

**The Employment, Investment and Current Account Effects of  
Exchange Rate Policies  
in a Cash-in-Advance Economy**

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Abstract

The effects of exchange rate policies are examined for a small open economy in an infinite horizon model with investment, labor/leisure choice, and cash-in-advance for consumption. An increase in the rate of depreciation of the domestic currency reduces consumption and labor supply, as the higher inflation increases the price of consumption relative to leisure. The fall in labor reduces the marginal productivity of capital, and a fall in investment. The country runs a current account surplus, despite the fall in output. The dynamics of the model for permanent, temporary and anticipated policy changes are fully worked out.

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## I. Introduction

The open economy literature has always shown ample interest in the effects of monetary policy on output, employment and the current account. The traditional literature used the Mundell-Fleming-Dornbusch model in order to work out the various implications of a variety of policy issues for an open economy. Although this approach is still widely used, it has been criticized for its lack of microfoundations.

The seminal papers which consider policy issues in an optimizing framework are by Obstfeld (1981a, 1981b). He assumes that instantaneous utility is a function of consumption and real money holdings, as in Sidrauski (1967). Also, as in Uzawa (1968), the rate of time preference is an increasing function of instantaneous utility. In his (1981b) paper he considers the policy effects for such an economy when the central bank fixes the rate of growth of money. However, the results in Obstfeld (1981a) reveal that the analysis is considerably simplified if one, instead, assumes that the central bank fixes the rate of devaluation of the domestic currency. He shows that in that setting, with perfectly flexible prices, a once and for all devaluation will not have any effect. On the other hand, an increase in the rate of devaluation will lead to a sharp fall in consumption and real money holdings in the short run, leading to a current account surplus. After that, both consumption and real money holdings increase along the adjustment path to the new steady state equilibrium.

These important contributions can be criticized on three grounds. First, as the model deals with an endowment economy, it precludes any discussion of the effects of monetary policy on employment, output, and investment which were the focus of the literature using the Mundell-Fleming-Dornbusch model. Second, the money-in-the-utility-function is used in a relatively narrow set of subfields in Macroeconomics. The cash-in-advance (CIA) approach to introducing

money into a model is more widely used, especially in the empirical asset pricing literature. Third, Uzawa preferences have sometimes been criticized for their assumption that the rate of time preference is an increasing function of instantaneous utility, which is necessary for the stability of the steady state for a small open economy.<sup>1</sup> Obstfeld uses Uzawa preferences for the following reason. In an infinite horizon model with endowments and a small open economy for a steady state to exist one should assume that the rate of time preference is equal to the world rate of interest. This assumption then precludes any dynamics for an endowment economy: after a disturbance the economy jumps to its new steady state.

The present paper attempts to deal with these three weaknesses. We use an infinite horizon model in which the representative household can make labour/leisure decisions. We also allow for investment decisions by the firms, subject to adjustment costs. We assume that households face a CIA constraint on their expenditures.<sup>2</sup> In this setting, changes in the inflation rate will change the price of consumption relative to leisure. This, by changing the labour input in the production process, changes the marginal productivity of capital, and, hence, investment.

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<sup>1</sup> It has been argued by some authors that this assumption is "arbitrary and even counter-intuitive" (Svensson and Razin, 1983, p.45). Blanchard and Fischer (1989, pp. 74- 75) warn their reader that "the Uzawa function with its assumption (that the rate of time preference is increasing in instantaneous utility) is not particularly attractive as a description of preferences and is not recommended for general use."

<sup>2</sup> Other related papers which employ CIA constraints in an open economy setting in order to study policy issues include Calvo and Vegh (1994, 1995) and Edwards and Vegh (1997). Calvo and Vegh do not allow for endogenous output in their (1995) paper. In their (1994) paper they allow for output endogeneity through a Phillips curve, without labour/leisure choice or capital. Edwards and Vegh (1997) allow for labour/leisure choice but not for capital or investment. They also do not derive the full adjustment dynamics. In the present paper, however, we shall derive the full adjustment dynamics. We will see that capital and investment play a crucial role in the adjustment of the model. The focus of the papers cited above are also very different from the current paper's.

With adjustment costs for investment the economy adjusts slowly to its long run equilibrium.

The dynamics of the model are, thus, rich enough even with a fixed rate of time preference.<sup>3</sup>

In accordance with Obstfeld (1981a), Calvo (1981) and Djajic (1982), it is assumed that the central bank targets the rate of change of the exchange rate (not the rate of growth of money *per se*).<sup>4</sup> This precludes complicated, yet no so crucial, off steady state effects, similar to those analyzed by Fisher (1979). It, thus, reduces the dimensions of the dynamic system corresponding to the model, facilitating the use of simple phase diagrams.<sup>5</sup> It is well known that the steady state policy effects are the same, regardless of whether the central bank fixes the rate of growth of money or the inflation rate.<sup>6</sup>

We show that an unanticipated permanent increase in the rate of depreciation of the domestic currency (i.e., the domestic inflation rate) will lead to a fall in consumption, as with CIA constraint on consumption higher inflation increase the price of consumption. The

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<sup>3</sup> The model has much in common with Sen and Turnovsky (1989a, 1989b) and Turnovsky and Sen (1991). They, however, abstract completely from money and monetary policy.

<sup>4</sup> This assumption is consistent with the assumptions in the literature concerned with the time consistency of monetary policy (e.g., Kydland and Prescott (1977), Backus and Drifill (1985), and Walsh (1995)), where it is also assumed that the central bank targets the inflation rate (not the rate of growth of money *per se*). Recently, Mishkin (2000) also argued that the central banks of most developed as well as emerging countries do indeed target the inflation rate rather than the rate of growth of money.

<sup>5</sup> When the central bank targets the rate of growth of money, instead, the rate of change of the exchange rate (i.e., the inflation rate) will be endogenous, and variable off the steady state. It is this fact which increases the dimension of the dynamic system, and precludes the use of simple phase diagrams.

<sup>6</sup> Hence, the assumption that the central bank targets the rate of change of the exchange rate implies that the central bank may possibly be intervening on the foreign exchange market to some extent *only* during the adjustment period before the steady state. Also, as we will see, instead of intervening on the foreign exchange market the central bank could adjust the rate of growth of money appropriately.

representative household substitutes leisure for consumption, reducing labour supply. The resulting fall in labour input in the production process reduces the marginal productivity of capital, leading to a fall in investment. With investment adjustment costs, capital adjusts slowly towards its long run level. As capital falls during the adjustment period, it reduces wages, and labour supply. Hence, along the adjustment path consumption falls, as the representative household substitutes leisure for consumption.

From this, one can also derive the adjustment of output and the current account in response to a permanent unanticipated increase in the rate of depreciation of the domestic currency. The fall in labour input reduces output on impact. Nevertheless, the current account goes into a surplus as the fall in consumption and investment dominate the fall in output. Along the adjustment path output falls, as both capital and labour input fall. The current account also continues to be in surplus until the steady state is reached.

We also derive the effects of an anticipated permanent, and an unanticipated temporary increase in the rate of depreciation of the domestic currency. In particular, it is shown that an unanticipated temporary increase in the rate of depreciation of the domestic currency, as with the unanticipated permanent increase, leads, on impact, to a fall in consumption, an increase in labour supply, a fall in investment, a fall in output, and a current account surplus. Such temporary changes in exchange rate policies have long term effects. The reason, basically, is that the model has two predetermined variables (capital and the net foreign asset position of the country). This means that the initial values of these predetermined variables determine the position of the stable path of the model.<sup>7</sup> Hence, the values of these predetermined variables at

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<sup>7</sup> This is because of hysteresis, or history dependence, which has also been emphasized by Sen and Turnovsky ( 1989a, 1989b, 1990).

the time the policy is revised, dictate the position of the new stable path.

The rest of the paper is organized as follows. The model is presented in section II. The effects of exchange rate policies are presented in section III. Some concluding remarks are made in section IV.

## II. The Model

The model has a lot in common with Sen and Tervovsky (1989a,b), in which money is completely left out of the model. The preferences of the representative household are given by

$$\int_0^{\infty} U(c_t, l_t) e^{-\beta t} dt \quad (1)$$

where  $c$  is his consumption,  $l$  his labor supply and  $\beta$  his (fixed) rate of time preference. The utility function has the following properties:

$$U_c(c, l) > 0, \quad U_l(c, l) < 0, \quad U_{cc}(c, l) < 0, \quad U_{ll}(c, l) < 0 \quad \text{and} \quad U_{cc}U_{ll} - U_{cl}^2 > 0.$$

Money is introduced into the model through a cash in advance constraint on consumption expenditures. Hence, at any point in time the household requires real money balances ( $m_t$ ) in order to finance his expenditures:

$$m_t \geq c_t \quad (2)$$

All domestic firms are owned by the representative household, who receives all their profits  $\pi_t$ . The representative household also receives monetary transfers from the government which has a real value of  $\tau_t$ . He faces the flow budget constraint

$$\dot{m}_t + \dot{b}_t = \pi_t + w_t l_t + r b_t + \tau_t - c_t - \varepsilon_t m_t, \quad (3)$$

where  $w_t$  is the wage rate,  $b_t$  the value of internationally traded bonds he holds,  $\varepsilon_t$  the inflation rate and  $r$  the interest rate, which is fixed abroad. Thus, the household's asset accumulations

$(\dot{m}_t + \dot{b}_t)$  is equal to his net income  $(\pi_t + w_t l_t + r b_t + \tau_t)$  minus his consumption and the inflation tax.

Now define the household's total accumulable assets as  $a_t (\equiv m_t + b_t)$ . Then

$\dot{a}_t = \dot{m}_t + \dot{b}_t$  and (3) can be re-written as

$$\dot{a}_t = r a_t + \pi_t + w_t l + \tau_t - c_t - (\varepsilon_t + r) m_t \quad (4)$$

The representative household also faces the intertemporal solvency condition

$$\lim_{t \rightarrow \infty} e^{-rt} a_t \geq 0 \quad (5)$$

and the initial condition given by his initial asset holdings  $a_0$ .

As money does not yield utility directly, and as the return on bonds completely dominate the return on money, it follows that equation (2) will always hold with strict equality – *i.e.*,  $m_t$  is residually determined once  $c_t$  is chosen. Thus, setting  $m_t = c_t$  in equation (4) we can write the Hamiltonian for the household's problem as

$$H^h = U(c, l) + \lambda [\pi + w l + r a + \tau - c - (r + \varepsilon) c],$$

where  $\lambda$  is a co-state variable.

The optimality conditions for this problem are:

$$H_c^h = 0 \Rightarrow U_c(c, l) = \lambda(1 + r + \varepsilon) \quad (6)$$

$$H_l^h = 0 \Rightarrow U_l(c, l) = -\lambda w \quad (7)$$

$$\lambda \beta - H_a^h = \dot{\lambda} \Rightarrow \lambda(\beta - r) = \dot{\lambda} \quad (8)$$

and the standard transversality condition

$$\lim_{t \rightarrow \infty} e^{-\beta t} \lambda_t a_t = 0. \quad (9)$$

Note that as  $\beta$  and  $r$  are both fixed, from equation (8) for a steady state to exist we will require  $r = \beta$ . This is a standard assumption made in the literature and it implies that  $\lambda$  is always at its steady state level,  $\bar{\lambda}$ .

Now consider the problem of the representative firm. It has the standard neo-classical constant return to scale production function with capital and labor as inputs:

$$Y_t = F(K_t, l_t) \quad (10)$$

where,  $F_K(K, l) > 0$ ,  $F_l(K, l) > 0$ ,  $F_{KK}(K, l) < 0$ ,  $F_{ll}(K, l) < 0$  and  $F_{KK}l_{ll} - F_{Kl}^2 = 0$

The profit function ( $\pi_t$ ) net of investment expenditures is

$$\pi_t = F(K_t, l_t) - w_t l_t - \Phi(I_t) \quad (11)$$

where  $\Phi(I)$  is total costs associated with the investment  $I_t$ :

$$\Phi(I) = I + \Psi(I) \quad (12)$$

where  $\Psi(I)$  are the adjustment costs associated with  $I_t$ . The function  $\Psi(I)$  is assumed to be a non-negative convex function of investment. This convexity implies  $\Psi' \geq 0$  and  $\Psi'' > 0$ . By choice of units we may set  $\Psi(0) = 0$ ;  $\Psi'(0) = 0$ , which implies that  $\Phi(0) = 0$  and  $\Phi'(0) = 1$ .

The firm's problem will be to maximize the present value of its profits.

$$\text{Max} \int_0^{\infty} \pi_t e^{-rt} dt = \int_0^{\infty} [F(K_t, l_t) - w_t l_t - \Phi(I_t)] e^{-rt} dt \quad (13)$$

$$\text{Subject to } \dot{K}_t = I_t \quad (14)$$

and the initial condition  $K_0$ .

Dropping the time subscripts, the Hamiltonian for this problem is<sup>8</sup>

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<sup>8</sup> Here,  $q$  is a co-state variable, also known as 'Tobin's  $q$ '.



$$H^f = F(K, l) - wl - \Phi(I) + qI \quad (15)$$

The optimality conditions for this problem are

$$H_l^f = 0 \Rightarrow F_l(K, l) = w \quad (16)$$

$$H_I^f = 0 \Rightarrow \Phi'(I) = q \quad (17)$$

$$qr - H_k^f = \dot{q} \Rightarrow \dot{q} = qr - F_k(K, l) \quad (18)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} q_t k_t e^{-rt} = 0. \quad (19)$$

Finally, consider the government side of the model. The role played by the government is kept as simple as possible. The government controls the real lump sum transfer ( $\tau$ ) such that the inflation rate  $\varepsilon$  is kept at a constant level. Hence,<sup>9</sup>

$$\dot{m} + \varepsilon m = \tau. \quad (20)$$

We are now in a position to study the equilibrium for this economy, by which we mean a state of equality between the planned demand and supply derived from the optimization problems for households and firms for given government policies. Combining the optimality conditions of households and firms, and the government budget constraint (20) we obtain the following set of equations:

$$U_c(c, l) = \bar{\lambda}(1 + r + \varepsilon) \quad (21)$$

$$U_l(c, l) = -\bar{\lambda}F_l(K, l) \quad (22)$$

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<sup>9</sup> An implicit assumption here is that we have a flexible exchange rate system in which the foreign exchange reserves of the government are constant. Of course, the government could alternatively target the inflation rate by intervening on the foreign exchange market. As argued in the introduction, such intervention on the foreign exchange market may be necessary only along the idjustment path to the steady state, because in steady state  $\varepsilon$  is equal to the rate of growth of money.

$$\Phi'(I) = q \quad (23)$$

$$F(K, l) = \pi + wl + \Phi(I) \quad (24)$$

$$\dot{q} = qr - F_k(K, l) \quad (25)$$

$$\dot{K} = I \quad (26)$$

$$\dot{b} = F(K, l) + rb - c - \Phi(I) \quad (27)$$

We can solve (21) and (22) for  $c$  and  $l$  to obtain

$$c = c(K, \lambda, \varepsilon) \quad (28)$$

$$l = l(K, \lambda, \varepsilon) \quad (29)$$

The partial derivatives of private consumption demand and labor supply implicit in (28) and (29) are found by differentiating equations (21) and (22) with respect to  $K, \lambda$  and  $\varepsilon$ . This gives

$$\begin{aligned} c_k &= \frac{\partial c}{\partial k} = \frac{\bar{\lambda} F_{lk} U_{cl}}{D} < 0 & l_k &= \frac{\partial l}{\partial k} = \frac{-\bar{\lambda} U_{cc} F_{lk}}{D} > 0 \\ c_\lambda &= \frac{\partial c}{\partial \lambda} = \frac{(1+r+\varepsilon)(U_{ll} + \bar{\lambda} F_{ll}) + F_l U_{cl}}{D} & l_\lambda &= \frac{\partial l}{\partial \lambda} = \frac{-[F_l U_{cc} + U_{lc}(1+r+\varepsilon)]}{D} \\ c_\varepsilon &= \frac{\partial c}{\partial \varepsilon} = \frac{\bar{\lambda}[U_{ll} + \lambda F_{ll}]}{D} < 0 & l_\varepsilon &= \frac{\partial l}{\partial \varepsilon} = \frac{-\bar{\lambda} U_{lc}}{D} \end{aligned}$$

where  $D = U_{cc}(U_{ll} + \bar{\lambda} F_{ll}) - U_{cl}^2 > 0$ .

Now substituting (29) into (25), we obtain

$$\dot{q} = qr - F_k(K, l(K, \bar{\lambda}, \varepsilon)) \quad (30)$$

Next, note that from equation (23) we can write  $I = I(q)$  (where  $I'(q) > 0$ ). Substituting this into equation (26) we obtain

$$\dot{K} = I = I(q) \quad (31)$$

As  $\lambda$  is always at its steady state level, equations (30) and (31) give us a system of differential equations which can be solved for  $K_t$  and  $q_t$ . Linearizing these equations around the steady state equilibrium results in the following differential equation system

$$\begin{bmatrix} \dot{K} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} K - \tilde{K} \\ q - \tilde{q} \end{bmatrix} \quad (32)$$

where,  $a_{11} = 0$ ,  $a_{12} = \frac{1}{\Phi''}$ ,  $a_{21} = -[F_{kk} + F_{kl}l_k]$ , and  $a_{22} = r$ . Also,  $\tilde{K}$  and  $\tilde{q}$  are steady state values of  $K$  and  $q$ . Since the determinant of the matrix of the coefficients in equation (32) is negative ( $a_{11}a_{22} - a_{21}a_{12} < 0$ ), the long run equilibrium exhibits saddle point stability with the saddle path given by

$$K = \tilde{K} + (K_0 - \tilde{K})e^{\mu t}, \text{ and} \quad (33)$$

$$q = \tilde{q} + \mu\Phi''(K - \tilde{K})e^{\mu t}, \quad (34)$$

where  $\mu$  is the negative eigenvalue of the coefficient matrix in (32).

Combining (33) and (34) we obtain

$$q = \tilde{q} + \mu\Phi''(K - \tilde{K}), \quad (35)$$

which is the negatively sloping schedule XX in Figure 1.<sup>10</sup>

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<sup>10</sup> The following comments with respect to Figure 1 are in order. First, from (30) and (31) it is clear that in the Figure the  $\dot{q} = 0$  will be upward sloping and the  $\dot{K} = 0$  will be horizontal.

Second, again, from (30) and (31) it is clear that  $\frac{\partial \dot{q}}{\partial K} > 0$ , and  $\frac{\partial \dot{K}}{\partial q} > 0$ . This explains the directions of the changes in  $K$  and  $q$  off the  $\dot{q} = 0$  and the  $\dot{K} = 0$  schedules.

To determine the dynamics of the current account first consider both household's and government's budget constraints (4) and (20) and use the definition of profit ( $\pi$ ) given in equation (11), in order to obtain the current account balance of the economy

$$\dot{b} = F(K, l) + rb - c - \Phi(I). \quad (36)$$

Now substitute for  $c_t$  and  $l_t$  from (28) and (29) into (36) and linearize the resulting equation around the steady state, noting that  $\lambda_t = \bar{\lambda}$  always, to obtain

$$\dot{b} = [F_k + F_l l_k - c_k](K - \tilde{K}) - \Phi'(\tilde{I}) I'(\tilde{q})(q - \tilde{q}) + r(b - \tilde{b}) \quad (37)$$

We know in a steady state  $\tilde{I} = 0$ . Also, by assumption  $\Phi'(0) = 1$ . Hence, (37) can be rewritten as:

$$\dot{b} = [F_k + F_l l_k - c_k](K - \tilde{K}) - I'(\tilde{q})(q - \tilde{q}) + r(b - \tilde{b}) \quad (38)$$

The solution to this differential equation is

$$b_t = \tilde{b} + \frac{\Omega_1(K_0 - \tilde{K})}{\mu_1 - r} e^{\mu_1 t} + \left[ (b_0 - \tilde{b}) - \frac{\Omega_1(K_0 - \tilde{K})}{\mu_1 - r} \right] e^{rt} \quad (39)$$

where  $\Omega_1 = F_k + F_l l_k - c_k - \mu > 0$ .

For (39) to converge, the coefficient of  $e^{rt}$  must be zero:

$$b_0 = \tilde{b} + \frac{\Omega_1(K_0 - \tilde{K})}{\mu - r}, \quad (40)$$

which, for given values of  $K_0$  and  $b_0$ , explain how  $\tilde{K}$  and  $\tilde{b}$  must be related for saddle point stability.<sup>11</sup>

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<sup>11</sup> Also note that (40) ensures that the No Ponzi Game condition (5) is satisfied. To see this multiply both sides of (39) by  $e^{-rt}$  and then let  $t \rightarrow \infty$ . It would then be clear that without (40) condition (5) would not be satisfied.

Hence, the solution for  $b_t$  is

$$b_t = \tilde{b} + \frac{\Omega_1(K_0 - \tilde{K})}{\mu - r} e^{\mu t}, \quad (41)$$

which upon using equation (33) gives us

$$b_t - \tilde{b} = \frac{\Omega}{\mu - r} (K_t - \tilde{K}). \quad (42)$$

This is the equation of the negatively sloping schedule ZZ in Figure (2.B). This schedule describes the relationship between  $K$  and  $b$  along the stable path of the complete model. One important point to note is that, as both  $K$  and  $b$  are predetermined variables, the position of ZZ depends very much on the initial conditions  $(K_0, b_0)$ . In particular, if the initial conditions of another country are such that it does not start on ZZ, then that country will end up at a different steady state equilibrium. The model, therefore, exhibits hysteresis.

### III. The Effects of Exchange Rate Policies

In this section we discuss the effects of a permanent unanticipated, a temporary unanticipated and an anticipated permanent changes in the rate of depreciation of domestic currency  $\varepsilon$  on the important macroeconomic variables. As in Obstfeld (1981a), a once and for all depreciation of the domestic currency will not have any effect, as all prices are fully flexible.

#### A. A Permanent Unanticipated Increase in Inflation

We first consider the steady state effects of an increase in  $\varepsilon$ . The steady state is given by equations (6), (7), (16), (17), and by (14), (30) and (36) with  $\dot{K} = \dot{q} = \dot{b} = 0$ . Noting that  $\Phi'(0) = 1$ , these equations can be written completely as

$$U_c(\tilde{c}, \tilde{l}) = \bar{\lambda}(1 + r + \varepsilon) \quad (43)$$

$$U_l(\tilde{c}, \tilde{l}) = -\bar{\lambda} F_l(\tilde{K}, \tilde{l}) \quad (44)$$

$$\tilde{q} = 1 \quad (45)$$

$$F_k(\tilde{K}, \tilde{l}) = r \quad (46)$$

$$F(\tilde{K}, \tilde{l}) - \tilde{c} + r\tilde{b} = 0 \quad (47)$$

$$b_0 - \tilde{b} = \frac{\Omega_1}{\mu - r} (K_0 - \tilde{K}) \quad (48)$$

These equations jointly determine the steady state equilibrium levels of  $\tilde{c}, \tilde{l}, \tilde{K}, \tilde{q}, \bar{\lambda}$  and  $\tilde{b}$ .

Differentiating them totally, we obtain the steady state effects of an increase in  $\varepsilon$  as:

$$\frac{d(K/l)}{d\varepsilon} = 0, \quad (49)$$

$$\frac{1}{\tilde{K}} \cdot \frac{d\tilde{K}}{d\varepsilon} = \frac{1}{\tilde{l}} \cdot \frac{d\tilde{l}}{d\varepsilon} = -\frac{\frac{\lambda}{K} F_l F_{kk}}{D_1} = \frac{\frac{\lambda}{l} F_l F_{kk}}{D_1} < 0, \quad (50)$$

$$\frac{d\tilde{c}}{d\varepsilon} = \frac{\lambda F_l \left[ F_{kl} F_k \left( \frac{\Omega_1}{r - \mu} - 1 \right) + F_l F_{kk} \right]}{D_1} < 0, \quad (51)$$

$$\frac{d\tilde{q}}{d\varepsilon} = 0, \quad (52)$$

$$\frac{d\bar{\lambda}}{d\varepsilon} = \frac{-\lambda \left[ U_{cl} \left\{ F_{kl} F_k \left( \frac{\Omega_1}{r - \mu} - 1 \right) + F_l F_{kk} \right\} + F_{kk} U_{ll} \right]}{D_1} < 0, \quad (53)$$

$$\frac{d\tilde{b}}{d\varepsilon} = \frac{\lambda \left( \frac{\Omega_1}{r - \mu} \right) F_l F_{kl}}{D_1} = -\left( \frac{\Omega_1}{r - \mu} \right) \frac{d\tilde{K}}{d\varepsilon} > 0, \quad (54)$$

$$\text{where } D_1 \equiv \left[ \{F_l U_{cc} + (1+r+\varepsilon)U_{cl}\} \left\{ F_{kk} F_l + F_{kl} r \left( \frac{\Omega_1}{r-\mu} - 1 \right) \right\} + F_{kk} \{F_l U_{cl} + (1+r+\varepsilon)U_{ll}\} \right] > 0.$$

In order to provide the intuition for these results, we first work out the transition path to the new steady state. Suppose that the initial equilibrium is at points A and B in Figures 2.A and 2.B, respectively. From (50), (52) and (54) it is clear that the increase in  $\varepsilon$  will shift the XX schedule to the left, and it will leave the ZZ schedule unaffected. Hence, immediately after the increase in  $\varepsilon$  there will be a sharp decline in  $q$  (point J in Figure 2.A). After this there will be a fall in  $K$  and an increase in  $b$  as the equilibrium moves along  $X'X'$  and ZZ to the long run equilibrium at  $A'$  and  $B'$ .

The intuition for these results is as follows. With CIA constraints on consumption, the increase in  $\varepsilon$  increases the cost of consumption relative to leisure. Thus, the representative household reduces his labour supply. This results in a fall in  $q$  and in investment, as it reduces the marginal productivity of capital.<sup>12</sup> The fall in labour supply also results in a fall in output. Despite the fall in output the current account turns into a surplus ( $b$  rises) as consumption and investment have both fallen.

One can easily derive the adjustments of employment, consumption and output as we move to the new steady state equilibrium. With  $l_k > 0$  and  $c_k < 0$ , along the adjustment path labour supply falls and consumption rises. The reason is that the fall in capital reduces the real

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<sup>12</sup> Note that  $q$  is the present value of the dividend payments on a unit of capital (the market value of capital) divided by the replacement cost of capital (which is unity in terms of the consumption good). Hence, changes in current and expected future marginal productivities of capital will affect  $q$ . If  $q$  falls below 1 we will have  $\dot{K} < 0$ , and if  $q$  rises above 1 we will have  $\dot{K} > 0$ , as in Tobin's  $q$  theory of investment.

wage. This reduces labour supply, and increase leisure taken by the representative household. Thus, the representative household substitutes leisure for consumption along the adjustment path.

### B. A Temporary Increase in Inflation

Suppose initially the economy is in a steady state with the inflation rate at  $\varepsilon_0$ , along with the corresponding capital stock  $K_0$  and net foreign bonds  $b_0$ . At time 0, the inflation rate is raised from  $\varepsilon_0$  to  $\varepsilon_1$ . It is understood that at time  $T > 0$ , the inflation rate will be reverted to its initial value. The mathematical details of the effects of such a temporary increase in  $\varepsilon$  are available upon request. It is relatively straightforward to explain the effects using the phase diagrams we have already derived.

If the increase in  $\varepsilon$  is expected to be only temporary then  $q$  will fall by a smaller amount than with a permanent increase in  $\varepsilon$ . The reason is that it will be expected that the labour supply will increase in the future, when the policy is reversed, which will then tend to increase the marginal productivity of capital.<sup>13</sup> Hence, when the policy is implemented at time 0 the equilibrium jumps from A to D in Figure 2.A, and between times 0 and T the economy moves along the unstable trajectory DH. As  $K$  and  $b$  are both predetermined, the implementation of the policy at time 0 does not affect the instantaneous equilibrium in Figure 2.B. However, while the economy moves along the trajectory DH in Figure 2.A, the equilibrium in Figure 2.B moves along BF. Intuitively, along DH there is less capital decumulation than there would have been had the increase in  $\varepsilon$  been permanent (along  $XX'$ ); and, moreover, this capital decumulation

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<sup>13</sup> Recall, from footnote 9, that  $q$  is the present value of the dividend payments on a unit of capital (the market value of capital) divided by the replacement cost of capital (which is unity in terms of the consumption good).



tends to decelerate as we get closer to the date the policy will be reversed. As a result the trajectory BF deviated from ZZ as shown in Figure 2.B.

When the policy is reversed at time T,  $K_T$  and  $b_T$  dictate the position of the stable path from then on. This is  $Z'Z'$  in Figure 2.B. As  $K_T < K_0$  and  $b_T > b_0$  the new steady state equilibrium level of  $K$  will be less than  $K_0$ , and the new steady state level of  $b$  higher than  $b_0$ . This implies that in Figure 2.A the saddle path for time T will be  $X''X''$ , with a steady state level of  $K$  less than  $K_0$ , and a steady state level of  $q$  the same as  $q_0$  (unity). The new steady state equilibrium will be given by points L and N in Figure 2.A and 2.B, respectively.

Hence, in the new steady state equilibrium after a temporary increase in  $\varepsilon$  the economy will have a lower level of capital and a better net foreign asset position. The model exhibits hysteresis. When the economy moves along the trajectory BF, in Figure 2.B, the combinations of capital and bonds which are held by the households change. As capital affects the wage rate while bonds do not, the temporary change in the combination of the households' assets affects their long run budget constraint, as described by the ZZ schedules in Figure 2.B.

### C. An Anticipated Permanent Increase in Inflation

Finally, note that an anticipated permanent increase in  $\varepsilon$ , with the government announcing at time 0 that it will permanently increase  $\varepsilon$  at time T, will have the same effects as an unanticipated temporary fall in  $\varepsilon$ . The effects of an anticipated permanent increase in  $\varepsilon$  will, therefore, be the reverse of what was discussed above.

Consider Figures 3.A and 3.B, where initially the economy is in equilibrium at A and B, respectively. The announcement of the policy at time 0 will reduce  $q$  instantly, as it is anticipated that the future increase in the inflation rate will reduce the marginal productivity of capital. Clearly,  $q$  will fall by a smaller amount than if the announcement and implementation of

the policy coincided. Hence, when the intentions of the government regarding its future policies is announced at time 0 the equilibrium jumps from A to N in Figure 3.A, and between times 0 and T the economy moves along the unstable trajectory NQ. As  $K$  and  $b$  are both predetermined, the implementation of the policy at time 0 does not affect the instantaneous equilibrium in Figure 3.B. However, while the economy moves along the trajectory NQ in Figure 3.A, the equilibrium in Figure 3.B moves along BS. Intuitively, along NQ there is less capital decumulation than there would have been had the policy been implemented at time 0 rather than at T (along  $XX'$ ); and, moreover, the capital decumulation accelerates as we get closer to the actual implementation time for the policy. As a result the trajectory BS deviated from ZZ as shown in Figure 3.B.

When the policy is actually implemented at time T,  $K_T$  and  $b_T$  dictate the position of the stable path from then on. This is  $Z''Z''$  in Figure 3.B. This implies that the saddle path for time T on in Figure 3.A will be  $X''X''$ . After time T the economy moves along  $X''X''$  and  $Z''Z''$  to the new steady state equilibrium at points F and G, respectively.

The degree of capital decumulation between times 0 and T is less than what it would have been had the policy been implemented at the same time that it was announced (at time 0). Also, as the model exhibits hysteresis, the new steady state level of capital is larger than it would have been had the policy been implemented at time 0.

#### IV. Conclusions

Two very distinct models have been used in the literature in order to analyze the effects of monetary policies in open economies. On the one hand, the aggregative Mundell-Fleming-Dornbush model has been used to analyze the effects of monetary policies on employment, output, investment, the current account, and other macroeconomic variables. On the other hand, intertemporal utility maximizing models have employed endowment economies, or models with

relatively incomplete production structures, in order to analyze the effects of monetary policies on, mainly, consumption, real money holdings and the current account. This paper has made an attempt to study the effects of exchange rate policies in an intertemporal optimising model with employment, investment and output effects.

We have employed an infinite horizon model in which the representative agent has labour/leisure choice, and firms can make investment decisions. Money is introduced into the model through a CIA constraint on consumption. A permanent unanticipated increase in the inflation rate increased the price of consumption in terms of leisure. This reduced consumption and labour supply, as the representative agent substituted leisure for consumption. The resulting fall in labour input reduced output and the marginal productivity of capital, leading to a fall in investment. Despite the fall in output, the current account turns into a surplus because of the fall in consumption and investment. The model also showed that during the transition period to the new steady state there were further falls in consumption and labour supply, as the reduction in capital led to reductions in the wage rate.

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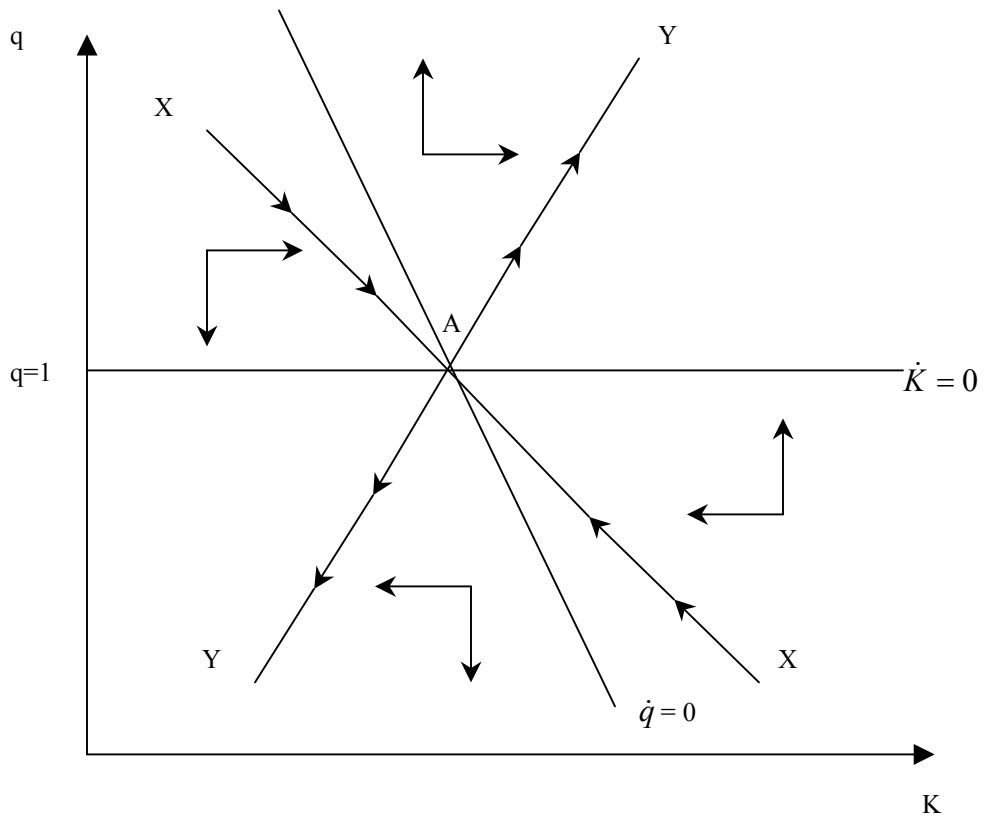
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**Figure 1:** The Phase Diagram

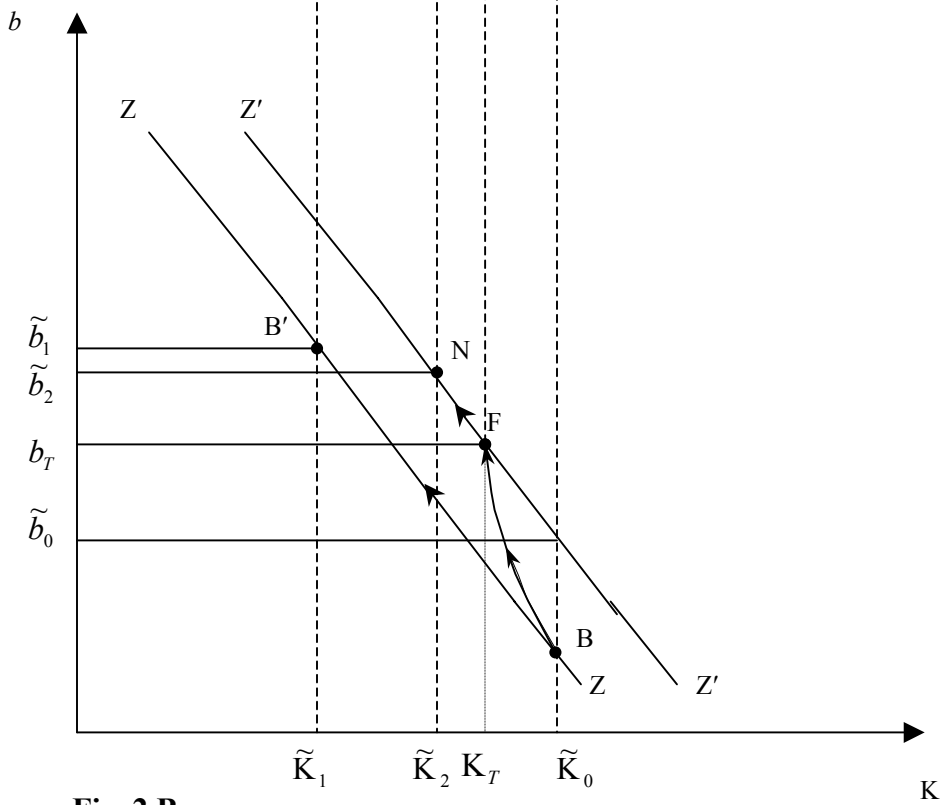
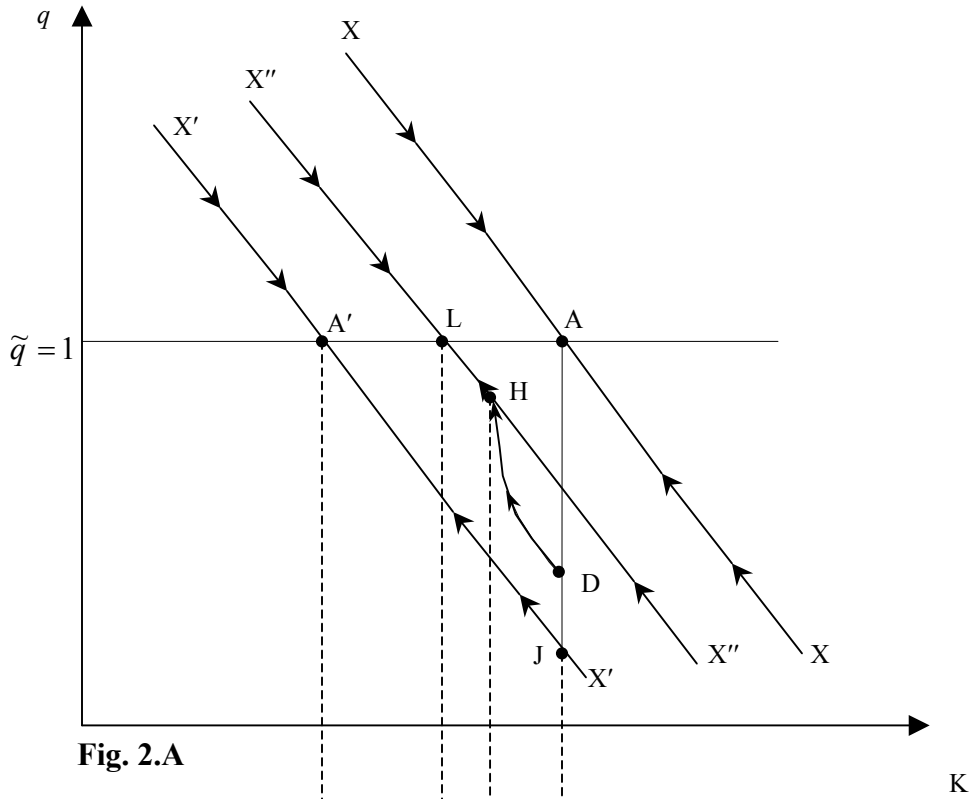


Figure 2: The Effects of an Increase in  $\varepsilon$  - Temporary vs. Permanent

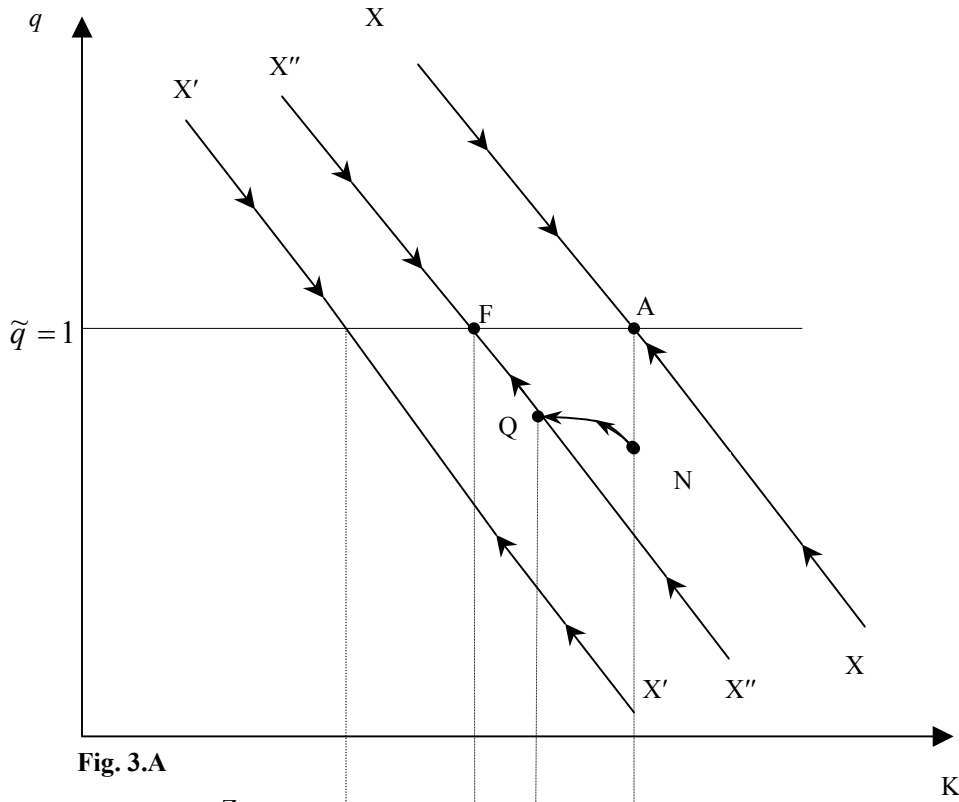


Fig. 3.A

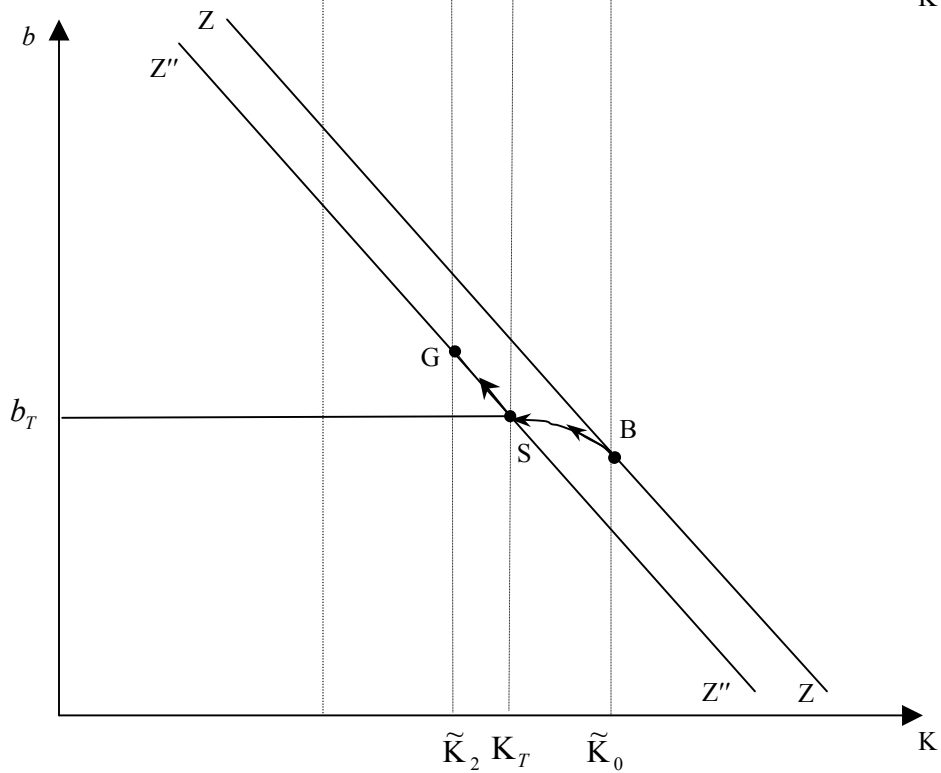


Fig. 3.B

Figure 3: The Effects of an Anticipated Increase in  $\epsilon$



