Risk Sharing in a Federation with Population Mobility and Long Horizons

by

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Abstract

This paper considers risk sharing among individuals within and across regions in a federation with population mobility and infinite horizons. It is shown that the regional authorities will not fully exploit gains from inter-regional risk sharing when population mobility is imperfect. However, in the Nash equilibrium there is complete risk sharing among the individuals within each region, which corresponds to the policies of the central authority. Regional authorities who care about their reputation may be able to commit to an efficient allocation. It is possible that improvements in the degree of mobility will make such commitments *less* likely.

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I. Introduction

There has long been an interest in Canada, and more recently in the European Union (EU), on issues pertaining to a federation with a mobile population. One class of models that can be used to discuss such issues is the fiscal externality economy, which involves multiple jurisdictions and a freely mobile population in a static setting with no uncertainty.[1] In this setting there is need for inter-regional transfers in order to obtain an efficient distribution of population in the federation. Flatters, Henderson and Mieszkowski (1974), and Boadway and Flatters (1982) argued that these transfers should be administered by a central authority. Recently, however, Myers (1990) and Mansoorian and Myers (1993) (henceforth referred to as MM) have shown that in the fiscal externality economy with two regions the regional authorities will make the inter-regional transfers that are necessary for efficiency. Thus, in that environment there is no efficiency role for a central authority. The central authority's role is to decide on the degree of redistribution in the federation.

The MM model allows for imperfect population mobility by assuming that individuals in the economy derive different degrees of non-pecuniary benefits from residing in the two regions that form the federation (attachment to home). This is a realistic characterization of a federation consisting of culturally diverse communities, and allows a discussion of the importance of the degree of population mobility on the equilibrium.

An important implication of the MM results is that their model needs to be extended in various directions in order to identify the genuine sources of inefficiency in a federation with a mobile population. Their model has been extended by Burbidge and Myers (1994a) to include capital tax competition, and by Wellisch (1994) to include spillouts of public goods. The purpose of the present paper is to construct the model of a federation in an infinite horizon setting with stochastic technological shocks in order to discuss the possible risk sharing arrangements in a federation..

The role of risk sharing arrangements in a federation has recently been discussed by Persson and Tabellini (1996a, b). In particular, in their (1996a) paper, they construct the model of a federation with two regions, and an immobile population. They have an endowment economy in

[1] Hercowitz and Pines (1991) provide an interesting exception to this.

which each individual faces a probability of having a positive endowment (employed) and a probability of having no endowments (unemployed). The unemployment rate in the two regions are stochastic and independent of each other. There is, therefore, scope for risk sharing among individuals within each region (intra-regional risk sharing) and among individuals across regions (inter-regional risk sharing). They abstract completely from private financial markets.[2] The regional authorities affect the degree of intra-regional risk sharing through their tax/transfer policies, and also decide on the provision of local public goods which affects the probability of having a smaller unemployment rate in their region. The federal authorities, on the other hand, decide on the degree of inter-regional risk sharing through their tax/transfer policies. Regional policies are decided by the voters in each region, while the federal policies are decided by all the voters in the federation. Persson and Tabellini compare and contrast the political equilibria with various constitutional arrangements ! for example, whether all voting takes place simultaneously, or whether votes on federal policies are taken before the votes on regional policies, and so on.

These political economy considerations are swept aside in the present paper. I consider a stripped down version of the Persson and Tabellini model with the following important modifications, which make it more akin to the models in the fiscal federalism literature. For simplicity, I abstract from public goods. Unlike Persson and Tabellini, I assume that there is imperfect population mobility, as in MM. Endowments of the lucky in each region are assumed to be decreasing in the population size in that region. In the static version of the model, congestion effects, together with free mobility, induce the regional authorities to make voluntary inter-regional transfers.[3] In the present model, the

[2]There is a large literature in Macroeconomics which has modeled incomplete markets endogenously, by explicitly introducing financial intermediaries with costly state verification. Gertler (1988) provides a comprehensive survey of this literature. Gertler and Rogoff (1990) construct a two country model with financial intermediaries in order to discus the often reported sub-optimal allocation of capital between the rich and poor countries. An important finding of this literatures is that modelling incomplete markets tends to substantially complicate the analysis. Instead of working through the complicated interaction between fiscal policies and endogenous market incompleteness, Persson and Tabellini (1996a, p. 625, and 1996b, p. 984) close the financial markets.

[3] In fiscal federalism models, it is the diminishing marginal productivity of labour together with free mobility that is the main inducement for interregional transfers; because in the absence of such transfers the rich region will have an inefficiently large population size. In the static version of the present model, with endowments, congestion together with free mobility acts in a similar way.

regional authorities are allowed to decide on the degree of inter-regional risk sharing, because they are allowed to make voluntary inter-regional transfers. Finally, the infinite horizon allows a discussion of the role of reputation in inducing the regional authorities to implement a set of efficient policies; and it facilitates discussion of the effects of improvements in the degree of population mobility on the possibility of efficient outcomes.

At the beginning of each period, before the state of productivity in each region is revealed, the regional authorities announce their policies contingent on the states of nature. These are the tax/transfer policies which determine the degree of risk sharing among individuals in each region, and also the inter-regional transfers contingent on the states of nature. After the levels of productivity in the two regions are revealed, the individuals in the economy decide where to reside, and the policies that were announced are carried out. It is shown that the regional authorities provide the efficient (full) degree of risk sharing among the individuals within their regions (intra-regional risk sharing), regardless of the degree of population mobility. Inter-regional risk sharing requires a coordinated set of state contingent transfers that flow from one region to another in some states, and vice versa in other states. With imperfect population mobility, the regional authorities disagree over the sizes of these state contingent transfers. Since each regional authority chooses the transfers in the states in which it experiences a more favourable shock, the disagreement among them precludes coordination of these state contingent transfers that are necessary for socially optimal risk sharing. However, with perfect mobility there are no disagreements over inter-regional redistribution, and all gains from risk sharing are fully exploited.

In a recent paper Burbidge and Myers (1994b) consider a model in which there are two types of individuals with different abilities, and perfect mobility by individuals of each type. In their model, the regional authorities disagree over the degree of redistribution between the more able and less able individuals, without any reference to whether these individuals have a preference for residing in any particular region:[4] they are concerned with *intra-regional* redistribution. They show that

[4] Indeed, in their model, with perfect mobility, in equilibrium every individual of each type is indifferent between living in either region.

disagreement over *intra-regional* redistribution will lead to inefficiency even in a deterministic environment.

On the other hand, in the attachment-to-home model the regional authorities disagree over the degree of redistribution between the individuals according to their attachments to the regions; each regional authority has a preference for individuals more attached to that region. Hence, in the attachment-to-home model the focus is on *inter-regional* redistribution. This was the main focus of Mansoorian and Myers (1992). There, it was emphasised that there is a need to introduce imperfect mobility into the standard fiscal externality economy. Otherwise, with free mobility the same level of utility will prevail throughout the federation and the regional authorities will not disagree over the distribution of resources in the federation, eliminating any role for a central authority (Myers (1990)). The attachment-to-home model led the regional authorities to disagree over the distribution of resources in the federation. It was shown that this disagreement over *inter-regional* redistribution will *not* lead to inefficiency in the deterministic environment with two regions. In the present paper, it is shown that disagreements over inter-regional redistribution will lead to inefficiency in a stochastic environment.

The infinite horizon setting facilitates a discussion of the possibility of the regional authorities signing contracts to implement an efficient set of policies. The reason is that, then, unlike in the static setting, one can discuss sanctions which can be imposed on a regional authority if it breaks the contract, even in the absence of a central authority. Consider the simplest reputation game in which if one region does not implement the policies specified in the contract it suffers sanctions imposed by the other region by never again enjoying its cooperation. Also, suppose the efficient allocation under consideration is the one that places equal weights on the objectives of the regional authorities. In that case, in a symmetric model, a regional authority will have to make a transfer to the other region only if it has experienced a more favourable shock. Once such a state is revealed the region will make the necessary transfers voluntarily if the present value of the losses it suffers if it loses reputation (i.e., the present value of its gains from risk sharing) are larger than the instantaneous gains from not making the transfers. It is not possible to say *a priori* whether improvements in the degree of population mobility will make such commitments more likely.

The paper is organised as follows. The static version of the model is discussed in sections II and III. The infinite horizon version of the model is discussed in section IV. Some concluding remarks are made in section V.

II. The Static Model with Decentralisation

The federation consists of two regions, indexed by i (i=1, 2). I consider an endowment economy which has much in common with that used by Persson and Tabellini. p_i is the fraction of the population in region i that are lucky and receive **R**(N_i) units of the good, where N_i is the population size in the region and **R**N(@)<0.[5] (1 ! p_i) is the fraction of the population in the region that are unlucky and receive nothing.

Assume that p_i can take one of two values, **(** or **$**, with **(**>**$**. The state of nature is denoted by (p_1, p_2). There are two possible states: (**(**, **$**) with probability **B**(**(**, **$**), and (**$**, **(**) with probability **B**(**$**, **(**). Of course, **B**(**(**, **$**)+**B**(**$**, **(**)=1. Henceforth, for any variable z that depends on (p_1, p_2) we will denote = z(p_1, p_2).

The sequence of events is as follows. First, the regional authorities announce their policies contingent on the states of nature. These are the tax/transfer policies that determine the degree of risk sharing among individuals in each region, and also the inter-regional transfers contingent on the states of nature. Then, the p_i are chosen by nature. After this, events (i) ! (iii) take place simultaneously, in analogy with the deterministic model: (i) individuals decide where to reside; (ii) a fraction p_i of the population that decide to reside in region i becomes lucky, a fraction 1 ! p_i become unlucky; and (iii) the policies that were promised are implemented.

Population size in the federation is normalised to unity. Thus, in each state (p_1, p_2) we have

$$_1 + _2 = 1. \qquad (1)$$

Intra-regional risk sharing in each region i is determined by the regional authority's choice of the consumption of the lucky ($_i$) and the unlucky ($_i$) in different states, through the appropriate tax/transfer policies. Inter-regional risk sharing, on the other hand, is determined by their choices of

[5] Persson and Tabellini, on the other hand, assume that **R**(N_i)=1 œ N_i. The assumption **R**N(@)<0 captures the notion of congestion. Its role in the present model was discussed in the Introduction, and is worked out in footnote 9 below.

the state contingent inter-regional transfers. The transfers from region i to region j in each state (p_1, p_2) are denoted by $_i$.

In each state (p_1, p_2) region i has the resource constraint

$$ {}_i p_i \mathbf{R}({}_i) = {}_i p_{ii} + (1 \, ! \, p_i)_{ii} + {}_i - {}_j, \qquad (2) $$

where the left hand side is total endowments in region i in that state, while the elements on the right hand side are, respectively, total consumption by the lucky, total consumption by the unlucky, transfers paid to the other region, and transfers received from the other region.

Next, consider the determination of the distribution of population in the federation. Suppose state (p_1, p_2) is given. Then for any individual residing in region i there is a probability p_i that he will be lucky and receive a consumption of $_i$. There is also a probability ($1 \, ! \, p_i$) that the same individual will be unlucky, and have a consumption of $_i$. The utility individuals derive from consuming x units of the good is given by U(x), which is a von Neumann-Morgenstern utility function. Thus, the expected value of U([illegible]) for any individual residing in region i, conditional on (p_1, p_2), is

$$ V_i(p_1, p_2) = p_i U({}_i) + (1 \, ! \, p_i) U({}_i). \quad (3) $$

To discuss the implications of the degree of population mobility, I assume that, as in MM, individuals derive non-pecuniary benefits from residing in the two regions. They are, moreover, heterogeneous only with respect to their degrees of attachment to the two regions. There is one individual of each type, denoted by n, and individuals are distributed uniformly over the interval [0, 1]. Individual n will derive a non-pecuniary benefit of k(1 ! n) if he resides in region 1, and kn if he resides in region 2. k measures the degree of population mobility: k=0 implies perfect mobility, and k=4 implies no mobility.[6]

After the state (p_1, p_2) is revealed the total expected utility for individual n will be $_1$+k(1 ! n) if he resides in region 1, and $_2$+kn if he resides in region 2. Individuals are free to choose their region of residence. Thus, in equilibrium, after (p_1, p_2) is revealed, there will be one individual, denoted by , that will be indifferent between living in either region. For this marginal individual we will have

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[6] Myers and Papageorgiou (1997) introduce imperfect mobility through *pecuniary* migration costs. The attachment-to-home model is a much more tractable means of introducing imperfect mobility.

$$_1 + k(1!) = _2 + k. \qquad (4)$$

All individuals with n< will choose to reside in region 1, while all others will reside in region 2. Hence, from (1), we will have

$$_1=, \text{ and } _2=1!. \qquad (5)$$

Now, (p_1, p_2) is (**(**, **$**) with probability **B**(**(**, **$**) and (**$**, **(**) with probability **B**(**$**,**(**). Hence, using (3), the *unconditional* expected value of U(@) for any individual who resides in region i in both states of nature (referred to as a permanent resident of region i) will be

func { W sub i `` =`` pi (gamma , ` beta) `` V sub i ( gamma , ` beta ) `` + `` pi ( beta , ` gamma ) `` V sub i (beta , ` gamma) .} (6)

I assume that the objective of the regional authority i is to maximize W_i by choosing the optimal degrees of intra- and inter-regional risk sharing. There are various objective functions one can use in this model with heterogeneous agents. For example, one can maximize the sum of the utilities of all the agents in the region, or the average utility of all the agents in the economy, or total land rents. Maximizing W_i is the simplest objective function compared to these alternatives, and it allows me to compare my results with those of MM, where the regional authorities also maximize the part of the utility of their residents that is from consumption alone.[7]

Moreover, in the two extreme cases of no attachment (k=0) and complete attachment (k=4) we really have a model with homogeneous agents in each region. Thus, in these cases, W_i is the most sensible objective function, because by maximizing W_i regional authority i will be maximizing the utility of everyone in its region. A slightly weaker version of this argument can be extended to the more general case in which 0<k<4, as discussed in footnote 8 below.

The problem of regional authority i is to choose $_i$, $_i$, and $_i$ to maximize W_i subject to its

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[7] Mansoorian and Myers (1997) compare and contrast the implications of these alternative objective function in a deterministic setting.

resource constraint (2), the migration equilibrium conditions (4), and condition (5) for all (p_1, p_2). In addition, it has the constraint $_i$\$0, which prevents it from forcing the other region to make transfers to it. To perform this optimisation problem, first solve (2) for $_i$, substitute the result into (3), and use (5) to eliminate $_i$, to obtain $_i$ as a function of $_i$, $_i$, $_j$, and . Then substitute for $_i$ into (6) to obtain W_i as a function of the same variable.

The optimality conditions for this problem are

func { { partial W sub i } over {partial c tilde sub i} `` + `` {partial W_i} over {partial N tilde } ` { partial N tilde } over {partial c tilde sub i}`` = ``0, ~~~ forall~ (p sub 1, `` p sub 2),

} (7)

func { { partial W sub i } over {partial S tilde sub i } `` + `` {partial W_i} over {partial N tilde } ` { partial N tilde } over {partial S tilde sub i } `` <= `` 0, ~~~~ S tilde sub i `` >= `` 0, ~~~~ a n d }

(8)

func { left \[` { partial W sub i } over {partial S tilde sub i } `` + `` {partial W_i} over {partial N tilde } ` { partial N tilde } over {partial S tilde sub i } ` right \] ``S tilde sub i `` = `` 0, ~~~~ forall (p sub 1 , `` p sub 2). }(9)

The migration responses that appear in these conditions are obtained from (4). As discussed above, $_i$ can be expressed as a function of $_i$, $_i$, $_j$, and . Hence, for each state (p_1, p_2), (4) describes as an implicit function of $_i$, and $_i$ (i=1, 2). Totally differentiating it, we obtain

func { { partial N tilde } over {partial c tilde sub 1 } `` = `` - ``
{ p sub 1 `` left \[ ` U'( c tilde sub 1 ) `` -`` U' ( b tilde sub 1 ) right \]} over { line `` J tilde `` line }
, }

(10)

func { { partial N tilde } over {partial c tilde sub 2 } `` = ``{ p sub 2 `` left \[ ` U'( c tilde sub 2 ) `` -`` U' ( b tilde sub 2 ) right \]} over { line `` J tilde `` line }
, }

(11)

and

func { { partial N tilde } over {partial S tilde sub 1 } `` = ``- `{ partial N tilde } over {partial S tilde sub 2 } `` = ``
{ U'(b tilde sub 1) / N tilde `` + `` U' (b tilde sub 2) / (1`-`N tilde)} over { line `` J tilde `` line }
, }

(12)

where | | is the determinant of the Jacobian of the migration equilibrium in a particular state (p_1, p_2), and should be negative for stability (as in Boadway (1982)).

We have , , = 1 ! 2 ! 2k, where
func { Lambda tilde sub 1 `` =`` {partial V tilde sub 1 } over {partial N tilde } ``= `` U'(b tilde sub 1)`` left \[` p sub 1 ` psi ' ( N tilde ) `` + `` {S tilde sub 1 `` - ``S tilde sub 2 } over {N tilde sup 2} right \]
} (13)
and
func {Lambda tilde sub 2 `` =`` {partial V tilde sub 2 } over { partial N tilde }``=`` U' (b tilde sub 2)`` left \[`-` p sub 2 ` psi ' ( 1`-`N tilde ) `` + `` {S tilde sub 1 `` - `` S tilde sub 2 } over {(1`-`N tilde) sup 2} right \] .
}(14)

Substituting from (10) and (11) into (7), noting that
func { { partial W sub i } / {partial c tilde sub i} `` = `` pi tilde ` p sub i `` left\[ U'( c tilde sub i ) `` - `` U'(b tilde sub i)` right \] }

and MW_i/M=_i, we can show that

_i = _i for all (p_1, p_2), (i = 1, 2). (15)

Hence, in the Nash equilibrium we will have complete risk sharing among the individuals within each region, regardless of the degree of population mobility (i.e., regardless of the value of k).

To work out the degree of inter-regional risk sharing in the Nash equilibrium substitute from (12) into (8), noting that
func { { partial W sub i } /{partial S tilde sub i } `` = ``{ -` pi tilde } over {N tilde sub i} ` U'(b tilde sub i) , }
to obtain
func{ {Lambda tilde sub 1 } over { 1 `` - `` N tilde } `` {U'( b tilde sub 2 ) } `` + `` { Lambda tilde sub 2} over {N tilde } `{U'(b tilde sub 1) ~ >= ~ {- 2k U'(b tilde sub 1) } over {N tilde } , } }
(16)
and $_1$\$0 œ (p_1, p_2) for region 1, and
func{ {Lambda tilde sub 1 } over { 1 `` - `` N tilde } `` {U'( b tilde sub 2 ) } `` + `` { Lambda tilde sub 2} over { N tilde } `{U'( b tilde sub 1 ) ~ <= ~ { 2kU'( b tilde sub 2 ) } over {1``-``N tilde } , } } (17)

and $_2$\$0 œ (p_1, p_2) for region 2.[8] At this point it is important to point out that by maximizing $_i$ when the state of nature is revealed the regional authority i will be maximizing the utility of everyone who *ends*

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[8]As is well known, in dynamic settings it is possible that promises made by the government may not be implemented. Such time inconsistency problems come about because promises influence the behaviour of the private agents by affecting their expectations. Hence, once the expectations are formed the government's constraints are modified. The policies we have derived here are time consistent. To see this note that once (p_1, p_2) is revealed regional authority i will be maximizing $_i$, subject to (2), (4), (5), and $_i$\$0, by choosing $_i$, $_i$, and $_i$ for that particular state. As MW_i/M=M_i/M, for any variable in state (p_1, p_2) conditions (7) ! (9) will hold for all i after (p_1, p_2) is revealed.

up living in that region. The reason is that the non-pecuniary part of utility is a parameter in an agent's utility function. This, then, gives us another justification for employing W_i as the objective function of the regional authority i. ' [9]

Our next task is to determine whether the central authority can offer a degree of intra- and inter-regional risk sharing that will increase the value of the objective function of at least one regional authority without reducing the other.

III. The Planner's Problem in the Static Model

The central authority will choose ${}_1$, ${}_2$, ${}_1$, ${}_2$, and the net transfers from region 1 to 2, (/ ${}_1$! ${}_2$), in order to maximize a weighted sum of the objectives of the two regional authorities, $*W_1+(1!*)W_2$ for *0[0,1], subject to (2), (4) and (5) for all (p_1, p_2).

As discussed above, using (2), (3), (5) and (6), W_i can be expressed as a function of ${}_i$, ${}_i$, ${}_j$, and . Thus, the optimality conditions for this problem can be written as:

func { delta `` left \[ ` { partial W sub 1 } over {partial c tilde sub i} `` + `` {partial W_1} over {partial N tilde } ` { partial N tilde } over {partial c tilde sub i}` right \] `` + `` (1 `` -`` delta) `` left \[` { partial W sub 2 } over {partial c tilde sub i} `` + `` {partial W_2} over {partial N tilde } ` { partial N tilde } over {partial c tilde sub i } ` right \] `` = ``0, ~~~ (i `` = `` 1, `` 2) ~~~ forall~ (p sub 1, `` p sub 2),
} (18)

and

func { delta `` left \[ ` { partial W sub 1 } over {partial S tilde } `` + `` {partial W_1} over {partial N tilde } ` { partial N tilde } over {partial S tilde } ` right \] `` + `` (1``- delta) `` left \[ ` { partial W sub 2 } over {partial S tilde } `` + `` {partial W_2 } over {partial N tilde } ` { partial N tilde } over {partial S tilde } ` right \]`` = `` 0, ~~~ ~ forall (p sub 1 , `` p sub 2). }
(19)

Substituting from (10) and (11) into (18), noting that $MW_i/M={}_i$, $MW_i/M_j=0$ for j…i, and

func { { partial W sub i } / {partial c tilde sub i} `` = `` pi tilde ` p sub i `` left\[ U'( c tilde sub i ) `` - `` U'(b tilde sub i)` right \] }

, it can be shown that ${}_i={}_i$ (i=1,2) for all (p_1, p_2). Hence, with the central authority, as in the Nash

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[9]In the Introduction I discussed the role played by the assumption RN<0. At this point note that if RN=0 then from (13) and (14) at ${}_1={}_2=0$ we will have ${}_1={}_2=0$, and so $MW_i/M={}_i=0$. Now $MW_i/M_i=!UN(b_i)/{}_i<0$. Thus, if RN=0 then, from (8) and (9), no transfers will be made by either region in any state.

equilibrium, we will have complete risk sharing among individuals in both regions.

Now substitute from (12) into (19), and use the fact that
func { { partial W sub 1 } /{partial S tilde } `` = ``{ -` pi tilde } over {N tilde } ` U'(b tilde sub 1) }
and
func { { partial W sub 2 } /{partial S tilde } `` = ``{ ` pi tilde } over {1 `` -`` N tilde } ` U'(b tilde sub 2) }
to obtain
func{ {Lambda tilde sub 1 } over { 1 `` - `` N tilde } `` {U'( b tilde sub 2 ) } `` + `` { Lambda tilde sub 2} over {N tilde } `{U'( b tilde sub 1 ) ~ = ~ delta `` {- 2k U'(b tilde sub 1) } over { N tilde } `` + `` (1``- delta) `` { 2k U'(b tilde sub 2) } over {1``-``N tilde } } ~~~~ forall ~ (p sub 1 , `` p sub 2). } (20)

This is not consistent with (16) and (17) for all (p_1, p_2) when population is only imperfectly mobile (i.e., when k>0). To see this suppose, for simplicity, that in state (**(**, **$**) region 1 makes a transfer to region 2, because region 1 has experienced a more favourable shock (recall that **(**>**$**). Thus, in that state (16) will hold with equality, and (17) with inequality. By the symmetry of the model, it follows that, then, in state (**$**, **(**) region 2 will be making a transfer to 1. Thus, in that state (17) will hold with equality and (16) with inequality. There is no * that will be consistent with these results. If *=1 then (20) will be consistent with (16) and (17) in state (**(** , **$**), but not in state (**$**, **(**). Similarly, if *=0 then (20) will be consistent with (16) and (17) in state (**$**, **(**), but not in (**(**, **$**). Finally, if 0<*<0 then (20) will be inconsistent with (16) in state (**(**, **$**) and inconsistent with (17) in (**$**, **(**).[10, 11]

To see the reason for inefficiency in the Nash equilibrium first note that inter-regional risk sharing and inter-regional redistribution are inextricably linked. Inter-regional redistribution involves inter-regional transfers that are independent of the state (p_1, p_2), and are designed to increase the

[10]If in the Nash equilibrium no region makes a transfer in either state, then (16) and (17) will hold with strict inequalities in all states. It is clear that for each state we will be able to find a 0<*<0 such that (20) will hold for that particular state at the Nash equilibrium allocation. However, the *'s that we would require for different states will not be the same, again because there will be a coordination failure, as the regional authorities disagree over the degree of interregional redistribution.

[11]The extreme case of no population mobility (k=4) will further illustrate the point. In that case the migration responses (12) will all be zero. Thus, as $MW_i/M_i<0$, from (8) and (9), we will have ${}_1={}_2=0$, for all (p_1, p_2), and no interregional risk sharing. However, by the symmetry of the model we know that complete interregional risk sharing will be a Pareto superior allocation.

expected utility in one region at the expense of the other. Inter-regional risk sharing, on the other hand, involves transfers that are contingent on the state, and are, moreover, *coordinated* across states. The regional authorities do not have the instruments to coordinate their state contingent inter-regional transfers.

With attachment, regional authorities disagree over the degree of inter-regional redistribution. To see this, abstract completely from inter-regional risk sharing by considering a world in which there is only one possible state (p_1, p_2). Then the problem of region 1 will be to maximize V_1 subject to (4), and the other constraints. Region 1 will thus, in effect, be maximizing $V_2+2kN!k$. On the other hand, region 2 will be maximizing V_2. Clearly, region 1 will desire a larger N than region 2. In such a world the degree of inter-regional redistribution in the Nash equilibrium will be determined by the rich region *alone*, through its choice of the transfer. In that environment, strategic interaction will not result in an inefficient outcome essentially because the poor region is passive in the game (it is constrained by $S_i\$0$).[12] (See MM, p. 129, for a full discussion.)

On the other hand, in a world with more than one possible state of nature we need some *coordinated* state contingent transfers in order to exploit gains from inter-regional risk sharing. In the present, symmetric, model the state contingent transfers should flow from region 1 to 2 in state ((, $) (when in the Nash equilibrium the expected marginal utility of consumption in region 1 is lower), and from 2 to 1 in state ($,() (when in the Nash equilibrium the expected marginal utility of consumption in region 2 is lower). As the regional authorities disagree over the degree of inter-regional redistribution in the federation, they disagree over the sizes of these transfers in different states. Each region determines the size of the transfer in the state in which it experiences a more favourable shock than the other region. Thus, the authorities' disagreement over inter-regional redistribution leads to a failure to coordinate the state contingent transfers, and, as a result, some

[12]With only one state of nature we can always choose a *0[0,1] such that (20) is satisfied, and is consistent with (16) and (17), at the Nash equilibrium allocation. If in the Nash equilibrium region 1 makes a transfer then we set *=1; and if region 2 makes a transfer then *=0. If in the Nash equilibrium neither region makes a transfer then we can choose a 0<*<1.

gains from inter-regional risk sharing are not exploited.[13]

The regional authorities cannot sign contracts amounting to trading in state contingent claims in order to overcome this problem. This is because they will not honour such contracts once the state of nature is revealed (see footnote 8). To fully appreciate the reason for this it is important first to be clear on the difference between contracts signed by private individuals and contracts signed by governments (the regional authorities). With contracts signed by private individuals we assume that heavy sanctions will be imposed by governments if an individual repudiates a contract. It is this threat of sanctions by governments that prevents default by private individuals, even though these contracts are the outcome of free markets. The main problem with contracts signed by regional authorities is that for them to be honoured there must be a powerful central authority to impose heavy sanctions in case of repudiation. In deriving the decentralised equilibrium we have followed the literature and assumed away such a powerful central authority.[14] (The role of contracts between the regional authorities, and sanctions in case of repudiation will be taken up in the next section.)[15]

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[13]As pointed out in the Introduction, Burbidge and Myers (1994b) consider a model in which there are two types of individuals with different abilities. In their deterministic model there is perfect mobility by individuals of each type, and the regional authorities disagree over the degree of redistribution between the two types within each region (*intra-regional* redistribution). This disagreement leads to an inefficient outcome in their model. (The related papers by Wildasin (1991, 1995) are also concerned with intra-regional redistribution). The reason for inefficiency in the present model is fundamentally different. In the present model all individuals are identical in terms of their abilities before the state of nature is revealed; each facing the same probability of being lucky or unlucky. Moreover, both regional authorities are in complete agreement as to the degree of redistribution between the lucky and the unlucky within their region: both authorities treat the individuals equally, guaranteeing them equal levels of consumption in each state. The regional authorities in the present model disagree over the degree of *inter-regional* redistribution when population is imperfectly mobile, each wanting a higher income for all of its own residents in every state. (Persson and Tabellini (1996b) also concentrate on interregional redistribution.)

[14]At this point it is important to emphasize that the externalities which have been identified in the local public economics literature can often be corrected through side payments and enforceable contracts between the regions. Enforcement of such contracts, however, would require a powerful central authority. In the literature, it is assumed that in the presence of such a powerful central authority these externalities will in fact be corrected by direct central intervention. The assumption that is made in the present paper is consistent with this literature.

[15]Empirical estimates suggest that a substantial portion of the interregional risk sharing in Canada and the U.S. is carried out by the central authorities. Eichengreen (1993, pp. 1336-1338) provides a survey of this empirical literature. Eichengreen advocates similar risk sharing arrangements for the EU. (Also see Eichengreen and Friden (1994, pp. 183-188).) The inefficiency result derived in the present paper provides a justification for such a centralised arrangement for interregional risk sharing..

Some extreme cases will serve to further highlight the source of inefficiency in this model. First consider the case of an economy with asymmetric regions. Suppose one of the regions (region 1, say) is so rich that it makes transfers to region 2 in *all* the states. Then (16) will hold with equality, and (17) with inequality, in all the states. Then (20) will hold at the Nash equilibrium allocation with *=1. In this case the degree of inter-regional redistribution (and, hence, of inter-regional risk sharing) will be determined by region 1 (the rich region) alone. Region 2 will be essentially passive in the game, as in the deterministic model. Thus, in this case strategic interaction between the regions will not lead to inefficient degrees of risk sharing.

Finally, consider the case of perfect mobility (i.e., when k=0). Then, (16) and (17) will imply (20): there will be only one efficient allocation, and the Nash equilibrium will coincide with it. In this case the regional authorities know that with free mobility the same level of expected utility will prevail throughout the federation. They, thus, do not disagree over the degree of inter-regional redistribution; and there is no coordination failure of the type described above.

IV. The Infinite Horizon Model

In this section I consider a very simple extension of the model to an infinite horizon setting. This will facilitate a discussion of a simple reputation game, and its implication for the ability of the regional authorities to sign contracts which will allow them to implement an efficient allocation.

Assume that the preferences of individual n are given by

func { E `` left \{ `` Sum from {t``=``0} to {inf} `` (1`` + `` theta) sup {-t} `` left \[``   U({x}_t^ n ) `` + `` {I} sub t sup n    `` right \] `` right\},}

where x is consumption of the private good by this individual at time t, U(@) is a von Neumann-Morgenstern utility function, 2 is the rate of time preference, and I measures the non-pecuniary benefit the individual derives solely from its residence in a region. I is equal to k(1!n) if this individual resides in region 1 in period t, and it is kn if he resides in region 2 in period t.

Productivities (i.e., (p_1, p_2)) are assumed to be intertemporally independent. This, together with the fact that there are no predetermined variables in the model, means that the value of a variable in any period is independent of its values in other periods. Hence, in most of what follows time subscripts will be suppressed. The sequence of events in each period is as before.

In this dynamic setting the choice of the objective function is even harder than before. For the sake of simplicity, and in order to draw heavily on the results that have already been derived, I assume that in each period regional authority i maximizes W_i. Again, as mentioned above, in the two extreme cases of perfect mobility (k=0) and no mobility (k=4) this is the most sensible objective function. In the less extreme cases, by maximizing W_i the regional authority i would certainly maximize the lifetime utility of its permanent residents ! i.e., the agents who decide to reside in the region in all states. Furthermore, with relatively mild assumptions about the characteristics of the equilibrium outcome, it can be argued that by maximizing W_i the regional authority i will be doing its best in order to maximize the expected utility of its median voter in all periods.[16] Next consider the case in which in a particular period region 1 has experienced a less favourable shock (state ($, ()). Then, if in the subsequent period the same productivity is experienced all individuals in region 1 would agree that they would like to receive a larger transfer from 2. On the other hand, if in the subsequent period region 1 experiences a more favourable shock, then the conditions outlined above would ensure that the existing residents of region 1 would agree on the size of the transfers $_1$. Thus, in this case again, by maximizing W_1 in the subsequent period, regional authority 1 will be doing its best in order to maximize the expected utility of its median voter. By the symmetry of the model, similar arguments will hold for region 2.

If the regional authorities do not care about their reputation, then the policies they will be pursuing will be described by (7) ! (9). The question is whether in this dynamic setting the regional

[16] The details of what is needed for this to be true are as follows. Suppose in a particular period region 1 has experienced a more favourable shock (i.e., the state is ((, $)), and it has made a transfer to 2, as discussed in Section II. Then, the marginal individual in this region (individual N((, $)) would like to see a larger transfer; because then it would migrate to region 2 and collect part of the transfer without paying for it. Suppose individual N((, $)/2 does *not* like to see a larger transfer. Then, the majority of voters in that region would like to see the same policy if in the next period region 1 continues to experience the more favourable outcome. Moreover, if in the next period region 1 experiences a less favourable shock (state ($, ()) then that region will be constrained by the condition $_1$$0. Thus, if in a particular period region 1 has experienced a more favourable shock then by maximizing W_1 in the subsequent period the regional authority will be doing the best it can in order to maximize the expected utility of its median voter.

authorities can credibly sign contracts that will allow them to implement an efficient set of policies. The efficient allocation we will be concerned with here will be the one that attaches equal weights on the objectives of both regional authorities (*=0.5). This will be the allocation most acceptable to both authorities, and will also preserve the symmetry of the model.

To facilitate a discussion of the sanctions that can be imposed by a regional authority if the other authority repudiates the contract, consider the simplest reputation game. Assume that if a region (region i, say) that has experienced a more favourable shock does not make the necessary transfers for risk sharing, then it will lose its reputation and will never obtain the cooperation of the other region (region j). Thus, in that case, if region i does not make the necessary transfers then its expected utility for all the following periods will be W (the expected utility with no coordination). On the other hand, if the necessary transfers are made then region j will cooperate in all successive periods, and the expected utility for region i in all those periods will be W (the expected utility with coordination). Thus, the present value of the losses the regional authority i will suffer if it refrains from making the necessary transfers in the state in which it experiences a more favourable shock will be (W ! W)/2. On the other hand, the instantaneous gains from withholding the transfers necessary for risk sharing is U() ! U(), where and are, respectively, consumption per person in region i with and without coordination if that region experiences a more favourable shock than the other. Clearly, if 2 is sufficiently small then the present value of the losses will dominate the instantaneous gains from withholding the transfers necessary for risk sharing. In that case, the necessary state contingent transfers will be made voluntarily. The regional authorities will then be able to credibly sign contracts in order to implement the efficient policies.

The next question is whether improvements in the degree of population mobility will make it more likely that the regional authorities will be able to commit to an efficient allocation. To answer this question first note that as the degree of attachment to home decreases (k falls) the disagreement between the regional authorities narrows. This then reduces the gains from risk sharing (W ! W). Thus, a fall in k reduces the costs to a region of losing reputation (i.e., (W ! W)/2) by not making the necessary state contingent transfers when it experiences a more favourable shock. On the other hand, as k falls the gains to such a region from withholding the transfers which are over and above those without coordination also fall for the following two reasons. First, the size of the transfer that the region has to make beyond those without coordination falls with k, because the disagreement between the regional authorities narrows. Second, from (12), as k decreases the emigration associated with a transfer increases. Thus, both the costs and benefits of withholding the transfers necessary for risk sharing fall as the degree of population mobility increases. It is, therefore, not possible to say *a priori* whether improvements in the degree of population mobility will make it more likely that the regional authorities will be able to commit to an efficient allocation.

Let [W ! W]/[U() ! U()] be the critical value 2* such that if 2<2* then the efficient allocation will be implemented by the regional authorities. The above reasoning suggests that 2* will be a non-monotonic function of k. Numerical evaluations of the model indicate that 2* is highly non-monotonic is k (see the Appendix.)

V. Conclusions

This paper has considered the issue of risk sharing in a federation with population mobility. It was shown that there is some scope for inter-regional risk sharing that is not fully exploited by the regional authorities when population is only imperfectly mobile. Intra-regional risk sharing is, however, perfect even with decentralisation. It was also shown that in the infinite horizon setting the regional authorities may credibly commit to an efficient set of state contingent policies without any need for central intervention. Surprisingly, improvements in the degree of population mobility may make such commitments *less* likely.

There are various directions in which this basic model could be extended. The reputation game that was used in this paper was the simplest possible game. More elaborate games have been used with regard to monetary and fiscal policies ! see Persson and Tabellini (1990). One could allow the regional authorities to run budget deficits. The technological shocks were assumed to be intertemporally independent. It would be interesting to work out the importance of policy coordination when shocks persist. Finally, it would be fruitful to consider the possibility of capital accumulation. This would add a predetermined variable to the model, and also allow the possibility of capital tax competition.

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Appendix A

In this appendix, I describe the procedure for a numerical evaluation of the effects of improvements in the degree of population mobility on the likelihood of the regional authorities being able to successfully commit to the efficient allocation which places equal weights on the objective functions of the two regional authorities.

I assume that the function **R**(") is of the simple form

R($_i$)= ! /**"** (A.1)

while the utility function is
func{ U (b tilde sub i) `` = `` log (b tilde sub i)

.

} (A.2)

In Appendix B, I show that with perfect population mobility (k=0) if **R**(") is given by (A.1) then no region will make an inter-regional transfer. From this, one can conclude that with (A.1) there will be no inter-regional transfers for any value of k. The reason is that as k increases the incentives of the richer regional authority to make inter-regional transfers are weakened. If at k=0 the rich regional authority does not make transfers, then they will not make transfers with k>0.

The numerical examples are consistent with the preceding proposition. In the numerical example, I set **"**=0.5, p_1=0.95, p_2=0.7, and **B**=0.5. With p_1>p_2 we must have ${}_2$=0. Now note that from equations (2), (5) and (15)
func{ b tilde sub 1 `` = `` p sub 1 ` psi (N tilde ) `+ ` {S tilde sub 2 ` - ` S tilde sub 1} over {N tilde }
~~~~~~~~ a n d ~~~~~~~~ b tilde sub 2 `` =`` p sub 2 ` psi (1`-`N tilde )` + ` {S tilde sub 1 ` - ` S tilde sub 2} over {1 `- `N tilde }.

} (A.3)

Next, note that **if ${}_1$>0** then (16) would hold with equality; and the solution for and ${}_1$ would be given by (16) (with equality), (13), (14), (A.3) and (4). The numerical solution to these equations, however, give us ${}_1$<0. These solutions are, therefore, consistent with the above proposition that with (A.1) no region will make a transfer regardless of the degree of population mobility.

Equipped with this result, we can obtain the value of in the Nash equilibrium by simply
~~~~~~~~

setting $_1$=$_2$=0, and then substituting for $_1$ and $_2$ from (A.3) into (4). Once   is obtained as a solution to (4), we can substitute it back into (A.3) and obtain the Nash equilibrium levels of $_1$ and $_2$. Henceforth, call these   and . Because the model is symmetric, the values of $_1$ and $_2$ for when p_1=0.7 and p_2=0.95 can be obtained by switching   and . We can, thus, calculate the expected values of instantaneous utility without cooperation, W and W.

In order to obtain the cooperative solution, first set $_1$! $_2$= in (A.3). Then solve   (20) (with *****=0.5), (13), (14), (A.3) and (4) for   and . After this, substitute for   and   into (A.3) in order to obtain the values of $_1$ and $_2$ with cooperation. Henceforth, call these   and . Because the model is symmetric, the values of $_1$ and $_2$ for when p_1=0.7 and p_2=0.95 can be obtained by switching   and . We can, thus, calculate the expected value of instantaneous utility with cooperation, W and W.

We are now in a position to calculate

2*=[W ! W]/[U() ! U()],

and plot the values of **2*** against k.

We can see that the resulting plot is highly non-monotonic, especially when k is small (i.e., when population is more mobile). The numerical results also show that when k is very large **2*** tends towards 0. Hence, it is hard to implement the efficient outcome when population is not mobile.

Appendix B

In this appendix, I prove that if the function **R**(") is of the simple form

R($_{i}$)=!/"   (B.1)

then with perfect mobility (k=0) in the Nash equilibrium no region will make a transfer to the other region. To prove this, first note that with perfect mobility the migration equilibrium condition (4) implies that we should have

$_{1} = _{2}$.   (B.2)

With perfect mobility the conditions (16) and (17) reduce to

func{ {Lambda tilde sub 1 } over { 1 `` - `` N tilde } `` {U'( b tilde sub 2 ) }    `` = `` {   Lambda tilde sub 2}   over { N tilde }   `{U'(b tilde sub 1) ~ } . }   (B.3)

Now note that from equations (2), (5) and (15)

func{ b tilde sub 1 `` = `` p sub 1 ` psi (N tilde ) `+ ` {S tilde sub 2 ` - ` S tilde sub 1} over {N tilde } ~~~~~~~~ a n d ~~~~~~~~ b tilde sub 2 `` =`` p sub 2 ` psi (1`-`N tilde )` + ` {S tilde sub 1 ` - ` S tilde sub 2} over {1 `- `N tilde }.

}(B.4)

Suppose $p_1>p_2$. In that case, we know that =0. Substituting from (B.4) into (B.2) and (B.3), we will get two equations in two unknowns, $_1$ and . If **R**(") is given by (B.1), then it follows that both (B.2) and (B.3) will be satisfied with $_1$==0, and with   given by

p_1"   = p_2(1!)".   (B.5)

Similarly, if $p_2>p_1$ then with perfect mobility (k=0), and with (B.1), at the Nash equilibrium we will have $_1$==0, and   given by (B.5). One can conclude that if (B.1) holds then at the Nash equilibrium there will not be any interregional transfers with perfect mobility (k=0).

Values of 2* for Different k

k	2*
0.4000000E-03	0.5920286E-04
0.4500000E-03	0.6657890E-04
0.5000000E-03	0.5001479
0.5500000E-03	0.5001627
0.6000000E-03	0.8875749E-04
0.6500000E-03	1.000288
0.7000000E-03	0.1035103E-03
0.7500000E-03	1.000333
0.8000000E-03	0.2501774
0.8500000E-03	0.1256908E-03
0.9000000E-03	0.5002661
0.9500000E-03	0.5002809
0.1000000E-02	0.1668637
0.1050000E-02	0.3335919
0.1100000E-02	0.2502438
0.1150000E-02	0.2002379
0.1200000E-02	0.3002836
0.1250000E-02	0.2002585
0.1300000E-02	0.3003072
0.1350000E-02	0.2502991
0.1400000E-02	0.2503101
0.1450000E-02	0.3336901
0.1500000E-02	0.2860623
0.1550000E-02	0.2503432
0.1600000E-02	0.3128838
0.1650000E-02	0.2781567
0.1700000E-02	0.2781681
0.1750000E-02	0.2503874
0.1800000E-02	0.2503984
0.1850000E-02	0.2731491
0.1900000E-02	0.3186405
0.1950000E-02	0.2504315
0.2000000E-02	0.2696846
0.2050000E-02	0.3081807
0.2100000E-02	0.2862009
0.2150000E-02	0.3219494
0.2200000E-02	0.3219614
0.2250000E-02	0.2504975
0.2300000E-02	0.2817797
0.2350000E-02	0.2652356
0.2400000E-02	0.3241119
0.2450000E-02	0.2637089
0.2500000E-02	0.2900552
0.2550000E-02	0.3006011
0.2600000E-02	0.2863161

0.2650000E-02	0.2863276
0.2700000E-02	0.2614746
0.2750000E-02	0.2832425
0.2800000E-02	0.2606266
0.2850000E-02	0.3006713
0.2900000E-02	0.2599075
0.2950000E-02	0.2784532
0.3000000E-02	0.2685351
0.3050000E-02	0.2765585
0.3100000E-02	0.2840480
0.3150000E-02	0.2910551
0.3200000E-02	0.2663459
0.3250000E-02	0.2734661
0.3300000E-02	0.2801684
0.3350000E-02	0.2721887
0.3400000E-02	0.2785554
0.3450000E-02	0.2845790
0.3500000E-02	0.2771147
0.3550000E-02	0.2828675
0.3600000E-02	0.2691058
0.3650000E-02	0.2935334
0.3700000E-02	0.2865687
0.3750000E-02	0.2622027
0.3800000E-02	0.2786463
0.3850000E-02	0.2897813
0.3900000E-02	0.2774856
0.3950000E-02	0.2821565
0.4000000E-02	0.2866374
0.4050000E-02	0.2754311
0.4100000E-02	0.2797839
0.4150000E-02	0.2839730
0.4200000E-02	0.2880075
0.4250000E-02	0.2777549
0.4300000E-02	0.2904702
0.4350000E-02	0.2854843
0.4400000E-02	0.2843449
0.4450000E-02	0.2879128
0.4500000E-02	0.2913661
0.4550000E-02	0.2822929
0.4600000E-02	0.2856742
0.4650000E-02	0.2889535
0.4700000E-02	0.2764311
0.4750000E-02	0.2796560
0.4800000E-02	0.2827904
0.4850000E-02	0.2858381
0.4900000E-02	0.2888027
0.4950000E-02	0.2774422
0.5000000E-02	0.2803628
0.5050000E-02	0.2896285

0.5100000E-02	0.2923216
0.5150000E-02	0.2913157
0.5200000E-02	0.2843257
0.5250000E-02	0.2928843
0.5300000E-02	0.2770666
0.5350000E-02	0.2853196
0.5400000E-02	0.2821338
0.5450000E-02	0.2869683
0.5500000E-02	0.2893125
0.5550000E-02	0.2885132
0.5600000E-02	0.2825308
0.5650000E-02	0.2819032
0.5700000E-02	0.2813012
0.5750000E-02	0.2834944
0.5800000E-02	0.2801680
0.5850000E-02	0.2822891
0.5900000E-02	0.2817208
0.5950000E-02	0.2837692
0.6000000E-02	0.2877437
0.6050000E-02	0.2826323
0.6100000E-02	0.2820944
0.6150000E-02	0.2772573
0.6200000E-02	0.2853193
0.6250000E-02	0.2829406
0.6300000E-02	0.2865698
0.6350000E-02	0.2860090
0.6400000E-02	0.2877610
0.6450000E-02	0.2788089
0.6500000E-02	0.2844327
0.6550000E-02	0.2877642
0.6600000E-02	0.2834625
0.6650000E-02	0.2867103
0.6700000E-02	0.2825502
0.6750000E-02	0.2857185
0.6800000E-02	0.2816909
0.6850000E-02	0.2847832
0.6900000E-02	0.2843354
0.6950000E-02	0.2839001
0.7000000E-02	0.2834767
0.7050000E-02	0.2863829
0.7100000E-02	0.2840888
0.7150000E-02	0.2836829
0.7200000E-02	0.2832876
0.7250000E-02	0.2860340
0.7300000E-02	0.2808002
0.7350000E-02	0.2882717
0.7400000E-02	0.2831236
0.7450000E-02	0.2827619
0.7500000E-02	0.2824090

Values of 2* for Large k

k	2*
1.000000	0.1777427
2.000000	0.1444939
3.000000	0.1283825
4.000000	0.1188946
5.000000	0.1126446
.	
.	
.	
10.00000	0.9866853E-01
11.00000	0.9728818E-01
12.00000	0.9612200E-01
13.00000	0.9512535E-01
14.00000	0.9426250E-01
15.00000	0.9350840E-01
.	
.	
.	
95.00000	0.8416210E-01
96.00000	0.8414280E-01
97.00000	0.8412433E-01
98.00000	0.8410604E-01
99.00000	0.8408775E-01
100.0000	0.8407001E-01