High-Order Consumption Moments and Asset Pricing*

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Abstract

We propose the consumption CAPM, in which the pricing kernel depends on the moments of the cross-sectional distribution of consumption. Using data from the U.S. Consumer Expenditure Survey, we find that this model explains the observed equity premium and risk-free return simultaneously with realistic values of both risk aversion and the time preference discount factor when the first four moments of the cross-sectional distribution of consumption are taken into consideration. This result shows that not only the mean, as in the representative-agent consumption CAPM, but also the variance, third, and fourth moments of the cross-sectional distribution of consumption are important determinants of the equilibrium asset returns.

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1 Introduction

Over the last two decades numerous studies have focused on the Lucas (1978) consumption-based capital asset pricing model, hereafter consumption CAPM. Lucas (1978) considers a one-good, pure exchange economy with a single consumer, interpreted as a representative for a large number of identical infinitely-lived consumers. The single power utility maximizing representative consumer can freely trade in perfect capital markets without transaction costs or other capital market imperfections. In equilibrium of this economy, the representative agent's consumption equals aggregate consumption per capita, so that average consumption can be used in place of the consumption of any particular individual and hence the stochastic discount factor (SDF), or pricing kernel, in the implied Euler equation is the average consumption growth rate raised to the power $-\gamma$, where γ is the coefficient of relative risk aversion.

Empirical evidence is that this basic consumption CAPM is inconsistent with data on consumption and asset returns in many respects. E.g., a reasonably parameterized representative-agent model generates a mean equity premium, which is substantially lower than that observed in data. This model is inconsistent with the observed mean excess return on the market portfolio over the risk-free rate unless the representative consumer is assumed to be implausibly risk averse. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others. Another anomaly with the same model is that it yields the risk-free rate, which is too high compared to the observed mean return on the risk-free asset. As a consequence, a time preference discount factor greater than one is required to fit the mean risk-free rate of return. This is the risk-free rate puzzle as described in Weil (1989).

Faced with this empirical evidence, financial economists have explored several ways to improve the ability of the consumption CAPM to fit the data. One straightforward response to the difficulties with this model is to try different functional forms for utility (e.g., Aschauer 1985; Eichenbaum *et al.* 1988; Epstein and Zin 1989, 1991; Startz 1989; Sundaresan 1989; Abel 1990, 1999; Constantinides 1990; Harvey 1991; McCurdy and Morgan 1991; Chou *et al.* 1992; Bakshi and Chen 1996; Campbell and Cochrane 1999; Gordon and St-Amour 2004). Another suggestion is that market frictions, such as transactions costs and limits on borrowing or short sales, can make aggregate consumption in the economy an inadequate proxy for the consumption of asset holders (e.g., Campbell and Mankiw 1990; Mankiw and Zeldes 1991; Basak and Cuoco 1998; Alvarez and Jermann 2000; Constantinides *et al.* 2002). Bewley (1982), Mehra and Prescott (1985), Mankiw (1986), Constantinides and Duffie (1996), Brav *et al.* (2002), Jacobs and Wang (2004), Balduzzi and Yao (2005), and Kocherlakota and Pistaferri (2005) argue that consumer heterogeneity can be relevant for asset pricing.

Another strand of the literature alters the specification of probability distributions in the representative-agent consumption CAPM (e.g., Kahneman and Tversky 1979; Rietz 1988; DeLong *et al.* 1990; Barberis *et al.* 1998; Abel 2002). Although substantial progress has been made, there is no yet a model of the SDF that would be generally accepted.

One plausible way to explain the poor empirical performance of the Lucas (1978) consumption CAPM is to argue that the empirical failure of this model can be due, in part, to the fact that this model abstracts from consumer heterogeneity. The potential for the model with heterogeneous consumers to explain the equilibrium behavior of stock and bond returns, both in terms of the level of equilibrium rates and the discrepancy between equity and bond returns, was first suggested by Bewley (1982), Mehra and Prescott (1985), and Mankiw (1986).

The empirical evidence on whether consumer heterogeneity plays an important role in explaining asset returns is somewhat contradictory. Cogley (2002) uses a Taylor approximation of the individual's intertemporal marginal rate of substitution and like Jacobs (1999), who investigates Euler equations at the disaggregate level, does not find empirical evidence that the assumption of consumer heterogeneity can improve substantially the empirical performance of the consumption CAPM.

Brav *et al.* (2002), Jacobs and Wang (2004), Balduzzi and Yao (2005), and Kocherlakota and Pistaferri (2005) however reach an opposite conclusion. Brav *et al.* (2002) take a Taylor approximation of the equally weighted average of the investors' intertemporal marginal rates of substitution and find that when the third moment of the cross-sectional distribution of the individual consumption growth rate is taken into account, the model is able to fit the observed excess return on the market portfolio over the risk-free rate with low risk aversion. This result provides some evidence for the importance of consumer heterogeneity for explaining the market premium. When testing the Euler equation for the risk-free rate, they find that observation error in individual consumption severely biases downward the estimated time preference discount factor and renders the estimates meaningless. Jacobs and Wang (2004) investigate the performance of a pricing kernel linear in the first and the second moment of the cross-sectional distribution of consumption growth and find that the consumption CAPM with this SDF significantly outperforms the capital asset pricing model in explaining the cross-section of asset returns. Empirical evidence in Balduzzi and Yao (2005) is that the pricing kernel that depends on changes in the cross-sectional variance of log consumption reconciles the observed equity premium with economically plausible values of risk aversion. Kocherlakota and Pistaferri (2005) find that the SDF given by the reciprocal of the gross growth of the γth moment of the consumption distribution, where γ is the coefficient of relative risk aversion, can explain the equity premium, but not bond and stock market returns simultaneously.

In this paper, we also investigate the role of consumer heterogeneity in explaining asset returns. In contrast to Brav *et al.* (2002), we take a Taylor approximation of the cross-sectional average of investors' marginal utilities of consumption around average consumption and derive the SDF that is the discounted ratio of the Taylor approximations of the average of investors' marginal utilities of consumption at two successive dates. This allows to express the SDF in terms of the cross-sectional moments of consumption instead of the cross-sectional moments of the intertemporal marginal rate of substitution as in Brav *et al.* (2002).

To test the ability of this SDF to explain the equity premium and the risk-free rate, we use individual-level quarterly consumption data from the U.S. Consumer Expenditure Survey (CEX). Although our approach is different from those in Brav *et al.* (2002) and Kocherlakota and Pistaferri (2005), we also find that taking account of consumer heterogeneity helps explain the equity premium. Our result is that when the Taylor approximation of the average of investors' marginal utilities of consumption captures the first three moments of the cross-sectional distribution of consumption, the model proposed in this paper fits the observed excess return on the market portfolio over the risk-free rate with an economically plausible value of risk aversion.

An important result is that, in contrast to the Brav *et al.* (2002) and Kocherlakota and Pistaferri (2005) SDF's, our pricing kernel explains the observed equity premium and risk-free return simultaneously with realistic values of both risk aversion and the time preference discount factor when the fourth moment of the cross-sectional distribution of consumption is taken into consideration. In Monte Carlo simulations, we find that this result is robust to the measurement error in reported individual consumption.

The main result of this paper is that not only the mean, as in the representative-agent consumption CAPM, but also the variance, third, and fourth moments of the cross-sectional distribution of consumption are important determinants of the equilibrium asset returns. When the first four cross-sectional moments of consumption are considered, the consumption CAPM can explain both the equity premium and the risk-free rate with economically plausible values of the agent's preference parameters.

The rest of the paper proceeds as follows. Section 2 derives the pricing kernel. Section 3 describes the data and presents the empirical results on the equity premium and the risk-free rate. Section 4 concludes.

2 An Approximate Equilibrium Asset Pricing Model

Consider the intertemporal choice problem of an investor, who can freely trade in perfect capital markets without frictions, short-sale restrictions, or taxes and who maximizes expected lifetime discounted utility:

$$E_t\left[\sum_{j=0}^{\infty}\delta^j u\left(C_{k,t+j}\right)\right], \tag{1}$$

where δ is the time preference discount factor, $C_{k,t}$ is the individual k's consumption in period t, $u(\cdot)$ is a current period utility function,[1] and $E_t[\cdot]$ is an expectations operator. Expectations in (1) are taken conditional on information available at time t.

The investor's optimal consumption profile must satisfy the following first-order condition, or Euler equation:

$$E_t\left[\delta u'\left(C_{k,t+1}\right)R_{i,t+1}\right] = u'\left(C_{k,t}\right), \tag{2}$$

where $R_{i,t+1}$ is the simple gross return on asset i, in which the investor has a positive position. The right-hand side of (2) is the marginal utility cost of decreasing consumption by one dollar at time t. The left-hand side of (2) is the increase in expected utility at time $t+1$, which results from investing the dollar in asset i at time t, selling it at time $t+1$ for $R_{i,t+1}$ dollars, and consuming the proceeds.

The first-order condition (2) holds for each investor at date t and hence the equally weighted average of equation (2) over all investors in the economy must also hold at date t:

$$E_t\left[\delta K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t+1}\right)R_{i,t+1}\right] = K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right). \tag{3}$$

If we divide the both sides of (3) by $K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right)$, we get

$$E_t\left[\delta\frac{K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t+1}\right)}{K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right)}R_{i,t+1}\right] = 1. \tag{4}$$

In this model, the SDF is the discounted ratio of the averages of investors' marginal utilities of consumption at two successive dates:

$$M_{t+1} = \delta\frac{K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t+1}\right)}{K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right)}. \tag{5}$$

Since the ratio of the cross-sectional averages of marginal utilities of consumption is not generally the same as the cross-sectional average of the ratios of marginal utilities of consumption at two successive dates, SDF (5) generally differs from

$$M_{t+1} = \delta K^{-1}\sum_{k=1}^{K}\frac{u'\left(C_{k,t+1}\right)}{u'\left(C_{k,t}\right)}. \tag{6}$$

[1] The utility function is supposed to be increasing ($u'(\cdot) > 0$), capturing the desire for more consumption, and concave ($u''(\cdot) < 0$), reflecting the declining marginal value of additional consumption. Besides, we assume an agent to be prudent ($u'''(\cdot) > 0$).

In the case of CRRA preferences, e.g., SDF's (5) and (6) coincide only if all the agents in the economy are risk neutral and/or if the agents have the same consumption growth rate. When all agents have CARA preferences, SDF (5) coincides with (6) only if any individual's consumption is constant over time.

From equation (2), it follows that for the excess return on a risky asset over the risk-free rate we have to take into account the marginal utility of consumption and not the ratio of marginal utilities at two successive dates. Therefore, we prefer to use SDF (5), rather than (6). This distinguishes our approach from that in Brav *et al.* (2002), whose approach is built on the pricing kernel given by equation (6).

Since the agent's utility function is concave, the average of investors' marginal utilities of consumption is very vulnerable to the measurement error in reported individual consumption when there are individuals with low consumption. The degree of this vulnerability depends on the degree of concavity of the utility function. This can make it difficult to test directly the hypothesis that (5) is a valid SDF, especially when the number of individuals in the cross-section is small.

Suppose that the agent's utility function $u(\cdot)$ is $N+1$ times differentiable and take an N-order Taylor approximation of the average of investors' marginal utilities of consumption around average consumption $\overline{C}_t$:

$$K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right)\approx\sum_{n=0}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)K^{-1}\sum_{k=1}^{K}\left(C_{k,t}-\overline{C}_t\right)^n \tag{7}$$

or equivalently

$$K^{-1}\sum_{k=1}^{K}u'\left(C_{k,t}\right)\approx\sum_{n=0}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)Z_{n,t}=u'\left(\overline{C}_t\right)+\sum_{n=2}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)Z_{n,t}, \tag{8}$$

where $Z_{n,t}=K^{-1}\sum_{k=1}^{K}\left(C_{k,t}-\overline{C}_t\right)^n$ is the nth moment the cross-sectional distribution of consumption. Here, $u^{(n)}(\cdot)$ denotes the nth derivative of $u(\cdot)$.

Because marginal utility is strictly convex, a direct consequence of Jensen's inequality is that the average of investors' marginal utilities of consumption is greater than or equal to the marginal utility of average consumption. The inequality is strict unless all agents in the economy have identical consumption, $C_{k,t}=\overline{C}_t$, as in the Lucas (1978) representative-agent model.

Substituting (8) into (4) yields the following approximate Euler equation:

$$E_t\left[\delta\frac{u'\left(\overline{C}_{t+1}\right)+\sum_{n=2}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_{t+1}\right)Z_{n,t+1}}{u'\left(\overline{C}_t\right)+\sum_{n=2}^{N}\frac{1}{n!}u^{(n+1)}\left(\overline{C}_t\right)Z_{n,t}}R_{i,t+1}\right]\approx 1. \tag{9}$$

or equivalently

$$E_t\left[\delta\frac{u'\left(\overline{C}_{t+1}\right)}{u'\left(\overline{C}_t\right)}\frac{A_{t+1}}{A_t}R_{i,t+1}\right]\approx 1, \tag{10}$$

where $A_t = 1 + \sum_{n=2}^{N} \frac{1}{n!} \frac{u^{(n+1)}(\overline{C}_t)}{u'(\overline{C}_t)} Z_{n,t}$.

This is the approximate equilibrium asset pricing model. In this model, the SDF is the SDF in the representative agent consumption CAPM times a new term, which is a function of the moments of the cross-sectional distribution of consumption at dates t and $t+1$:

$$M_{t+1} = \delta \frac{u'\left(\overline{C}_{t+1}\right)}{u'\left(\overline{C}_t\right)} \frac{A_{t+1}}{A_t}. \tag{11}$$

An attractive feature of model (10) is that the set of factors in A_t obtains endogenously from the Taylor approximation and the signs of the factor coefficients are driven by preference assumptions. If the estimation of the cross-sectional consumption moments is less susceptible to measurement error than the average of investors' marginal utilities of consumption, then SDF (11) is less vulnerable to the measurement error in reported individual consumption compared with SDF (5).

When in equilibrium all investors have identical consumption, $C_{k,t} = \overline{C}_t$, by Jensen's inequality $K^{-1} \sum_{k=1}^{K} u'\left(C_{k,t}\right) = u'\left(\overline{C}_t\right)$ and therefore model (10) reduces to the representative-agent model

$$E_t \left[\delta \frac{u'\left(\overline{C}_{t+1}\right)}{u'\left(\overline{C}_t\right)} R_{i,t+1} \right] = 1, \tag{12}$$

in which the pricing kernel is the discounted representative-agent intertemporal marginal rate of substitution in consumption between dates t and $t+1$:

$$M_{t+1} = \delta \frac{u'\left(\overline{C}_{t+1}\right)}{u'\left(\overline{C}_t\right)}. \tag{13}$$

Hence the representative-agent consumption CAPM (12) can be viewed as a special case of model (10) when $C_{k,t} = \overline{C}_t$ for any investor k. Clearly, the assumption that all investors in the economy have the same consumption is not realistic. It follows that $u'\left(\overline{C}_t\right)$ is only a rough approximation of the average of investors' marginal utilities of consumption, especially when the values of $C_{k,t}$ differ significantly from $\overline{C}_t$. This suggests that the second and higher moments of the cross-sectional distribution of consumption are potentially important determinants of the equilibrium asset returns.

An important question is how many moments of the cross-sectional distribution of consumption should be taken into account or, in other words, at which order the Taylor approximation in model (10) should be truncated. To approximate the average of investors' marginal utilities of consumption as better as possible, we may wish to consider as many cross-sectional moments of consumption as possible. One problem here is that, to expand the average of investors' marginal utilities of consumption up to the nth cross-sectional moment of consumption, we must be able to sign $(n+1)$th derivative of the utility function. Another problem is that, like the average of investors' marginal utilities of consumption, high-order consumption moments may be vulnerable to the measurement error in reported individual consumption what can make it difficult to test model (10) when a high-order Taylor approximation is used. This suggests that the Taylor approximation of the average of investors' marginal utilities of consumption should be truncated at the cross-sectional moment of consumption, for which first we are still able to sign the coefficient and second which is not vulnerable to the measurement error in reported individual consumption.

When signing the derivatives of the agent's utility function, as is conventional in the literature, we can assume that the marginal utility of consumption is positive ($u'(\cdot) > 0$) and an agent is risk averse ($u''(\cdot) < 0$). Decreasing absolute risk aversion implies $u'''(\cdot) > 0$. Assume further that absolute prudence, $AP(\cdot) = -u'''(\cdot)/u''(\cdot)$, is decreasing.[2]

Proposition 1. *Absolute prudence is decreasing (DAP) if and only if* $u''''(\cdot) < -AP(\cdot)\, u'''(\cdot)$. *The condition* $u''''(\cdot) < 0$ *is necessary for DAP* (see Appendix for the proof).

Thus, assuming positive marginal utility, risk aversion, decreasing absolute risk aversion, and decreasing absolute prudence, we can sign the first four derivatives of the utility function what allows to expand the average of investors' marginal utilities of consumption up to the third cross-sectional moment of consumption.

To capture the potential effect of consumer heterogeneity on equilibrium asset returns, it might be useful to expand the average of investors' marginal utilities of consumption up to the fourth moment of the cross-sectional distribution of consumption. To sign the coefficient of the fourth polynomial term in the Taylor approximation, we must determine the sign of the fifth derivative of the utility function. A natural assumption is that, like absolute risk aversion, absolute prudence is convex, i.e. the absolute level of precautionary savings is decreasing in wealth at a decreasing rate.

Proposition 2. *Absolute prudence is convex (CAP) if and only if* $u'''''(\cdot) > -2AP'(\cdot)\, u'''(\cdot) - AP(\cdot)\, u''''(\cdot)$. *If preferences exhibit prudence and decreasing absolute prudence, then* $u'''''(\cdot) > 0$ *is the necessary condition for CAP* (see Appendix for the proof).

The restriction of decreasing absolute prudence allows us to sign the fifth derivative of the utility function. Combined with the conditions $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'''(\cdot) > 0$, and $u''''(\cdot) < 0$, this makes it possible to investigate the role of the first four cross-sectional moments of consumption in explaining asset returns.

A class of utility functions widely used in the literature is the set of utility functions

[2] Kimball (1990) defines "prudence" as a measure of the sensitivity of the optimal choice of a decision variable to risk (of the intensity of the precautionary saving motive in the context of the consumption-saving decision under uncertainty). A precautionary saving motive is positive when $-u'(\cdot)$ is concave ($u'''(\cdot) > 0$) just as an individual is risk averse when $u(\cdot)$ is concave. Intuitively, the willingness to save is an increasing function of the expected marginal utility of future consumption. Since marginal utility is decreasing in consumption, the absolute level of precautionary savings must also be expected to decline as consumption rises.

exhibiting an harmonic absolute risk aversion (HARA). HARA utility functions take the following form: $u(C_t) = a(b + zC_t/\gamma)^{1-\gamma}$, where a, b, and z are constants, $b + zC_t/\gamma > 0$, and $az(1-\gamma)/\gamma > 0$ (the last inequality is necessary to insure that $u'(\cdot) > 0$). There are several well-known utility specifications that can be obtained as special cases of HARA utility functions. CRRA utility functions, $u(C_t) = a(zC_t/\gamma)^{1-\gamma}$, can be obtained by selecting $b = 0$ (in the special case when $a = (z/\gamma)^{\gamma-1}/(1-\gamma)$, we get $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$, where γ is the relative risk aversion coefficient, $\gamma \neq 1$). A logarithmic utility specification, $u(C_t) = log(C_t)$, corresponds to the case when $b = 0$, $a = (z/\gamma)^{\gamma-1}/(1-\gamma)$, and $\gamma = 1$. When $\gamma \to \infty$, we obtain CARA utility functions, $u(C_t) = -exp(-zC_t/b)$. By selecting $\gamma = -1$, we get quadratic utility functions, $u(C_t) = a(b - zC_t)^2$.

When the first five derivatives of a HARA utility function exist, $u'(\cdot) > 0$ and $u''(\cdot) < 0$ imply $u'''(\cdot) > 0$, $u''''(\cdot) < 0$, and $u'''''(\cdot) > 0$. Thus, any HARA class utility function, for which the first five derivatives exist, can be used in model (10) when the Taylor approximation is expanded up to the fourth moment of the cross-sectional distribution of consumption.

In what follows, assume, as in Lucas (1978), that different agents in the economy have identical CRRA preferences $u(C_{k,t}) = (C_{k,t}^{1-\gamma} - 1)/(1-\gamma)$. For this utility specification, $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'''(\cdot) > 0$, $u''''(\cdot) < 0$, and $u'''''(\cdot) > 0$, as required. With the CRRA utility function, model (4) becomes

$$E_t\left[\delta\frac{K^{-1}\sum_{k=1}^{K} C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K} C_{k,t}^{-\gamma}} R_{i,t+1}\right] = 1. \tag{14}$$

The SDF in this model is

$$M_{t+1} = \delta\frac{K^{-1}\sum_{k=1}^{K} C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K} C_{k,t}^{-\gamma}}. \tag{15}$$

Under the CRRA preference specification, we can rewrite the approximate equilibrium asset pricing model (10) as

$$E_t\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t} R_{i,t+1}\right] \approx 1, \tag{16}$$

where

$$A_t = 1 + \sum_{n=2}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma + l)\right)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^n. \tag{17}$$

In model (16), the pricing kernel is

$$M_{t+1} = \delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t}. \tag{18}$$

As we noted in Section 2, with CRRA preferences the ratio of the cross-sectional averages of marginal utilities of consumption equals the cross-sectional average of the ratios of marginal utilities of consumption at two successive dates only when all the agents in the economy are risk neutral and/or when the agents have the same consumption growth rate. Since Brav *et al.* (2002) use a Taylor approximation of the cross-sectional average of the ratio of marginal utilities, it follows that their pricing kernel is generally different from ours. This may lead to different quantitative results when empirically testing these two SDF's.

Define $\varepsilon_t \equiv \sum_{n=2}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t}}{\overline{C}_t} - 1 \right)^n$. As ε_t approaches zero,

$$A_t \approx exp \left(\sum_{n=2}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t}}{\overline{C}_t} - 1 \right)^n \right) \tag{19}$$

and hence

$$\begin{aligned} A_{t+1}/A_t \quad &\approx \quad exp \left(\sum_{n=2}^{N} \frac{1}{n!} (-1)^n \left(\prod_{l=0}^{n-1} (\gamma + l) \right) \right. \\ &\left. \times \left\{ K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t+1}}{\overline{C}_{t+1}} - 1 \right)^n - K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t}}{\overline{C}_t} - 1 \right)^n \right\} \right) \end{aligned} \tag{20}$$

For the second-order Taylor approximation, e.g.,

$$\begin{aligned} A_{t+1}/A_t \quad &\approx \quad exp \left(\frac{1}{2} \gamma(\gamma + 1) \left\{ K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t+1}}{\overline{C}_{t+1}} - 1 \right)^2 - K^{-1} \sum_{k=1}^{K} \left(\frac{C_{k,t}}{\overline{C}_t} - 1 \right)^2 \right\} \right) \\ &= \quad exp \left(\frac{1}{2} \gamma(\gamma + 1) \left\{ var \left(\frac{C_{k,t+1}}{\overline{C}_{t+1}} \right) - var \left(\frac{C_{k,t}}{\overline{C}_t} \right) \right\} \right). \end{aligned} \tag{21}$$

This follows that the pricing kernel in the approximate equilibrium asset pricing model is

$$M_{t+1} = \delta \left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} \frac{A_{t+1}}{A_t} \approx \delta \left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} exp \left(\frac{1}{2} \gamma(\gamma + 1) \left\{ var \left(\frac{C_{k,t+1}}{\overline{C}_{t+1}} \right) - var \left(\frac{C_{k,t}}{\overline{C}_t} \right) \right\} \right) \tag{22}$$

and therefore depends on changes in the cross-sectional variance of the ratio of individual consumption to average consumption, rather than changes in the cross-sectional variance of log consumption, as in Balduzzi and Yao (2005), or the variance of changes in log consumption, as in Constantinides and Duffie (1996). Conditioning directly on individual consumption data makes unnecessary any assumptions about the nature of idiosyncratic shocks to individual consumption. This distinguishes our approach from that of Constantinides and Duffie (1996).[3]

If we take a Taylor approximation of order higher than two, we SDF will be a function of changes in higher-order moments of the cross-sectional distribution of $C_{k,t}/\overline{C}_t$. When the Taylor approximation is truncated after the fourth moment of the cross-sectional distribution

[3] Constantinides and Duffie (1996) show that one of the conditions, under which a model with heterogeneous consumers could resolve the equity premium puzzle, is that idiosyncratic shocks to individual consumption must be persistent.

of consumption, e.g., the pricing kernel in the approximate equilibrium asset pricing model will be a function of changes in the first four cross-sectional moments of $C_{k,t}/\overline{C}_t$ and not only the variance, as in Balduzzi and Yao (2005).

When in equilibrium all investors have the same consumption and hence for all t $C_{k,t} = \overline{C}_t$, by Jensen's inequality $K^{-1}\sum_{k=1}^{K} C_{k,t}^{-\gamma} = \overline{C}_t^{-\gamma}$ and therefore model (14) reduces to the Lucas (1978) representative-agent consumption CAPM

$$E_t\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{i,t+1}\right] = 1, \tag{23}$$

in which the SDF is the average consumption growth rate between dates t and $t+1$ raised to the power $-\gamma$, where γ is the coefficient of relative risk aversion:

$$M_{t+1} = \delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}. \tag{24}$$

Empirical evidence is that the consumption CAPM (23) is inconsistent with data on consumption and asset returns in many respects. E.g., a reasonably parameterized (with the relative risk aversion coefficient γ less than 10) representative-agent model generates a mean equity premium, which is substantially lower than that observed in data. This model can explain the mean excess return on the market portfolio over the risk-free rate only if the representative agent is assumed to be implausibly averse to risk. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others. Another problem with the representative-agent model is that it yields the risk-free rate, which is too high compared to the observed mean return on the risk-free asset, and, as a consequence, a time preference discount factor δ greater than one is required to fit the mean risk-free rate of return. This is the risk-free rate puzzle as described in Weil (1989).

To understand whether model (16) helps explain the equity-premium puzzle, consider the case when the average of investors' marginal utilities of consumption is expanded up to the fourth moment of the cross-sectional distribution of consumption and ε_t approaches zero. In this case, the SDF in model (16) will be a function of the changes in the mean, variance, third, and fourth cross-sectional moments of consumption, where the fourth cross-sectional moment has a positive sign, while the sign of the third moment will be negative.[4] This implies that model (16) has the potential to explain the equity premium if changes in the cross-sectional variance and the fourth cross-sectional moment are negatively correlated and changes in the third moment are positively correlated with the equity return.

Another way to assess the potential of model (16) to explain the equity premium and risk-free rate puzzles, is to assume that asset returns, consumption growth, and the rate of

[4] See equation (22) for the second-order Taylor approximation.

growth of A_t are homoskedastic and jointly lognormal. This implies that the risk-free real interest rate is

$$r_{f,t+1} = -log\delta + \gamma E_t\left[\Delta c_{t+1}\right] - \frac{1}{2}\gamma^2\sigma_c^2 - E_t\left[\Delta a_{t+1}\right] - \frac{1}{2}\sigma_a^2 + \gamma\sigma_{ca} \tag{25}$$

and the excess return on risky assets, including the market portfolio itself, over the risk-free rate is

$$E_t\left[r_{i,t+1}\right] - r_{f,t+1} = -\frac{1}{2}\sigma_i^2 + \gamma\sigma_{ic} - \sigma_{ia}, \tag{26}$$

where Δc_{t+1}, Δa_{t+1}, and $r_{i,t+1}$ denote the logarithms of the average consumption growth rate, the growth rate of A_t, and the simple gross return on asset i, respectively, σ_{xy} denotes the unconditional covariance of innovations in x and y, and $\sigma_x^2 \equiv \sigma_{xx}$.

The first three terms on the right-hand side of (25) and the first two terms on the right-hand side of (26) are the same as for the representative-agent consumption CAPM. Equation (25) implies that the consumption CAPM (16) can explain the observed risk-free rate with a lower, relative to the representative-agent consumption CAPM, value of the time preference discount factor δ if the term $E_t\left[\Delta a_{t+1}\right] + \frac{1}{2}\sigma_a^2 - \gamma\sigma_{ca}$ is positive. According to equation (26), the same model can explain the observed equity premium with a lower, compared with the representative-agent model, value of risk aversion if the growth rate of A_t is negatively correlated with asset returns, i.e. $\sigma_{ia} < 0$. As ε_t approaches zero for all t, it again implies that model (16) can fit the equity premium with an economically plausible value of risk aversion if changes in the variance and the fourth cross-sectional moment are negatively correlated and changes in the third moment are positively correlated with the excess return on the risky asset over the risk-free rate.

3 Empirical Results

In this section, we investigate empirically whether taking into account consumer heterogeneity helps explain the equity premium and the risk-free rate. We start by calibrating the Lucas (1978) representative-agent consumption CAPM (23) to data from the CEX. This is our benchmark model. To investigate whether the assumption of consumer heterogeneity can improve the empirical performance of this model, we then examine two models. First, we assess the empirical performance of model (14). Second, we test the approximate equilibrium asset pricing model (16). When testing the models, we follow the approach in Brav *et al.* (2002).

3.1 The Data

The CEX. To construct a time series of the relevant moments of the cross-sectional distribution of consumption, we use quarterly consumption data from the CEX, produced by the

Bureau of Labor Statistics (BLS). The CEX data available cover the period from 1980 Q1 to 2003 Q4. It is a collection of data on approximately 5000 households per quarter in the U.S. Each household in the sample is interviewed every three months over five consecutive quarters (the first interview is practice and is not included in the published data set). As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on quarterly consumption expenditures for the previous three months made by households in the survey (demographic variables are based upon heads of households). Various income information is collected in the second and fifth interviews as well as information on the employment of each household member.

As opposed to the Panel Study of Income Dynamics, which offers only food consumption data on an annual basis, the CEX contains highly detailed data on quarterly consumption expenditures.[5] The CEX attempts to account for an estimated 70% of total household consumption expenditures. Since the CEX is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CEX consumption data compared to the Panel Study of Income Dynamics consumption data.

The Consumption Data. Our measure of consumption is consumption of nondurables and services. For each household, we calculate quarterly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 2005 dollars with the CPI's (not seasonally adjusted, urban consumers) for appropriate consumption categories.[6] Aggregating the household's quarterly consumption across these categories is made according to the National Income and Product Account definitions of consumption aggregates. Following Attanasio and Weber (1995), Brav *et al.* (2002), and Vissing-Jorgensen (2002), we drop all consumption observations for the years 1980 and 1981 because the quality of the CEX consumption data is questionable for this period. Thus, our sample covers the period from 1982 Q1 to 2003 Q4.

All the consumer units in the CEX can be categorized in three tranches: the January, February, and March tranches. For a given year, the quarterly consumption of the January tranche corresponds to consumption over (i) November and December last year, and January, (ii) February, March, and April, (iii) May, June, and July, or (iv) August, September, and October. For the February tranche, the quarterly consumption is that over (i) December last year, January, and February, (ii) March, April, and Mai, (iii) June, July, and August, or

[5] Food consumption is likely to be one of the most stable consumption components. Furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the Panel Study of Income Dynamics is noise due to the absence of interview training.

[6] The CPI series are obtained from the BLS through CITIBASE.

(iv) September, October, and November. The quarterly consumption of the March tranche corresponds to consumption over (i) January, February, and March, (ii) April, Mai, and June, (iii) July, August, and September, or (iv) October, November, and December. Since our sample covers the period from 1982 Q1 to 2003 Q4, there are 88 observations for each of the tranches.

In order to transform our consumption data to a per capita basis, we normalize the consumption of each household by dividing it by the number of family members in the household. To calculate the cross-sectional moments of consumption, we use data on household per capita consumption. The per capita consumption of a subset of households is calculated by averaging the normalized consumption expenditures of the households in the subset.

The Returns Data. The measures of the nominal quarterly market return are the value- and equally weighted returns (capital gain plus dividends) on all stocks listed on the NYSE, AMEX, and NASDAQ obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The nominal quarterly market capitalization-based decile portfolio returns are from CRSP.[7] The nominal quarterly risk-free rate of interest is the 3-month Treasury bill return from CRSP. The real quarterly returns are calculated as the nominal returns divided by the 3-month inflation rate based on the deflator defined for consumption of nondurables and services. We calculate the equity premium as the difference between the real equity return and the real risk-free rate.

Asset Holders. In the fifth (final) interview, the household is asked to report the end-of-period estimated market value of all stocks, bonds, mutual funds, and other such securities held by the consumer unit on the last day of the previous month as well as the difference in this estimated market value compared with the value of all securities held a year ago last month. Using these two values, we calculate each consumer unit's asset holdings at the beginning of a 12-month recall period. We limit our analysis to the consumer units designated as asset holders according to a criterion of asset holdings at the beginning of a 12-month recall period above a certain threshold. We consider three subsets of households defined as asset holders. The first subset consists of all consumer units that report an estimated market value of all stocks, bonds, mutual funds, and other such securities held a year ago last month equal to or greater than $1000 (in 2005 dollars). The second and third subsets consist of households that report total assets equal to or exceeding respectively $2000 and $10000 (in 2005 dollars)

[7] CRSP places all common NYSE, AMEX, and NASDAQ stocks (excluding Unit Investment Trusts, Closed-End funds, Real Estate Investment Trusts, Americus Trusts, Foreign stocks, and American Depository Receipts) into ten equally populated portfolios, or deciles, based on the market value of equity outstanding at the end of the previous year. CRSP portfolios 1-2 represent large cap stocks, portfolios 3-5 represent mid-caps, portfolios 6-8 are small caps, and portfolios 9-10 represent micro-caps stocks.

at the beginning of a 12-month recall period. Since the CEX reports only some limited information about asset holdings,[8] we consider households that report asset holdings equal to or exceeding \$1000, \$2000, and \$10000 (in 2005 dollars) in order to reduce the likelihood of including households, who do not participate in financial markets.

Sample Selection Criteria. We drop from the sample nonurban households, households residing in student housing, households with incomplete income responses, and households that do not have a fifth interview. Following Brav *et al.* (2002), in any given quarter we drop from the sample households that report in that quarter as zero their food consumption, consumption of nondurables and services, or total consumption, as well as households with missing information on the above items. Additionally, we keep in the sample only households whose head is between 19 and 75 years of age.

In Table I, we present summary statistics on the quarterly per capita consumption of nondurables and services in 2005 dollars for the period from 1982 Q1 to 2003 Q4 for different subsets of households defined as asset holders. For each subset of households, summary statistics are reported separately for the January, February, and March tranches, as well as for the combined tranches.

3.2 The Model Calibration Results

The Representative-Agent Model. The Lucas (1978) representative-agent consumption CAPM is given by the following Euler equation:

$$E_t\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{i,t+1}\right] = 1. \tag{27}$$

By the law of iterated expectations,

$$E\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{i,t+1}\right] = 1. \tag{28}$$

For the excess return on the market portfolio $R_{m,t+1}$ over the risk-free rate $R_{f,t+1}$, equation (28) becomes

$$E\left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} (R_{m,t+1} - R_{f,t+1})\right] = 0. \tag{29}$$

To test the Euler equation (29), we calculate the statistic v_m as

$$v_m = T^{-1}\sum_{t=1}^{T}\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} (R_{m,t+1} - R_{f,t+1}). \tag{30}$$

[8]E.g., see Cogley (2002) and Vissing-Jorgensen (2002) for more details.

When the relative risk aversion coefficient γ is set equal to zero, the term $\left(\overline{C}_{t+1}/\overline{C}_t\right)^{-\gamma}$ is identically equal to one and the statistic v_m equals the sample mean of the excess market portfolio return. When the value of γ increases, the value of this statistic decreases. Under the null hypothesis that the Euler equation for the equity premium (29) holds, v_m is zero. Following Brav *et al.* (2002), we interpret the statistic v_m as the unexplained mean equity premium.

From (28), the Euler equation for the risk-free rate is

$$E\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{f,t+1}\right] = 1. \tag{31}$$

We define the statistic v_f as

$$v_f = T^{-1}\sum_{t=1}^{T}\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma} R_{f,t+1} - 1. \tag{32}$$

With δ set equal to zero, the statistic v_f is identically equal to -1. When the value of δ increases, the value of the statistic v_f increases with a fixed γ. Under the null hypothesis that the Euler equation for the risk-free rate (31) holds, the statistic v_f is zero.

To jointly test the Euler equations for the equity premium and the risk-free rate, here and below first we find the value of the coefficient of relative risk aversion γ, at which the unexplained-premium statistic v_m is zero for the combined tranches of individuals classified as asset holders. Then, given that value of the agent's risk aversion γ, we find the value of the time preference discount factor δ, at which the statistic v_f is zero for the combined tranches. The found values of risk aversion and the time preference discount factor are those, at which the Euler equations for the equity premium and the risk-free rate hold simultaneously.

The statistic v_m for the combined tranches is calculated as follows. For the values of the risk aversion coefficient γ increasing from zero to 10, the maximum value considered plausible by Mehra and Prescott (1985), with increments of 0.001, first we calculate the unexplained mean equity premium for each tranche of individuals classified as asset holders and then calculate the unexplained mean equity premium for the combined tranches as the weighted average of the unexplained premia of the three tranches, where the unexplained premium for each tranche is weighted by a measure of its volatility.[9] The value of γ, at which the sign of the unexplained-premium statistic v_m for the combined tranches becomes negative is considered as that, at which the unconditional Euler equation for the equity premium (29) holds.

To find the value of the time preference discount factor δ, at which the statistic v_f for the combined tranches is zero, for the values of δ increasing from zero with increments of 0.0001

[9] See Brav *et al.* (2002).

we calculate the statistic v_f for each tranche of households defined as asset holders and then calculate the statistic v_f for the combined tranches as the weighted average of the statistics for the three tranches weighted by measures of their volatilities. The value of δ, at which the statistic v_f for the combined tranches becomes positive is considered as that, at which the unconditional Euler equation for the risk-free rate (31) holds.

Our result is that there is no economically plausible (less than 10) value of risk aversion, at which the representative-agent model explains the equity premium. The result is robust to the portfolio of risky assets and the threshold value in the definition of asset holders. Below, we test whether the assumption of consumer heterogeneity can improve the ability of the consumption CAPM to fit the data.

The Models with Heterogeneous Consumers. First, we test the consumption CAPM, in which the SDF is the discounted ratio of the cross-sectional averages of marginal utilities of consumption at two successive dates:

$$E_t\left[\delta\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}R_{i,t+1}\right]=1. \tag{33}$$

The unconditional version of this Euler equation is

$$E\left[\delta\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}R_{i,t+1}\right]=1. \tag{34}$$

For the excess return on the market portfolio over the risk-free rate, model (34) can be rewritten as

$$E\left[\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}\left(R_{m,t+1}-R_{f,t+1}\right)\right]=0. \tag{35}$$

For this model, the unexplained mean equity premium statistic v_m is specified as

$$v_m=T^{-1}\sum_{t=1}^{T}\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}(R_{m,t+1}-R_{f,t+1}). \tag{36}$$

From (34), the unconditional Euler equation for the risk-free rate is

$$E\left[\delta\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}R_{f,t+1}\right]=1 \tag{37}$$

and therefore the statistic v_f is

$$v_f=T^{-1}\sum_{t=1}^{T}\delta\frac{K^{-1}\sum_{k=1}^{K}C_{k,t+1}^{-\gamma}}{K^{-1}\sum_{k=1}^{K}C_{k,t}^{-\gamma}}R_{f,t+1}-1. \tag{38}$$

As for the representative-agent model, we find that for any threshold value in the definition of asset holders there is no value of the relative risk aversion coefficient γ less than 10, at which model (34) explains the excess return on risky assets over the risk-free rate.

Next, we consider the approximate equilibrium asset pricing model

$$E_t\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t}R_{i,t+1}\right]\approx 1, \tag{39}$$

where

$$A_t = 1+\sum_{n=2}^{N}\frac{1}{n!}(-1)^n\left(\prod_{l=0}^{n-1}(\gamma+l)\right)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t}-1\right)^n \tag{40}$$

and test whether the unconditional version of this model

$$E\left[\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t}R_{i,t+1}\right]\approx 1 \tag{41}$$

has the potential to explain the equity premium and the risk-free rate of return with economically plausible values of the agent's preference parameters.

For this model, we define the unexplained mean equity premium statistic v_m as

$$v_m = T^{-1}\sum_{t=1}^{T}\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t}(R_{m,t+1}-R_{f,t+1}) \tag{42}$$

and the statistic v_f as

$$v_f = T^{-1}\sum_{t=1}^{T}\delta\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\frac{A_{t+1}}{A_t}R_{f,t+1}-1. \tag{43}$$

Our result in Section 2 is that we can sign the first five derivatives of the agent's utility function what makes it possible to investigate the role of the first four moments of the cross-sectional distribution of consumption in explaining the equity premium and the risk-free return. We start with investigating whether the model, in which the average of investors' marginal utilities of consumption is expanded up to the cross-sectional variance of consumption, can improve the empirical performance of the consumption CAPM. When testing the approximate equilibrium asset pricing model with

$$A_t = 1+\frac{1}{2}\gamma(\gamma+1)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t}-1\right)^2, \tag{44}$$

we find that, as for the representative-agent model, there is no value of risk aversion less than 10 that allows to explain the observed equity premium. The result is robust to the portfolio of risky assets and the threshold value in the definition of asset holders.

Then, we pursue the expansion further and truncate the Taylor approximation after the third cross-sectional moment of consumption. When the model is tested with

$$A_t = 1 + \frac{1}{2}\gamma(\gamma+1)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^2 - \frac{1}{6}\gamma(\gamma+1)(\gamma+2)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^3, \quad (45)$$

we obtain that the parameter estimates are robust and indicate moderate risk aversion (see Table II). However, the obtained estimates of the time preference discount factor δ are too low to be recognized as economically realistic. This leads to conclude that when the third moment of the cross-sectional distribution of consumption is considered, the approximate equilibrium asset pricing model can explain the equity premium, but not the risk-free rate of return.

Finally, we take the fourth-order Taylor approximation of the average of investors' marginal utilities of consumption and assess the plausibility of the model, in which the SDF depends on the forth moment of the cross-sectional distribution of consumption. In this model,

$$\begin{aligned} A_t &= 1 + \frac{1}{2}\gamma(\gamma+1)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^2 - \frac{1}{6}\gamma(\gamma+1)(\gamma+2)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^3 \\ &\quad + \frac{1}{24}\gamma(\gamma+1)(\gamma+2)(\gamma+3)K^{-1}\sum_{k=1}^{K}\left(\frac{C_{k,t}}{\overline{C}_t} - 1\right)^4. \end{aligned} \quad (46)$$

When the Taylor approximation is expanded up to the fourth cross-sectional moment of consumption, the estimate of the relative risk aversion coefficient is in the conventional range (see Table II). The estimate of the time preference discount factor is greater than that obtained for the third-order approximation and may be recognized as economically plausible. We find that the implied coefficient of relative risk aversion decreases and the time preference discount factor increases as the threshold value in the definition of asset holders is raised. The result is robust across the risk premia on the value- and equally weighted market portfolios and the risk premia on the market capitalization-based decile portfolios. This suggests that taking into account the first four moments of the cross-sectional distribution of consumption (not only the mean, as in the representative-agent consumption CAPM) helps explain both the market premium and the return on the risk-free asset with economically plausible values of risk aversion and the time preference discount factor.

Measurement Error Issue. A well documented potential problem with using individual-level data is the large measurement error in reported individual consumption.[10] In this section, we conduct Monte Carlo simulations designed to investigate whether the calibration

[10] E.g., see Zeldes (1989) and Runkle (1991).

results for the approximate equilibrium asset pricing model, in which the Taylor approximation of the average of investors' marginal utilities of consumption is expanded up to the fourth cross-sectional moment of consumption, are robust to measurement error.

It would be reasonable to assume that an individual with a higher consumption level should, in general, misreport his consumption by a larger amount than an individual with a lower level of consumption. Therefore, we assume that measurement error is proportional to the true level of consumption and suppose that the observed consumption level is $C_{k,t} = \epsilon_{k,t} C^*_{k,t}$, where $C^*_{k,t}$ is the true level of the agent k's date t consumption and $\epsilon_{k,t}$ is independently identically normally distributed with mean one and variance σ^2_ϵ, $\epsilon_{k,t} \sim N\left(1, \sigma^2_\epsilon\right)$.[11] When generating random numbers $\epsilon_{k,t}$, we assume that with the probability of 95% an individual misreports his consumption by the amount that does not exceed 10% of the true consumption level. The experiment is repeated 1000 times providing sampling distributions of γ and δ. The results are presented in Table III.

The Monte Carlo simulation results show that when the cross-sectional average of marginal utilities is expanded up to the fourth cross-sectional moment of consumption, the estimates of risk aversion and the time preference discount factor do not differ significantly from the true values. Measurement error of the form assumed here only slightly biases upward the estimate of the coefficient of relative risk aversion and biases downward the estimate of the time preference discount factor. This confirms our intuition that the estimation of the cross-sectional consumption moments is less susceptible to measurement error than the average of investors' marginal utilities of consumption and therefore the SDF given by equation (11) is less vulnerable to measurement error than the SDF given by equation (5).

4 Concluding Remarks

We proposed the consumption CAPM, in which the pricing kernel is the discounted ratio of the Taylor approximations of the cross-sectional average of investors' marginal utilities of consumption around average consumption at two successive dates.

Using quarterly consumption data from the CEX, we found that this model explains the observed equity premium with a low and economically plausible value of the coefficient of relative risk aversion when the first three cross-sectional moments of consumption (and not only the mean of the cross-sectional distribution, as in the Lucas (1978) representative-agent model) are taken into account. This result is in line with that in Brav *et al.* (2002) and Kocherlakota and Pistaferri (2005), who also find that the assumption of consumer heterogeneity helps explain the equity premium puzzle. However, in contrast to the Brav *et*

[11] The independence assumption presumes that the primary source of measurement error is negligence in responding and not the survey design.

al. (2002) and Kocherlakota and Pistaferri (2005) models, the consumption CAPM proposed in this paper is able to explain not only the equity premium, but also the risk-free rate of return. When the first four moments of the cross-sectional distribution of consumption are considered, the model explains the equity premium and the risk-free rate simultaneously with economically plausible values of both risk aversion and the time preference discount factor. This result is robust across the risk premia on the value- and equally weighted market portfolios and the risk premia on the market capitalization-based decile portfolios, as well as the threshold value in the definition of asset holders.

Since the representative-agent consumption CAPM fails to explain the equity premium and the risk-free rate with economically plausible values of the agent's preference parameters, the evidence found in this paper underlines the importance of taking into account consumer heterogeneity for asset pricing. The result is that not only the mean, as in the representative-agent consumption CAPM, but also the variance, third, and fourth moments of the cross-sectional distribution of consumption are important determinants of the equilibrium asset returns.

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Appendix

Proof of Proposition 1. DAP implies that

$$AP'(\cdot) = -\frac{u''''(\cdot)\,u''(\cdot) - (u'''(\cdot))^2}{(u''(\cdot))^2} < 0. \tag{47}$$

In order to prove that the condition $u''''(\cdot) < 0$ is necessary for DAP suppose, in contrast, that $u''''(\cdot) \geqslant 0$. When $u''''(\cdot) \geqslant 0$, $u''''(\cdot)\,u''(\cdot) \leqslant 0$ and therefore $AP'(\cdot) > 0$, what contradicts the assumption that absolute prudence is decreasing.

Inequality (47) means that $u''''(\cdot)\,u''(\cdot) - (u'''(\cdot))^2 > 0$ is the necessary and sufficient condition for DAP. We can rewrite this condition as

$$u''''(\cdot) < \frac{(u'''(\cdot))^2}{u''(\cdot)} = -AP(\cdot)\,u'''(\cdot). \tag{48}$$

Since an agent is assumed to be prudent, the term on the right-hand side of (48) is negative. ■

Proof of Proposition 2. Absolute prudence is convex if the following condition is satisfied:

$$AP''(\cdot) = -\frac{A-B}{C} > 0, \tag{49}$$

where $A = (u''(\cdot))^2\,(u'''''(\cdot)\,u''(\cdot) - u'''(\cdot)\,u''''(\cdot))$, $B = 2u''(\cdot)\,u'''(\cdot)\,(u''''(\cdot)\,u''(\cdot) - (u'''(\cdot))^2)$, and $C = (u''(\cdot))^4$.

To prove that $u'''''(\cdot) > 0$ is necessary for CAP under prudence and DAP, assume that $u'''''(\cdot) \leqslant 0$. An agent is prudent ($AP(\cdot) > 0$) if and only if $u'''(\cdot) > 0$. By Proposition 1, we know that the necessary condition for DAP is that $u''''(\cdot) < 0$. Then, under prudence and DAP, $A > 0$. Since $u''''(\cdot)\,u''(\cdot) - (u'''(\cdot))^2 > 0$ is the necessary and sufficient condition for DAP, prudence and DAP also imply that $B < 0$. In consequence, $AP''(\cdot) < 0$, what contradicts the initial assumption that absolute prudence is convex.

It follows from (49) that the necessary and sufficient condition for CAP is $A - B < 0$. This condition can be written as follows:

$$u''''' > \frac{2u'''(\cdot)\left(u''''(\cdot)\,u''(\cdot) - (u'''(\cdot))^2\right)}{(u''(\cdot))^2} + \frac{u'''(\cdot)\,u''''(\cdot)}{u''(\cdot)} \tag{50}$$

or, equivalently,

$$u''''' > -2AP'(\cdot)\,u'''(\cdot) - AP(\cdot)\,u''''(\cdot). \tag{51}$$

Under prudence and DAP, the term $-2AP'(\cdot)\,u'''(\cdot) - AP(\cdot)\,u''''(\cdot)$ is positive.[12] ■

[12] If an agent exhibits prudence, then $AP(\cdot) > 0$ and $u'''(\cdot) > 0$. The condition $u''''(\cdot) < 0$ is necessary for DAP.

Table I

Summary Statistics

This table reports summary statistics on the number of households and the quarterly household per capita consumption of nondurables and services in 2005 dollars for different sets of households classified as asset holders. The summary statistics are reported separately for the January, February, March, and combined tranches of households. Data for the period 1982 Q1 - 2003 Q4.

	Number of Households			Consumption	
Tranches	Minimum	Median	Maximum	Mean	Std. Dev.
		NIPA			
Combined				5191.68	
		CEX: Asset Holdings $\geqslant$ \$1000			
January	95	140	199	5607.32	3716.88
February	79	132	183	5631.60	3652.14
March	26	145	215	5433.13	3383.57
Combined	26	138	215	5556.24	3586.80
		CEX: Asset Holdings $\geqslant$ \$2000			
January	85	131	190	5654.11	3729.75
February	77	126	177	5683.63	3691.36
March	23	135	207	5485.09	3410.33
Combined	23	130	207	5606.90	3613.45
		CEX: Asset Holdings $\geqslant$ \$10000			
January	51	93	159	5857.62	3899.89
February	52	97	144	5912.17	3896.64
March	17	97	169	5631.71	3509.45
Combined	17	96	169	5799.66	3773.78

Table II

Model Calibration Results

This table reports the values of the relative risk aversion coefficient γ and the time preference discount factor δ, at which the unconditional Euler equations for the equity premium and the risk-free rate hold jointly for the combined tranches of consumers identified as asset holders. The pricing kernel is the discounted ratio of the Taylor approximations of the average of investors' marginal utilities of consumption at two successive dates. The average of investors' marginal utilities of consumption is expanded up to the variance (Second-Order Taylor Approximation), third (Third-Order Taylor Approximation), and fourth (Fourth-Order Taylor Approximation) moments of the cross-sectional distribution of household per capita consumption. The table presents the results for the value-weighted (VW) market portfolio, the equally weighted (EW) market portfolio, and the market capitalization-based decile portfolio (DEC1-DEC10) returns.

Parameters	VW	EW	DEC1	DEC2	DEC3	DEC4	DEC5	DEC6	DEC7	DEC8	DEC9	DEC10
						Asset Holdings $\geqslant$ \$1000						
						Third-Order Taylor Approximation						
γ	0.4590	0.4540	0.4550	0.4550	0.4550	0.4540	0.4560	0.4540	0.4560	0.4570	0.4560	0.4590
δ	0.3849	0.6620	0.6279	0.6279	0.6279	0.6620	0.5866	0.6620	0.5866	0.5354	0.5866	0.3849
						Fourth-Order Taylor Approximation						
γ	1.1360	1.0390	1.4050	1.0790	0.9460	0.8910	0.9280	0.9610	0.9050	0.9770	1.0570	1.2290
δ	0.6835	0.7097	0.6213	0.6986	0.7366	0.7534	0.7421	0.7322	0.7491	0.7275	0.7047	0.6603
						Asset Holdings $\geqslant$ \$2000						
						Third-Order Taylor Approximation						
γ	0.4460	0.4410	0.4420	0.4420	0.4420	0.4410	0.4430	0.4410	0.4430	0.4450	0.4430	0.4470
δ	0.4068	0.6725	0.6396	0.6396	0.6396	0.6725	0.5999	0.6725	0.5999	0.4885	0.5999	0.2948
						Fourth-Order Taylor Approximation						
γ	1.0640	0.9820	1.3020	1.0210	0.8890	0.8420	0.8750	0.9260	0.8600	0.9290	0.9950	1.1440
δ	0.7066	0.7301	0.6463	0.7188	0.7583	0.7732	0.7627	0.7469	0.7674	0.7460	0.7263	0.6850

Table II (continued)

Parameters	VW	EW	DEC1	DEC2	DEC3	DEC4	DEC5	DEC6	DEC7	DEC8	DEC9	DEC10
						Asset Holdings $\geqslant$ \$10000						
						Third-Order Taylor Approximation						
γ	0.4300	0.4260	0.4270	0.4270	0.4270	0.4260	0.4280	0.4260	0.4270	0.4290	0.4280	0.4310
δ	0.4659	0.6711	0.6351	0.6351	0.6351	0.6711	0.5910	0.6711	0.6351	0.5361	0.5910	0.3733
						Fourth-Order Taylor Approximation						
γ	0.9710	0.8870	1.1130	0.8960	0.7860	0.7570	0.7850	0.8390	0.7920	0.8540	0.9150	1.0490
δ	0.7280	0.7544	0.6861	0.7515	0.7874	0.7971	0.7877	0.7699	0.7854	0.7650	0.7454	0.7045

Table III
Monte Carlo Simulation Results

This table reports the mean and the standard deviation (in parantheses) of the sampling distributions of the values of the relative risk aversion coefficient γ and the time preference discount factor δ, at which the unconditional Euler equations for the equity premium and the risk-free rate hold jointly for the combined tranches of consumers classified as asset holders. The average of investors' marginal utilities of consumption is expanded up to the fourth moment of the cross-sectional distribution of consumption. Measurement error in reported individual consumption is assumed to be proportional to the true level of consumption. An individual is assumed to misreport his consumption by the amount that with the probability of 95% does not exceed 10% of the true consumption level. Each experiment was repeated 1000 times. The table presents the results for the value-weighted (VW) market portfolio, the equally weighted (EW) market portfolio, and the market capitalization-based decile portfolio (DEC1-DEC10) returns.

Parameters	VW	EW	DEC1	DEC2	DEC3	DEC4	DEC5	DEC6	DEC7	DEC8	DEC9	DEC10
						Asset Holdings $\geqslant$ \$1000						
γ	1.2181	1.0984	1.4980	1.1328	0.9911	0.9304	0.9725	1.0125	0.9547	1.0388	1.1312	1.3292
	(0.3170)	(0.2411)	(0.3419)	(0.2253)	(0.1976)	(0.1772)	(0.1954)	(0.2131)	(0.2072)	(0.2478)	(0.2850)	(0.3714)
δ	0.6644	0.6909	0.6047	0.6810	0.7189	0.7361	0.7243	0.7134	0.7300	0.7076	0.6845	0.6416
	(0.0616)	(0.0547)	(0.0482)	(0.0495)	(0.0502)	(0.0474)	(0.0504)	(0.0529)	(0.0541)	(0.0593)	(0.0617)	(0.0626)
						Asset Holdings $\geqslant$ \$2000						
γ	1.1312	1.0316	1.3733	1.0667	0.9280	0.8759	0.9135	0.9696	0.9008	0.9812	1.0555	1.2201
	(0.2905)	(0.2162)	(0.2821)	(0.2032)	(0.1772)	(0.1611)	(0.1773)	(0.1978)	(0.1875)	(0.2255)	(0.2561)	(0.3135)
δ	0.6885	0.7125	0.6302	0.7021	0.7415	0.7571	0.7459	0.7298	0.7502	0.7274	0.7075	0.6674
	(0.0590)	(0.0521)	(0.0475)	(0.0478)	(0.0475)	(0.0448)	(0.0477)	(0.0509)	(0.0508)	(0.0563)	(0.0588)	(0.0603)
						Asset Holdings $\geqslant$ \$10000						
γ	1.0485	0.9342	1.1679	0.9343	0.8199	0.7858	0.8189	0.8803	0.8343	0.9124	0.9852	1.1404
	(0.3062)	(0.1976)	(0.2233)	(0.1698)	(0.1536)	(0.1412)	(0.1550)	(0.1792)	(0.1800)	(0.2285)	(0.2674)	(0.3488)
δ	0.7048	0.7347	0.6676	0.7339	0.7703	0.7814	0.7706	0.7513	0.7662	0.7427	0.7218	0.6810
	(0.0701)	(0.0570)	(0.0527)	(0.0490)	(0.0477)	(0.0442)	(0.0479)	(0.0537)	(0.0549)	(0.0645)	(0.0693)	(0.0730)