \mu \tag{7}$$

$$V'(l) = -\lambda F_l(K,l) \tag{8}$$

$$\dot{\lambda} = \lambda(\beta + \varepsilon) - \mu \tag{9}$$

$$\dot{\mu} = \mu(1 + \beta) - \lambda\big[F_K + \varepsilon\big] \tag{10}$$

and the standard transversality conditions

$$\lim_{t\to\infty} a_t \lambda e^{-\beta t} = 0$$

$$\lim_{t\to\infty} K_t \mu e^{-\beta t} = 0 .$$

Now note that, from (7) and (8), the equilibrium levels of c and l can be represented by the following equations:

$$c_t = c(\overset{-}{\lambda_t}, \overset{-}{\mu_t}) \tag{11}$$

$$l_t = l(\overset{+}{\lambda_t}, \overset{+}{K_t}) \tag{12}$$

with $c_\lambda < 0, c_\mu < 0, l_\lambda > 0$ and $l_K > 0$.[6]

---

[5] Clearly, in making these optimal decisions the representative agent takes the values of τ_t and ε as given to it exogenously.

The government side of the model is kept as simple as possible. As mentioned in the introduction, we assume that the government chooses the lump sum transfers τ_t in order to maintain the inflation rate ε_t at a constant level ε, according to its flow constrain

$$\dot{m}_t + \varepsilon m_t = \tau_t. \tag{13}$$

The right hand side of equation (13) is total government expenditures, while the left hand side is total government revenue from seigniorage.

We are now in a position to work out the dynamics of the model with rational expectations. To this end, first note that from equations (3) and (13) we obtain the product market clearing condition:

$$F(K_t, l_t) = c_t + \dot{K}_t \tag{14}$$

The dynamics of the economy are obtained by substituting for *c* and *l* from (11) and (12) into (9), (10), and (14). This gives

$$\dot{\lambda} = \lambda(\beta + \varepsilon) - \mu \tag{15}$$

$$\dot{\mu} = \mu(1+\beta) - \lambda\left[F_K(K, l(K,\lambda)) + \varepsilon\right] \tag{16}$$

$$\dot{K} = F(K, l(K,\lambda)) - c(\lambda,\mu). \tag{17}$$

To study the transitional dynamics linearize (15)–(17) around the steady state to obtain

$$\begin{bmatrix} \dot{\lambda} \\ \dot{\mu} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & -1 & 0 \\ \Phi_{21} & \Phi_{22} & \Phi_{23} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} \end{bmatrix} * \begin{bmatrix} \lambda - \tilde{\lambda} \\ \mu - \tilde{\mu} \\ K - \tilde{K} \end{bmatrix} \tag{18}$$

---

[6] These partial derivatives are as follows: $\frac{\partial c}{\partial \lambda} = \frac{1}{U''(c)} < 0$, $\frac{\partial c}{\partial \mu} = \frac{1}{U''(c)} < 0$, $\frac{\partial l}{\partial \lambda} = \frac{-F_l}{V''(l) + \lambda F_{ll}} > 0$ and $\frac{\partial l}{\partial K} = \frac{-\lambda F_{lK}}{V''(l) + \lambda F_{ll}} > 0$.

where tildes denote steady state values, $\Phi_{11} = \beta + \varepsilon\,(>0)$, $\Phi_{21} = -\left[F_K + \varepsilon + \lambda F_{Kl} l_\lambda\right](<0)$,

$\Phi_{22} = (1+\beta)\,(>0)$, $\Phi_{23} = -\left[F_{KK} + F_{Kl} l_\lambda\right](>0)$, $\Phi_{31} = F_l l_\lambda - c_\lambda\,(>0)$, $\Phi_{32} = -c_\mu\,(>0)$, and

$\Phi_{33} = F_K + F_l l_K\,(>0)$.

The stable path of the system is given by the following equations:[7]

$$K_t - \widetilde{K} = (K_0 - \widetilde{K})e^{\xi t} \tag{19}$$

$$\lambda_t - \widetilde{\lambda} = -\frac{\Phi_{23}}{\Phi_{21} + (\Phi_{22} - \xi)(\Phi_{11} - \xi)}(K_0 - \widetilde{K})e^{\xi t} \tag{20}$$

$$\mu_t - \widetilde{\mu} = -\frac{\Phi_{23}(\Phi_{11} - \xi)}{\Phi_{21} + (\Phi_{22} - \xi)(\Phi_{11} - \xi)}(K_0 - \widetilde{K})e^{\xi t} \tag{21}$$

where ξ is the negative eigenvalue of the coefficient matrix in (18).

Now, Linearizing (11)–(12) and using (19)–(21), we obtain the adjustments of consumption and employment along the optimum path:

$$c_t - \widetilde{c} = -\frac{c_\mu \Phi_{23}(\Phi_{11} - \xi) + c_\lambda \Phi_{23}}{\Phi_{21} + (\Phi_{22} - \xi)(\Phi_{11} - \xi)}(K_0 - \widetilde{K})e^{\xi t} \tag{22}$$

$$l_t - \widetilde{l} = \left(l_K - \frac{l_\lambda \Phi_{23}}{\Phi_{21} + (\Phi_{22} - \xi)(\Phi_{11} - \xi)}\right)(K_0 - \widetilde{K})e^{\xi t}. \tag{23}$$

In the following section we will derive the adjustment paths of the important macroeconomic variables for the case in which the CIA constraint is on consumption expenditures alone. After that we will compare the effects of an increase in the inflation rate in the two models.

---

[7] Since the above system has two jump variables and one predetermined variable, the stability condition requires that the system have one negative and two positive eigenvalues. The determinant of the above matrix is negative and so the product of three eigenvalues is negative. This indicates that either one or all the three roots have negative real parts. Now consider the trace of the matrix. Trace = $\Phi_{11} + \Phi_{22} + \Phi_{33} > 0$. Since the trace is the sum of the characteristic roots of the system; it being positive implies that at least one of the roots must be positive. Therefore, only one of the roots has a negative real part; and the steady state of the system exhibits saddle point stability.

III. The Model with CIA on Consumption Alone

In this section we consider the model with the alternative assumption that only consumption expenditures are subject to CIA constraints.[8] The CIA constraint in this case will be

$$m_t \geq c_t \qquad \forall t. \tag{24}$$

The agent's problem is to maximize (1) subject to (2)–(4), (24) and the initial condition a_0. Again, as money does not yield utility directly and as the return on bonds completely dominates the return on money, equation (24) will always hold with strict equality. Hence, m_t is residually determined once c_t is chosen. Thus, setting $m_t = c_t$ and also using (2), we can write the Hamiltonian for the representative agent's problem as

$$\mathrm{H} = U(c) + V(l) + \lambda\left[F(a-c,l) + \tau - c - \varepsilon c\right].$$

The optimality conditions for this problem are:

$$U'(c) = \lambda(1 + \varepsilon + F_K(K,l)) \tag{25}$$

$$V'(l) = -\lambda F_l(K,l) \tag{26}$$

$$\dot{\lambda} = \lambda(\beta - F_K(K,l)) \tag{27}$$

and the transversality condition

$$\lim_{t\to\infty} a_t \lambda_t e^{-\beta t} = 0.$$

Next, note that from (25) and (26), the equilibrium c and l will be represented by the following equations:

$$c_t = c(\overset{+}{K_t}, \overset{-}{\lambda_t}, \overset{-}{\varepsilon_t}) \tag{28}$$

---

[8] This case is also considered in Mansoorian and Mohsin (2001). In that paper the effects of inflation rate and nominal interest rate targeting are compared. Also in that paper the effects of monetary policy on the term structure of interest rates are discussed.

$$l_t = l(\overset{+}{K_t}, \overset{+}{\lambda_t}) \tag{29}$$

where $c_K > 0$, $c_\lambda < 0$, $c_\varepsilon < 0$, $l_K > 0$ and $l_\lambda > 0$.

As in the previous section, we assume that the government chooses the real lump sum transfers τ according to (13) in order to maintain the inflation rate ε at a constant level. Hence, from equations (3) and (13), we obtain the product market clearing condition:

$$F(K_t, l_t) = c_t + \dot{K}_t. \tag{30}$$

Next, substituting for c_t and l_t from (28) and (29) into (27) and (30), we obtain

$$\dot{K}_t = F(K_t, l(K_t, \lambda_t)) - c(K_t, \lambda_t, \varepsilon) \tag{31}$$

$$\dot{\lambda}_t = \lambda[\beta - F_K(K, l(K_t, \lambda_t)]. \tag{32}$$

These two equations jointly determines the dynamics of K and λ. Linearizing them around the steady state, we obtain

$$\begin{bmatrix} \dot{K}_t \\ \dot{\lambda}_t \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} * \begin{bmatrix} (K_t - \widetilde{K}) \\ (\lambda_t - \widetilde{\lambda}) \end{bmatrix}, \tag{33}$$

where, $\Lambda_{11} = F_K + F_l l_K - c_K$ (>,=,< 0), $\Lambda_{12} = F_l l_\lambda - c_\lambda$ (> 0), $\Lambda_{21} = -\lambda[F_{KK} + F_{Kl} l_K]$(>0), and $\Lambda_{22} = -\lambda F_{Kl} l_\lambda$(<0).

As K is predetermined while λ is a jump variable, for saddlepoint stability the determinant of the coefficient matrix in (33) should be negative. This condition will be satisfied with very mild assumptions. The equation of the saddle path is given by the following equations:

$$K_t - \widetilde{K} = (K_0 - \widetilde{K})e^{\varsigma t} \tag{34}$$

$$\lambda_t - \widetilde{\lambda} = \left(\frac{\varsigma - \Lambda_{11}}{\Lambda_{12}}\right).\left(K_0 - \widetilde{K}\right)e^{\varsigma t}, \tag{35}$$

where ς is the negative eigenvalue of the coefficient matrix in (33).

Linearizing (28)–(29), and using (34)–(35), we obtain the adjustments of consumption and employment along the optimum path:

$$c_t - \tilde{c} = \left(c_K + c_\lambda \left(\frac{\varsigma - \Lambda_{11}}{\Lambda_{12}} \right) \right) (K_0 - \tilde{K}) e^{\varsigma t} \tag{36}$$

$$l_t - \tilde{l} = \left(l_K + l_\lambda \left(\frac{\varsigma - \Lambda_{11}}{\Lambda_{12}} \right) \right) (K_0 - \tilde{K}) e^{\varsigma t} . \tag{37}$$

We are now in a position to compare and contrast the effects of an increase in the inflation rate on the two models.

IV. The Effects of an Increase in the Inflation Rate

To obtain the steady state effects of an increase in the inflation rate on capital, consumption, and employment for the model with CIA constraint on both consumption and investment totally differentiate equations (6)–(10) at the steady state, with $\dot{\mu} = \dot{\lambda} = \dot{K} = 0$. This gives

$$\frac{d\tilde{K}}{d\varepsilon} = \frac{\lambda\beta\{\Phi_{32}\Phi_{11} + \Phi_{31}\} + \lambda\Phi_{32}F_{kl}l_\lambda}{D} < 0 \tag{38}$$

$$\frac{d\tilde{c}}{d\varepsilon} = c_\lambda \frac{\lambda\{\Phi_{23}\Phi_{32} - \beta\Phi_{33}\}}{D} + c_\mu \frac{\lambda\{\Phi_{11}\Phi_{33} + \Phi_{21}\Phi_{33} - \Phi_{31}\Phi_{23}\}}{D} < 0 \tag{39}$$

$$\frac{d\tilde{l}}{d\varepsilon} = l_\lambda \frac{\lambda\{\Phi_{23}\Phi_{32} - \beta\Phi_{33}\}}{D} + l_K \frac{\lambda\beta\{\Phi_{32}\Phi_{11} + \Phi_{31}\} + \lambda\Phi_{32}F_{kl}l_\lambda}{D} < 0 , \tag{40}$$

where D (<0) is the determinant of the coefficient matrix in (18).

Similarly, in order to obtain the steady state effects on capital, consumption, and employment for the model with CIA constraint on consumption alone totally differentiate equations (25)–(27), and (30) at the steady state, with $\dot{\lambda} = \dot{K} = 0$. This gives

$$\frac{d\tilde{K}}{d\varepsilon} = \frac{c_\varepsilon \Lambda_{22}}{\Lambda_{11}\Lambda_{22} - \Lambda_{21}\Lambda_{12}} < 0 \tag{41}$$

$$\frac{d\tilde{c}}{d\varepsilon} = c_K \frac{c_\varepsilon \Lambda_{22}}{\Lambda_{11}\Lambda_{22} - \Lambda_{21}\Lambda_{12}} + c_\lambda \frac{-c_\varepsilon \Lambda_{21}}{\Lambda_{11}\Lambda_{22} - \Lambda_{21}\Lambda_{12}} + c_\varepsilon < 0 \tag{42}$$

$$\frac{d\tilde{l}}{d\varepsilon} = l_K \frac{c_\varepsilon \Lambda_{22}}{\Lambda_{11}\Lambda_{22} - \Lambda_{21}\Lambda_{12}} + l_\lambda \frac{-c_\varepsilon \Lambda_{21}}{\Lambda_{11}\Lambda_{22} - \Lambda_{21}\Lambda_{12}} < 0 \tag{43}$$

From (38)–(43) it is clear that an increase in the inflation rate will have qualitatively the same effect on the steady state levels of capital, employment and consumption. In both models the increase in the inflation rate will increase the cost of consumption relative to leisure. This will reduce the steady state consumption and employment, as the representative agent substitutes leisure for consumption. The fall in employment will reduce the marginal productivity of capital, which in turn will reduce capital accumulation and the steady state capital stock. In the case with CIA constraint on both consumption and investment there will be an additional effect on capital that was also present in the Stockman-Able setting without labour-leisure choice. The higher inflation will, in that model, act as a tax on the return on capital, which tends to reduce capital accumulation, and the steady state capital.

Hence, with endogenous labour supply the discussion of the qualitative effects of policy changes on the steady state equilibrium, which was central to the CIA models with fixed labour, is a rather mute point. The focus now should be on the quantitative analysis of the steady state effects, and the dynamics of the two models.

To execute a quantitative analysis of the model we follow the Real Business Cycle (RBC) literature and assume that instantaneous utility is logarithmic, $U(c_t, l_t) = (1-\alpha)\ln c_t + \alpha \ln(1-l_t)$, and the production function is Cobb-Douglas, $Y_t = K_t^\theta l_t^{1-\theta}$. The values of the coefficients we choose are also taken from the RBC literature (Cooley, 1995, chapter 1). Hence, we set $\alpha = 0.64$, $\theta = 0.3$ and $\beta = 0.042$.

In Table 1 we compare the effects of a 1% increase in the inflation rate on the steady state capital stock for the two models. For different initial inflation rates in the range 0% until 10%, a 1% increase in the inflation rate will result in approximately 2% fall in the steady state capital stock when both investment and consumption are subject to CIA constraints, while it results in an approximately 0.66% fall in the steady state capital when only consumption is subject to CIA constraints. The reason for such discrepancy is the fact that with CIA constraint on both consumption and investment the increase in the inflation rate acts as a tax on the return on capital. This reinforces the steady state fall in capital that results from the substitution of leisure for consumption.

In Table 2 we compare the effects of a 1% increase in the inflation rate on steady state consumption for the two models. For different initial inflation rates in the range 0% until 10%, a 1% increase in the inflation rate will result in approximately 1.06% fall in steady state consumption when both investment and consumption are subject to CIA constraints, while it results in an approximately 0.67% fall in steady state consumption when only consumption is subject to CIA constraints. The reason for this difference between the two models is that with CIA constraint on both consumption and investment an increase in the inflation rate directly reduces the return on capital. This reduces savings during the adjustment path to the steady state, which reinforces the steady state fall in consumption resulting from the substitution of leisure for consumption.

In Table 3 we compare the effects of a 1% increase in the inflation rate on steady state employment for the two models. For different initial inflation rates in the range 0% until 10%, a 1% increase in the inflation rate will result in approximately 0.67% fall in steady state employment. Surprisingly, the steady state effects on employment are the same for both models. This is so for different parameter values, as long as logarithmic utility is used. This is due to the functional forms used in the calibration process. One would expect steady state employment to fall by a larger amount when there are CIA constraints on both consumption and investment, because in that case steady state wage rate will be lower, with lower capital.

Figures 1 and 2 describe the adjustments of c, l and K for the two models, assuming an increase in the inflation rate by 1% from an initial inflation rate of 4%.[9] On impact, there will be a substantial fall in employment in both models, as the representative agent substitutes consumption for leisure. The fall in employment is substantially more with CIA on consumption and investment, as then the increase in the inflation rate directly reduces the return on investment, tilting the leisure profile to the present. The fall in employment would reduce the marginal productivity of capital, reducing capital accumulation. Hence, capital will be falling along the adjustment path to the new steady state.

With capital falling, there will be two competing effects on employment along the adjustment path. First, with capital falling the marginal productivity of labour will be falling, which tends to reduce employment along the adjustment path. Second, with capital falling, the marginal productivity of capital will be rising along the adjustment path. This would tend to increase employment over time, through the intertemporal substitution of leisure. In both cases, the second effect dominates, and employment rises over time along the adjustment path.

Consumption is the only important macroeconomic variable whose adjustment along the optimum path is different for the two models. Immediately after the increase in the inflation rate in both models there is a fall in consumption, because the representative agent substitutes leisure for consumption. Along the adjustment path, with falling capital, there are two forces impinging on the adjustment of consumption. First, with falling capital, there will be falls in the wage rate, inducing

[9] With CIA constraint on both consumption and investment the initial steady state level of consumption, capital stock and labour supply are 0.5990789, 3.9548438 and 0.2668134, respectively. With CIA constraint on consumption alone the initial steady state level of consumption, capital stock and labour supply are 0.6196592, 4.4261371 and 0.26681348, respectively.

the representative agent to substitute consumption for leisure. Second, with falling capital, there will be increases in the marginal productivity of capital, which would tend to tilt the consumption profile towards the future. With CIA constraint on consumption alone the first effect dominates; and consumption falls along the adjustment path. With CIA constraint on investment as well as consumption, the second effect dominates; and consumption rises along the adjustment path.

V. Conclusion

In this paper we have considered the effects of monetary policies in economies with endogenous labour supply in which money is introduced through CIA constraints. A considerable amount of tractability was attained by using a continuous time framework and by assuming that monetary policy was directed at controlling the inflation rate.

We compared and contrasted the effects of monetary policies in a model in which the CIA constraint was on both investment and consumption, with the case in which the CIA constraint was on consumption alone. It was shown that an increase in the inflation rate had qualitatively the same effect on the steady state levels of capital, employment and consumption. Only the dynamic adjustments of consumption were different for the two models. Some numerical evaluation of the two models were carried out in order to gauge the quantitative differences between them.

References

Abel, Andrew B. (1985): "Dynamic Behaviour of Capital Accumulation in a Cash-in-Advance Model." *Journal of Monetary Economics*, 16: 55-71.

Backus, D., and J. Driffill (1985): "Inflation and Reputation." *American Economic Review*: 530-538.

Blanchard, O., and S. Fischer (1989): *Lectures on Macroeconomics*. Cambridge, M.A.: MIT Press.

Cooley, T. (1995): *Frontiers of Business Cycle Research.* Princeton, N.J.: Princeton University Press

Fischer, S. (1979): "Capital Accumulation on the Transition Path in a Monetary Optimizing Model." *Econometrica*, 47: 1433-1439.

Kydland, F., and E. Prescott (1977): "Rules Rather than Discretion: the Inconsistency of Optimal Plans." *Journal of Political Economy*: 473-492.

Mansoorian, A., and M. Mohsin (2001): "Monetary Policy in a Cash-in-Advance Economy: Employment, Capital Accumulation and the Term Structure of Interest Rates." Mimeo: Department of Economics, York University.

Mishkin, Frederic S. (2000): "Inflation Targeting in Emerging-Market Countries." *American Economic Review Papers and Proceedings*, 90: 105-109.

Sidrauski, Miguel (1967): "Rational Choice and Patterns of Growth in a Monetary Economy". *American Economic Review Papers and Proceedings*, 30: 534-44.

Stockman, A.C. (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics.* 8: 387-393.

Turnovsky, Stephen J. (1997): *International Macroeconomic Dynamics*. MIT Press.

Turnovsky, Stephen J. (2000): *Methods of Macroeconomic Dynamics*. MIT Press.

Uzawa, Hirofumi (1968): "Time Preference, the Consumption Function, and Optimal Asset Holdings". In J.N. Wolfe (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks.* (Chicago: Aldine Publishing Company)

Walsh, C. (1995): "Optimal Contracts for Central Bankers." *American Economic Review*: 150-67.

Table 1: % change in steady state capital due to 1% increase in inflation rate

Increase in inflation rate	Model I (CIA on c and $\dot{K}$)	Model II (CIA on c only)
From 0% to 1%	-2.0374	-0.6916
From 1% to 2%	-2.0201	-0.6869
From 2% to 3%	-2.0031	-0.6822
From 3% to 4%	-1.9863	-0.6776
From 4% to 5%	-1.9699	-0.6730
From 5% to 6%	-1.9537	-0.6685
From 6% to 7%	-1.9378	-0.6641
From 7% to 8%	-1.9221	-0.6597
From 8% to 9%	-1.9067	-0.6554
From 9% to 10%	-1.8916	-0.6511

Table 2: % change in steady state consumption due to 1% increase in inflation rate

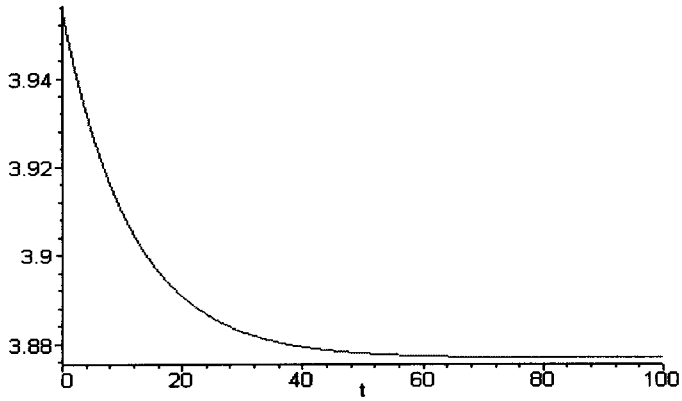
Increase in inflation rate	Model I (CIA on c and $\dot{K}$)	Model II (CIA on c only)
From 0% to 1%	-1.0973	-0.6916
From 1% to 2%	-1.0887	-0.6869
From 2% to 3%	-1.0803	-0.6822
From 3% to 4%	-1.0720	-0.6776
From 4% to 5%	-1.0639	-0.6730
From 5% to 6%	-1.0558	-0.6685
From 6% to 7%	-1.0479	-0.6641
From 7% to 8%	-1.0401	-0.6597
From 8% to 9%	-1.0324	-0.6554
From 9% to 10%	-1.0249	-0.6511

Table 3: % change in steady state employment due to 1% increase in inflation rate

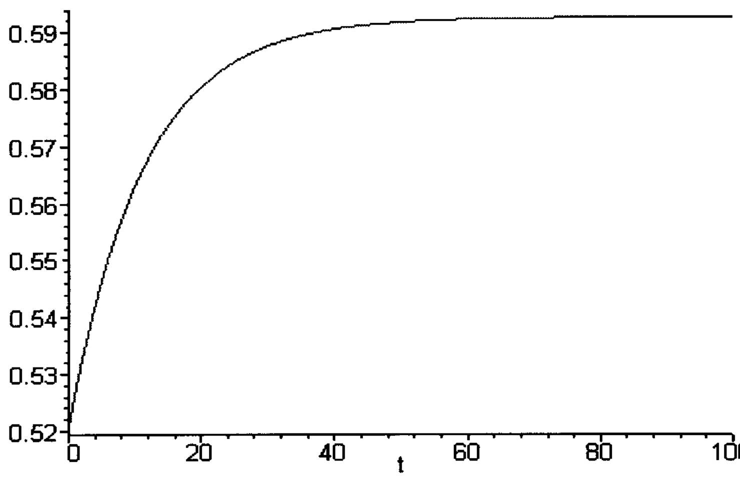
Increase in inflation rate	Model I (CIA on c and $\dot{K}$)	Model II (CIA on c only)
From 0% to 1%	-0.6916	-0.6916
From 1% to 2%	-0.6869	-0.6869
From 2% to 3%	-0.6822	-0.6822
From 3% to 4%	-0.6776	-0.6776
From 4% to 5%	-0.6730	-0.6730
From 5% to 6%	-0.6685	-0.6685
From 6% to 7%	-0.6641	-0.6641
From 7% to 8%	-0.6597	-0.6597
From 8% to 9%	-0.6554	-0.6554
From 9% to 10%	-0.6511	-0.6511

Figure 1: Adjustments of K, c and l with CIA on consumption and investment

The path of K during the adjustment period:



The path of c during the adjustment period (after initial jump at time 0):



The path of l during the adjustment period (after initial jump at time 0):

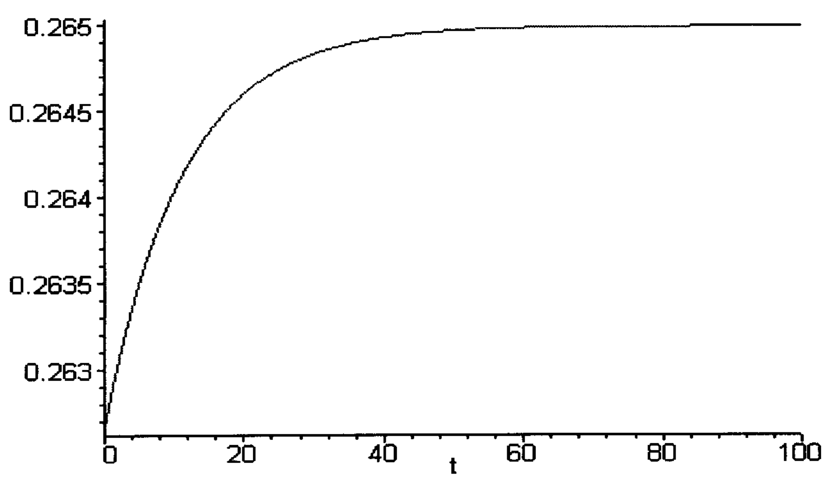
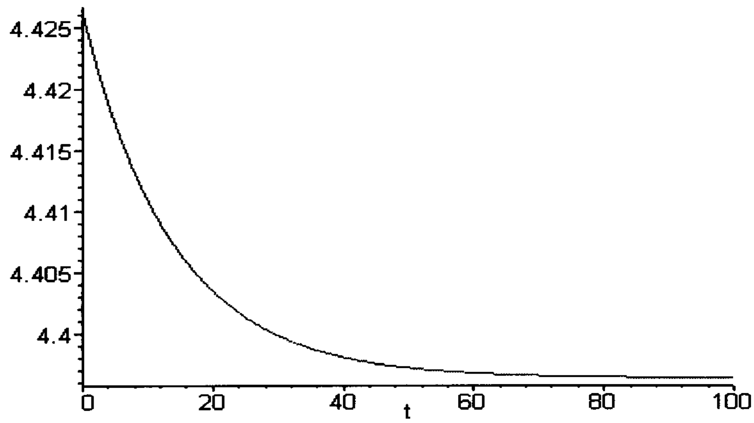
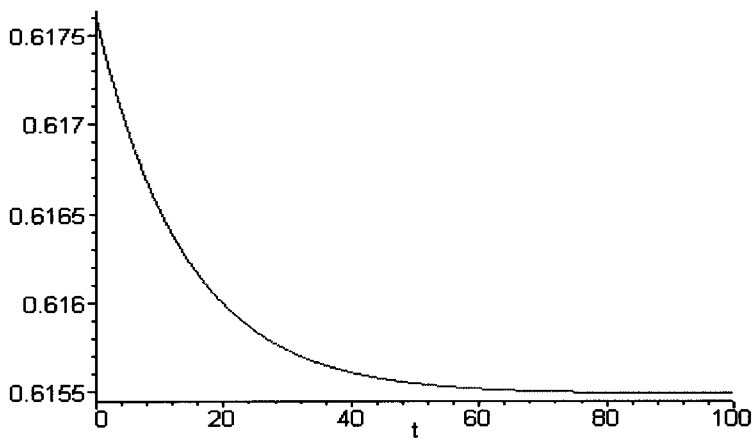


Figure 2: Adjustments of K, c and l with CIA on consumption alone

The path of K during the adjustment period:



The path of c during the adjustment period (after initial jump at time 0):



The path of l during the adjustment period (after initial jump at time 0):