Price Discrimination and Social Welfare with Demand Uncertainty

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Abstract

Price, output and welfare effects of third-degree price discrimination is analyzed in the context of a risk-averse monopolist, who commits to fixed prices before the revelation of random and potentially correlated demands. Assuming the disturbance term to be additive, white noise and the monopolist to have a quadratic (mean-variance) utility function, we show that price discrimination may occur with identical expected demands, the relatively risky but price insensitive market may be charged the lower price and despite linear demands, aggregate expected output may fall while social welfare rises. All of these results, which run counter to those in the deterministic model, are shown to be driven by the asymmetry in the revenue and risk characteristics of the markets and the willingness of the monopolist to trade increased level for reduced risk of expected profit in a manner similar to portfolio choice with risky and correlated assets.

Key Words: Monopoly (D42), Monopolization Strategies (L12), Decision Making under Risk and Uncertainty (D81)

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1 Introduction

The social desirability of third-degree price discrimination has been a topic of much research ever since Joan Robinson's (1933) pioneering analysis of the problem. The conventional wisdom has been that the welfare e®ects depend critically on the output e®ect of discrimination. It has been well known that the output e®ect, in turn, depends on the concavity of demand and in the limiting case of linear demand, discrimination does not change aggregate monopoly output. Following a series of paper (surveyed e.g., in Varian (1989)) on the issue, Schmalensee(1981) demonstrated that in the linear demand case, price discrimination inevitably leads to welfare loss. Subsequently, Varian (1985) generalized the result and showed that a necessary but not su±cient condition for social welfare to rise with discrimination is a rise in monopoly output.

The normative and positive analysis of third degree price discrimination has been extended to the context of a spatial economy with ¯xed production location by Greenhut and Ohta (1972), Holahan (1975) and Beckman (1976). These papers show that with linear demands, when radius of the monopolists market area is endogenous, spatial price discrimination raises monopoly output and, potentially, social welfare over f.o.b. mill pricing policy. However, with a ¯xed radius, discrimination does not change output precluding the possibility of welfare gain. When location is endogenous, Hwang and Mai (1990) show, by contrast, output and welfare e®ect of discrimination is indeterminate and depends on the parameters of the model. In particular, they demonstrate that welfare gain is possible even if spatial price discrimination were to reduce output.

In a recent contribution, Layson (1998) analyzes the price, output and welfare e®ect of third degree price discrimination when a monopolist sells in two markets with demand interdependence brought about by the substitutability and complementarity of the goods. The e®ects of price discrimination in this model are shown to depend on the degree of interdependence as well as convexity of demands and the slope of marginal cost.

The considerable literature on third degree price discrimination has, for most part, been confined to a deterministic world. A notable exception is the paper by Eckel and Smith (1993). They explore the pricing decision of a multi-product monopolist facing random, correlated demands. They assume convex cost so that expected cost can be reduced by reducing aggregate output variance. It is then demonstrated that if the monopolist, assumed to be risk neutral, were to maximize social welfare, the optimal prices may involve discrimination across markets. Price discrimination, in this case, reduces aggregate demand variance by exploiting covariance in market demands.

The paper by Eckel and Smith (1993) does not, however, address the traditional concern surrounding the price, output and welfare effects of price discrimination by a private welfare maximizing monopolist facing random demands. The purpose of the present paper is to fill this gap in the literature. We consider a model where a risk averse monopolist faces two markets with stochastic and potentially correlated demands. The monopolist is assumed to commit to an irreversible price in each market before the uncertainty is resolved. Third-degree price discrimination across markets in this setting is shown to trigger several unconventional positive as well as normative results. a) Price discrimination may occur even when price elasticities are identical across markets. b) Direction of price discrimination may be opposite to the conventional case. c) Discrimination may raise social welfare despite linear demands and negative output effect. All of these results, as we demonstrate, are driven by risk aversion inducing the monopolist to optimally trade return against risk, in a manner similar to portfolio choice with risky and correlated assets.

The organization of the paper is as follows: In section 2, we develop the basic stochastic model of third-degree price discrimination. Section 2 analyzes the price effects of third-degree discrimination under uncertainty. The impact on expected output and social welfare in the context of linear demands is developed in section 4. Section 5 presents the concluding remarks.

2 The Basic Model

Like the conventional third-degree price discrimination model, we consider a monopolist selling a product in two divisible markets. Unlike the orthodox model, however, we assume that demand in both markets is subject to random, white noise, disturbances. The monopolist is assumed to commit to prices before actual demands are revealed and then produce the quantities necessary to clear the markets. In order to make the role of demand variance and covariance meaningful, it is further assumed that prices, once set, will be maintained for a significant period.[1]

The demand functions (labeled by subscripts 1 and 2 respectively) are assumed to be

$$q_i = f_i(p_i) + e_i \tag{1}$$

where q_i and p_i are quantity demanded and the price in market i (i = 1 and 2) and $f_i' < 0$. We postulate that (e_1, e_2) has a multivariate distribution with $Ee_1 = Ee_2 = 0$, $E(e_i^2) = \sigma_i^2$, and $Cov(e_1, e_2) = \sigma_{12} \gtreqless 0$. The traditional models of price discrimination assume $\sigma_1 = \sigma_2 = \sigma_{12} = 0$ (i.e., demands are non-stochastic). As we will demonstrate, the volatility as well as covariance between markets have significant positive as well as normative implication for the third-degree price discrimination model.

For simplicity, the cost function is assumed to be linear in output,[2] i.e.,

$$C = F + cQ \tag{2}$$

where $Q = q_1 + q_2$ and F is the fixed cost. The profit function of the firm is thus given by

$$\pi = \pi(p_1, p_2, e_1, e_2) = (p_1 - c)(f_1(p_1) + e_1) + (p_2 - c)(f_2(p_2) + e_2) - F. \tag{3}$$

The monopolist is assumed to have a mean-variance utility function in profit, which. is given by[3]

$$U(\pi) = \pi(p_1, p_2, e_1, e_2) - (R/2)[\pi(p_1, p_2, e_1, e_2) - \pi(p_1, p_2, 0, 0)]^2, \tag{4}$$

where $R \geq 0$ is a risk-aversion index.

[1] This assumption is not uncommon in the theoretical literature in uncertainty and the behavior of the firm. Clearly, the assumption makes sense in markets where frequent price adjustments are very costly due, for example, to the cost of advertising.

[2] This simplified assumption rules out portfolio effect arising out of convex cost structure as assumed by Eckel and Smith (1993) in their model.

[3] Note that for the mean-variance analysis to be valid, it must be assumed that the joint density function is bivariate normal or the utility function is quadratic. For detail, see Huang and Litzenberger (1988).

3 Simple Monopoly vs. Price Discrimination

In this section, we begin by deriving the simple monopoly and then the price discriminating solutions.

3.1 Simple Monopoly

If the monopolist is unable to separate markets, a common price (i.e., $p_1 = p_2 = p$) must be set in the two markets, generating the simple monopoly solution. The expected profit function is accordingly,

$$EU(\pi) = f_1(p)(p - c) + f_2(p)(p - c) - F - (R/2)[(p - c)^2][\sigma_1^2 + \sigma_2^2 + 2\sigma_{12}] \tag{5}$$

The first-order condition for the monopolist's expected utility maximization is,

$$\frac{\partial EU(\pi)}{\partial p} = [f_1'(p) + f_2'(p)](p - c) + f_1(p) + f_2(p) - R(p - c)(\sigma_1^2 + \sigma_2^2 + 2\sigma_{12}) = 0. \tag{6}$$

Since $f_i' < 0$ and $\sigma_1^2 + \sigma_2^2 + 2\sigma_{12} > 0$, it is easily verified that the equilibrium price $p^* > c$. Thus, as in the standard model, the simple monopoly price must exceed marginal cost.

Let p^0 be the equilibrium price under certainty, which is determined according to $f_1'(p^0) + f_2'(p^0)](p^0 - c) + f_1(p^0) + f_2(p^0) = 0$. Evaluating (6) at p_0 yields

$$\frac{\partial EU(\pi)}{\partial p}\Big|_{p=p^0} = -R(p - c)(\sigma_1^2 + \sigma_2^2 + 2\sigma_{12}) < 0;$$

implying that $p^* < p^0$. This gives

Corollary 1: The non-discriminating price, p^*, with uncertainty is less than the corresponding non-discriminating price in the deterministic model, p^0.

This is not surprising. It is well known that a risk-averse monopolist will price closer to marginal cost when demand is uncertain. The negative impact on utility that the fall in mean profit creates is more than offset by the smaller variance due to the smaller markup.

3.2 Price Discrimination

In this case the firm is assumed to be able to set separate prices, p_1 and p_2, in the two markets to maximize the expected utility of profit,

$$EU(\pi) = f_1(p_1)(p_1 - c) + f_2(p_2)(p_2 - c) - F - (R/2)[(p_1 - c)^2\sigma_1^2 + (p_2 - c)^2\sigma_2^2 + 2(p_1 - c)(p_2 - c)\sigma_{12}] \tag{7}$$

The first order conditions for this maximization are given by,

$$\frac{\partial EU}{\partial p_1} = [(p_1 - c)f_1'(p_1) + f_1(p_1)] - R[(p_1 - c)\sigma_1^2 + (p_2 - c)\sigma_{12}] = 0 \tag{8}$$

$$\frac{\partial EU}{\partial p_2} = [(p_2 - c)f_2'(p_2) + f_2(p_2)] - R[(p_2 - c)\sigma_2^2 + (p_1 - c)\sigma_{12}] = 0 \tag{9}$$

where $f_i' = \partial f_i/\partial p_i$. Solving yields p_1^* and p_2^*.

Let (p_1^0, p_2^0) be the discriminating prices under certainty, which can be obtained from (8) and (9) by setting $\sigma_1^2 = \sigma_2^2 = \sigma_{12} = 0$. To see the relationship between (p_1^*, p_2^*) and (p_1^0, p_2^0), evaluate (8) and (9) at p_1^0 and p_1^0:

$$\frac{\partial EU}{\partial p_1}\Big|_{p_1^0,\, p_1^0} = -R[(p_1^0 - c)\sigma_1^2 + (p_2^0 - c)\sigma_{12}] \gtrless 0 \text{ if } \rho \lessgtr -\frac{(p_1^0 - c)\sigma_1}{(p_2^0 - c)\sigma_2} = v_1$$

$$\frac{\partial EU}{\partial p_2}\Big|_{p_1^0,\, p_1^0} = -R[(p_2^0 - c)\sigma_2^2 + (p_1^0 - c)\sigma_{12}] \gtrless 0 \text{ if } \rho \lessgtr -\frac{(p_2^0 - c)\sigma_2}{(p_1^0 - c)\sigma_1} = v_2$$

where rho is the correlation coefficient (i.e., $-1 \le \rho = \sigma_{12}/\sigma_1\sigma_2 \le 1$). Note that v_i (i=1,2) < 0 and $v_1 v_2 = 1$. Thus, if $v_i < -1$, then $-1 < v_j < 0$ $(i \neq j)$. Evidently, if $\rho > \max(v_1, v_2) < 0$, $\frac{\partial EU}{\partial p_1}|_{p_1^0,\, p_1^0} < 0$, $\frac{\partial EU}{\partial p_2}|_{p_1^0,\, p_1^0} < 0$. But if $\rho \in [-1, \max(v_1, v_2)]$, $\frac{\partial EU}{\partial p_i}|_{p_1^0,\, p_1^0} < 0$ and $\frac{\partial EU}{\partial p_j}|_{p_1^0,\, p_1^0} > 0$ $(i \neq j)$. Assuming that the system has a global maximum, we can therefore obtain

Corollary 2: The discriminating prices can be higher or lower under uncertainty than under certainty, depending on the degree of market correlations. Specifically,

1. If $\rho > \max(v_1, v_2)$, then $p_1^* < p_1^0$ and $p_2^* < p_2^0$;

2. If $\rho \in [-1, \max(v_1, v_2)]$, then either (a) $p_1^* > p_1^0$ and $p_2^* < p_2^0$ or (b) $p_1^* < p_1^0$ and $p_2^* > p_2^0$.

Clearly, when ρ (or σ_{12}) is positive or mildly negative, the discriminating prices with uncertainty are lower across the board compared to the prices in the deterministic model. But, when ρ is sufficiently negative, despite uncertainty, discriminating price in the relatively stable market may be higher than the corresponding price without uncertainty, contrary to the conventional wisdom. The intuition behind this price regime is that negative effect of increased profit volatility in the market where markup is raised is more than offset by the effect of smaller markup in the other market. The negative correlation between the markets strengthens this effect.

3.3 Comparison

Here, we examine the relationship of p_1^* and p_2^* to p^*. Evaluating (8) and (9) at $p_1 = p_2 = p^*$:

$$\frac{\partial EU}{\partial p_1}\Big|_{p_1=p^*} = [(p^* - c)f_1'(p^*) + f_1(p^*)] - R[(p^* - c)\sigma_1^2 + (p^* - c)\sigma_{12}] \quad (10)$$

$$\frac{\partial EU}{\partial p_2}\Big|_{p_2=p^*} = [(p^* - c)f_2'(p^*) + f_2(p^*)] - R[(p^* - c)\sigma_2^2 + (p^* - c)\sigma_{12}] \quad (11)$$

Rewrite (6) as $\frac{\partial EU}{\partial p_1}|_{p_1=p^*} + \frac{\partial EU}{\partial p_2}|_{p_2=p^*} = 0$. Given this, it is evident from (10) and (11) that $\frac{\partial EU}{\partial p_1}|_{p_1=p^*} \gtrless 0$ if and only if $\frac{\partial EU}{\partial p_2}|_{p_2=p^*} \lessgtr 0$. Assuming the system has a unique maximum, it easily follows that $p_1^* > p^* > p_2^*$ or $p_2^* > p^* > p_1^*$. This confirms the conventional wisdom that the price under a simple monopoly is bounded by the two discriminating prices. Two things should be noted: (i) In the conventional model, profitable price discrimination requires price elasticities of demand to differ in the non-discriminating equilibrium. In the present model, however, even if $f_1 = f_2$, there is still a basis for price discrimination. It is easily verified from (10) and (11) that $p_1^* \gtrless p_2^*$ if $\sigma_1 \lessgtr \sigma_2$; and (ii) optimal p_1^* and p_2^* depends not only on the elasticities of demands, but also on relative market volatility. Thus, in the stochastic model, the direction of price discrimination may go against the conventional grain. The elastic market may be charged the higher and the inelastic market the lower price. Clearly, for this unorthodox outcome to occur, the elastic market must be more risky so that the profits sacrificed by the perverse price discrimination is more than made up by the smaller aggregate profit variance this brings about. To summarize then, we have

Proposition 1 $p_1^* > p^* > p_2^*$ or $p_2^* > p^* > p_1^*$. This holds even if $f_1 = f_2$. Specifically, $p_1^* \gtrless p_2^*$ if $\sigma_1 \lesseqgtr \sigma_2$;

Figure 1 presents a graphical description of our model. For simplicity of exposition, we assume the following linear demand functions[4] and also, without loss of generality, zero cost.

$$Q_1 = a_1 - b_1 p_1 + e_1$$

$$Q_2 = a_2 - b_2 p_2 + e_2$$

Lines A_1B_1 and A_2B_2 are the linear demand functions under certainty, given above. MR_1 and MR_2 are the corresponding marginal revenue curves (defined in terms of prices). As shown, at a common price, market 1 has the flatter (inverse) demand curve and hence, is more elastic compared to market 2. In a non-stochastic world, with zero cost, evidently non-discriminating price would be p^0, where aggregate marginal revenue $MR_1(p^0) + MR_2(p^0) = 0$ (i.e., $p^0g_1 = p^0g2$). If markets can be divided, discriminating prices would be at p_1^0 and p_2^0 respectively where marginal revenues, $MR_1 = MR_2 = 0$. Clearly, $p_1^0 < p^0 < p_2^0$.

In the zero-cost stochastic model, however, the monopolist maximizes expected utility of revenue rather than revenue itself.[5] The relevant functions to look at are the marginal utility rather than marginal revenue functions. It is easily verified from (8) and (9) that given the assumed linear demand functions, while the marginal revenue functions are $MR_1 = a_1 - 2b_1p_1$ and $MR_2 = a_2 - 2b_2p_2$, marginal utility functions are $u_1 = (a_1 - p_2R\sigma_{12}) - (2b_1 + R\sigma_1^2)p_1$ (which is (8)) and $u_2 = (a_2 - p_1R\sigma_{12}) - (2b_2 + R\sigma_2^2)p_2$ (which is (9)). Assuming the markets to be stochastically independent (i.e., $\sigma_{12} = 0$), the marginal utility functions reduce to $u_1 = a_1 - (2b_1 + R\sigma_1^2)p_1$ and $u_2 = a_2 - (2b_2 + R\sigma_2^2)p_2$ and are given in Figure 1 by line

[4] It is easily verified that these demand functions can be derived from the following concave but quasi-linear indirect utility functions:

$$U = a_1p_1 + a_2p_2 - \frac{1}{2}b_1p_1^2 - \frac{1}{2}b_2p_2^2 + p_1e_1 + p_2e_2 + M;$$

where M is the consumption of a numeraire good, produced competitively.

[5] Since random shocks are assumed to be additive and $E(e_i) = 0$, line A_iB_i represents the expected demand function in the stochastic model.

u_1 and u_2, respectively. Clearly, these are steeper than the corresponding marginal revenue functions. In the stochastic model, raising prices increases the variance of revenues, and given the mean-variance utility function, monopolist's utility increases at a smaller rate than revenues. The relative slopes of the marginal utility functions depend on the slopes of marginal revenue functions (b_i) as well as variances (σ_i^2). If market 2 is sufficiently more volatile than market 1, it is clearly possible for u_2 to be steeper than u_1 even though MR_1 is steeper than MR_2, i.e., the market that is relatively more price elastic can become relatively less "utility elastic." This is the case illustrated in Figure 1. The non-discriminating price is p^* where $u_1(p^*) + u_2(p^*) = 0$ (i.e., $p^*d_1 = p^*d_2$) and the discriminating prices are at p_1^* and p_2^*, which are determined by $u_1 = 0$ and $u_2 = 0$ respectively. Two things may be noted in the context of this stochastic equilibrium. First, p^*, the non-discriminating price with uncertainty, is less than the corresponding non-discriminating price in the deterministic model, p^0, as claimed by Corollary 1. Also, the discreminating prices in the stochastic model, (p_1^*, p_2^*), are lower than the corresponding prices in the deterministic model, (p_1^0, p_2^0). This confirms Corollary 2 since the corvariance is assumed to be zero in Figure 1. Second, since u_2 is steeper than u_1 while MR_1 is steeper than MR_2, price is reduced (raised) in market 2 (1), which is the relatively less (more) price-sensitive market, reversing the orthodox direction of price discrimination. Furthermore, it is clear from figure 1 that even with identical demands, a basis for utility-improving price discrimination would arise if variances and, therefore, marginal utility functions were not identical across markets. This confirms proposition 1.

4 Outputs and Social Welfare with Linear Demands

In the non-stochastic case, Schmalensee (1981) and Varian (1985) have shown that with linear demand, aggregate output remains constant with price discrimination and Marshallian social welfare (the sum of consumer and producer surpluses) inevitably declines. We now reexamine this well-known conventional wisdom in the context of our stochastic model.

4.1 Aggregate Expected Output

We examines, in this subsection, the effect on aggregate expected output when the monopolist practices price discrimination. For simplicity, assume that demands are linear and $\sigma_{12} = 0$. It must be noted that since output is random in our model, the appropriate basis for comparison is expected output under discrimination and non-discrimination. From (6), (10), and (11), we know that $\frac{\partial EU}{\partial p_1}|_{p_1=p^*} + \frac{\partial EU}{\partial p_1}|_{p_1=p^*} = 0$, which, upon substitution, yields

$$\sum_{i=1}^{2}[\frac{1}{b_i}(2b_i + R\sigma_i^2)q_i^e(p^*) + a_i + (R\sigma_i^2 - b_i)] = 0; \tag{12}$$

where $q_i^e(p^*)$ is the expected delivery of output to market i at p^*. Total differentiation of (14) gives

$$b_2(2b_1 + R\sigma_1^2)\Delta q_1^e(p^*) + b_1(2b_2 + R\sigma_2^2)\Delta q_2^e(p^*) = 0 \tag{13}$$

where $\Delta q_i^e(p^*)$ denotes the change in expected output delivered to market i as the firm deviates from simple monopoly to price discrimination regime. Rewrite (13) as:

$$\theta_1[\Delta q_1^e(p^*) + \Delta q_2^e(p^*)] = (\theta_1 - \theta_2)\Delta q_2^e(p^*) \tag{14}$$

where $\theta_i = b_j(2b_i + R\sigma_i^2) > 0$ $(i, j = 1, 2;\ i \neq j)$. Evidently, under certainty (i.e., $\sigma_1 = \sigma_2 = 0$), $\theta_1 = \theta_2$. It is clear from (14) that $\Delta q_1^e(p^*) + \Delta q_2^e(p^*) = 0$, thus confirming the traditional result that aggregate output remains constant with discrimination. Under uncertainty, the effect on aggregate output under price discrimination can be either positive or negative depending on the relative market volatilities. From (14), we know that

$$\mathrm{sign}[\Delta q_1^e(p^*) + \Delta q_2^e(p^*)] = \mathrm{sign}[(\theta_1 - \theta_2)\Delta q_2^e(p^*)]$$

For $\Delta q_1^e(p^*) + \Delta q_2^e(p^*) < (>)\ 0$, it requires that $(\theta_1 - \theta_2)$ and $\Delta q_2^e(p^*)$ be opposite (identical) in sign. Note that $\theta_1 - \theta_2 = R(b_2\sigma_1^2 - b_1\sigma_2^2) \gtrless 0$ if $b_2\sigma_1^2 - b_1\sigma_2^2 \gtrless 0$. Further, we know from our discussion above that $\Delta q_2^e(p^*) \gtrless 0$ if $2b_1 + R\sigma_1^2 \lessgtr 2b_2 + R\sigma_2^2$.[6] Clearly, $\Delta q_1^e(p^*) + \Delta q_2^e(p^*) > 0$ when $b_2\sigma_1^2 - b_1\sigma_2^2 \gtrless 0$ and $2b_1 + R\sigma_1^2 \lessgtr 2b_2 + R\sigma_2^2$. Conversely, $\Delta q_1^e(p^*) + \Delta q_2^e(p^*) < 0$ when $b_2\sigma_1^2 - b_1\sigma_2^2 \gtrless 0$ and $2b_1 + R\sigma_1^2 \gtrless 2b_2 + R\sigma_2^2$. This is summarized in

[6] Note that $2b_1 + R\sigma_1^2 < (>)\ 2b_2 + R\sigma_2^2$ when u_1 is flatter (steeper) than u_2.

Proposition 2 Assume that demands are linear and $\sigma_{12} = 0$. Aggregate expected output rises when $b_2\sigma_1^2 - b_1\sigma_2^2 \gtrless 0$ and $2b_1 + R\sigma_1^2 \lessgtr 2b_2 + R\sigma_2^2$; but falls when $b_2\sigma_1^2 - b_1\sigma_2^2 \gtrless 0$ and $2b_1 + R\sigma_1^2 \gtrless 2b_2 + R\sigma_2^2$.

An implication of proposition 3, clearly, is that expected monopoly output must fall whenever the slopes of marginal revenue and marginal utility functions have opposite ranking across markets so that price is lowered (raised) in the relatively price inelastic (elastic) market, as shown in Figure 1.

4.2 Social Welfare

Given the quasi-linear indirect utility functions, it is well-known that the income effect is zero and that expected consumer welfare can be measured by the Marshallian consumer surplus in each market. The aggregate expected consumer surplus (ECS) is given by:[7]

$$ECS = \sum_{i=1}^{2} \frac{(a_i - b_i p_i)^2}{2b_i} + \sum_{i=1}^{2} \frac{\sigma_i^2}{2b_i} \tag{15}$$

Expected producer's surplus (utility) is given, from the monopolist's utility function (4), by:

$$EPS = \sum_{i=1}^{2} (p_i - c)(a_i - b_i p_i) - \frac{R}{2}[(p_1 - c)^2\sigma_1^2 + (p_2 - c)^2\sigma_2^2 + 2(p_1 - c)(p_2 - c)\sigma_{12}]. \tag{16}$$

Denoting social welfare under discrimination and non-discrimination by $W_D = ECS_D + EPS_D$ and $W_S = ECS_S + EPS_S$ respectively, it is easily verified that

$$\Delta W = W_D - W_S \gtrless 0 \tag{17}$$

the direction of the welfare change depending on the relative slope, size and risk characteristics of the markets. Since no simple general criteria for welfare change seems possible, we present a number of simulations with alternative parameter values. The results are summarized in Table

[7] At $p_i = p_i^*$, $q_i^* = a_i - b_i p_i^* + e_i$. At $p_i = (a_i + e_i)/b_i$, $q_i^* = 0$. Given e_i, consumer surplus is $CS = \sum_{i=1}^{2}(a_i - b_i p_i^* + e_i)(\frac{a_i + e_i}{b_i} - p_i^*)/2 = \sum_{i=1}^{2} \frac{(a_i - b_i p_i^* + e_i)^2}{2b_i}$. Taking the expected value of CS yields (15). Note that the second term of (15), $\sum_{i=1}^{2} \frac{\sigma_i^2}{2b_i} > 0$, captures the effect of uncertainty on aggregate expected consumer surplus (ECS).

1. Case I represents the benchmark. In this case, markets are identically risky and market 1 has the flatter inverse demand function. Not surprisingly, the direction of price, output and welfare change is identical to the conventional case. In case II, we make market 2 (the relatively less price-sensitive market) significantly more risky. The positive and normative effects of discrimination is dramatically reversed in this case. Price discrimination runs opposite to the conventional case. Aggregate expected output shrinks but producer's, consumer's and therefore social welfare rises. The intuition behind the output and price effects have already been explained in Figure 1. The intuition behind the welfare paradox is straightforward. Consider Figure 1 again. The change in consumer surplus is given, as usual, by the area under the demand curve bounded by the prices, $s_i k_i p_i^* p^*$ in each market. In the zero-cost non-stochastic model, change in producer surplus is identical to change in revenues. From the MR_i curves, it is easily seen that as price is raised (lowered) from p^* to p_1^* (p_2^*) in market 1 (2), social welfare decreases (increases) by $s_1 k_1 j_1 h_1$ ($s_2 k_2 j_2 h_2$). Clearly, $s_1 k_1 j_1 h_1 > s_2 k_2 j_2 h_2$ and social welfare declines. This is the conventional wisdom (due to Schmalensee (1981) and Varian (1985)) that discrimination is inevitably harmful if aggregate output fails to rise.

In the stochastic model, however, the change in producer welfare is measured by the change in expected utility rather than revenues. Thus, in market 1, expected producer utility rises by $d_1 p_1^* p^*$ and in market 2 by $p_2^* p^* d_2$. The expected social welfare loss (gain) in market 1 (2) is, therefore, $s_1 k_1 p_1^* d_1$ ($s_2 k_2 d_2 p_2^*$). As drawn, the expected social gain in market 2 outweighs the expected loss in market 1 and expected social welfare improves despite a fall in aggregate expected output. The reason for the reversal of orthodox intuition is easily seen. Since profit volatility is positively related to price in each market, compared to the deterministic models, expected monopoly utility falls (rises) by the additional area $h_1 j_1 p_1^* d_1$ ($h_2 j_2 d_2 p_2^*$) in market 1 (2). This area, bounded by the prices and between MR_i and u_i curves, represents the contribution of the change in the level of profit risk to produce utility. As market 2 is significantly more unstable, the fall in price in this market reduces profit volatility and consequently raises producer's utility sufficiently to reverse the orthodox direction of welfare change.

Figure 2 shows the sensitivity of welfare change, in the numerical simulations, to demand covariance. Clearly, when both markets are equally volatile (as in Case I), price discrimination produces the same welfare e®ect as in the certainty case (i.e, higher producer welfare, but lower consumer and social welfare), irrespective of market correlations. When market 2 is signi¯cantly riskier (as in Case II), discrimination raises consumer, producer, and social welfare for all feasible correlation coe±cients. When relative risk di®erential is less signi¯cant (as in Cases III, IV), however, correlation has to be su±ciently negative for social welfare to rise. The negative correlation in these cases serves to enhance the negative impact on overall variance of pro¯ts when price is lowered in the more volatile market and raised in the less. To sum up, we have

Proposition 3 : In the stochastic model, price discrimination can raise social welfare even if both demand curves are linear and expected aggregate output falls.

5 Concluding Remarks

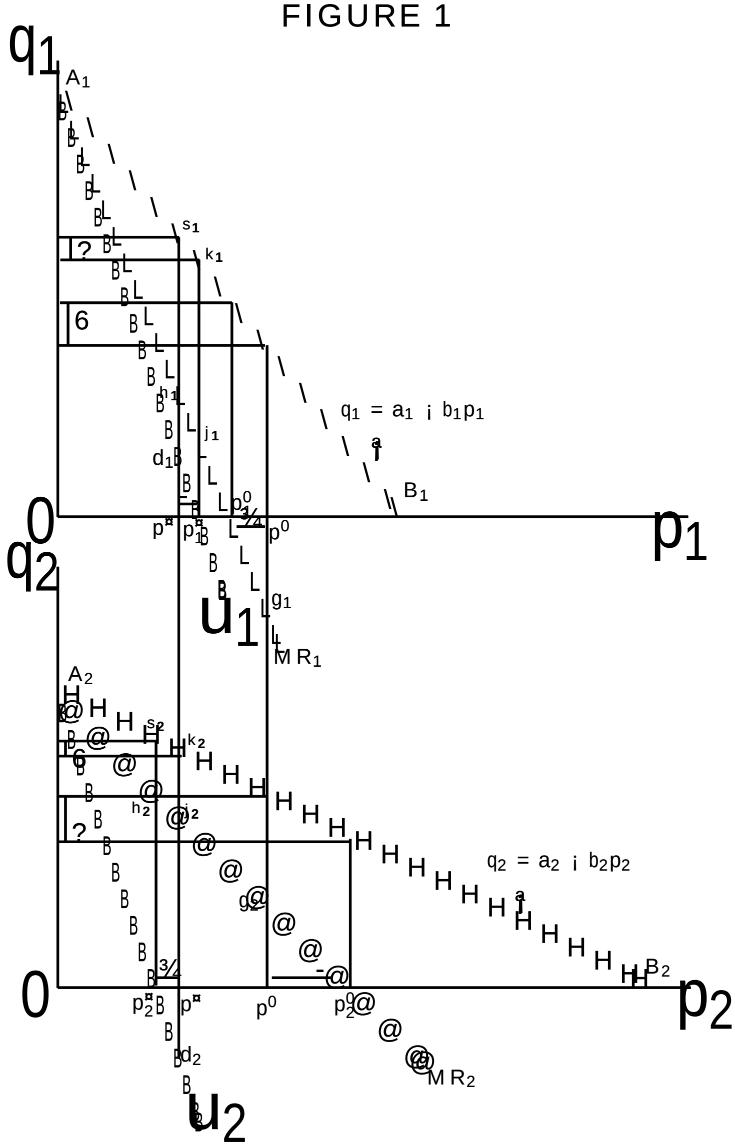
We have reexamined the price, output and welfare e®ects of third degree price discrimination when the discriminating monopolist faces two divisible, risky and potentially correlated markets. We assume that the monopolist is risk averse, has a mean-variance utility function, and commits to ¯xed prices in each market before the resolution of uncertainty. The demand uncertainty in each market is assumed to be additive in form. In the context of such a stochastic model, we demonstrate that the less price sensitive market may be charged the lower price and, despite linear demands, aggregate expected output may fall and social welfare may rise with price discrimination. These results, which run counter to the orthodox intuition, are shown to be driven by the di®erence between the risk and pro¯t characteristics of the two markets and the willingness of the monopolist to trade increased level for reduced risk of expected pro¯ts. Although the model is based on restrictive assumptions with respect to the nature of market uncertainty and the monopolist's utility function, like the spatial models, it casts doubt on the presumed social undesirability of third degree price discrimination.

Table 1: Numerical Examples

Parameter Values									Effect of Price Discrimination					
Case	c	a_1	a_2	b_1	b_2	R	σ_1^2	σ_2^2	ΔW	ΔECS	ΔEPS	$Q_{pd} - Q_s$	$p_1 - p_s$	$p_2 - p_s$
I	1	10	10	2	1	1	1	1	-	-	+	+	-	+
II	1	10	10	2	1	1	1	4	+	+	+	-	+	-
III	1	10	5	2	1	1	1	2	+/-	-	+	-	+	-
IV	1	10	10	1	1	1	1	2	+/-	-	+	-	+	-

Note: The sign pattern holds for all $-\sigma_1\sigma_2 \leq \sigma_{12} \leq \sigma_1\sigma_2$. The detail of the simulation is presented in Figure 2.

FIGURE 1



q1
A1
s1
k1
h1
j1
d1
q1 = a1 ¡ b1p1
B1
0
p1
q2
u1
g1
MR1
A2
s2
k2
h2
j2
g2
q2 = a2 ¡ b2p2
B2
0
p2
d2
MR2
u2

FIGURE 2

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