Asset Pricing with Idiosyncratic Consumption Risk and Limited Participation

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**Abstract**

A growing body of literature suggests limited asset market participation as a plausible explanation of the empirical failure of the standard consumption capital asset pricing model (CCAPM). Correct identification of capital markets investors is, however, often impossible due to imperfection of available information on assetholding status. As a plausible solution to the problem of sample classification when available information is an imperfect sample separation indicator, we propose the CCAPM in which the pricing kernel is calculated as the weighted average of individual households’ marginal rate of substitution, with the weights being the probabilities of holding the asset in question. The asset holding probabilities are conditional on available sample separation information and estimated from a binary response model as a function of demographic and family characteristics of consumers simultaneously with the parameters of Euler equations. The CCAPM with probability-weighted agents is less susceptible to sample misclassification compared to when available imperfect information on asset holding status is used to separate assetholders from nonassetholders. Using data from the U.S. Consumer Expenditure Survey (CEX), we find that, in contrast to when the reported in the CEX financial information is regarded as a perfect sample separation indicator, the model with probability-weighted agents is not rejected statistically both under conventional normal and weak-identification asymptotics and yields precise and economically realistic estimates of the coefficient of relative risk aversion (RRA). The hypothesis that the households’ market participation behavior exhibits considerable persistence is not rejected statistically. Empirical evidence is that the decision to own assets is likely to be endogenous with respect to the consumption and savings decisions and that allowing for this fact is important for estimating risk aversion.

*JEL classification:* G12  
*Keywords:* equity premium puzzle, Euler equation, limited asset market participation, probit model, risk-free rate puzzle.

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1 Introduction

The basic CCAPM treats asset prices as being determined by the consumption and savings decisions of a single representative agent with conventional time- and state-separable power utility. Numerous empirical studies show that the representative-agent model does not perform well when estimated on data for all households. For instance, the covariance of overall per capita consumption growth with the excess return on the market portfolio over a risk-free asset is only large enough to explain the market premium if the typical investor is implausibly averse to risk. This is the equity premium puzzle described by Mehra and Prescott (1985). Another anomaly is that, given the lack of variability of aggregate consumption growth, the representative agent must have a negative rate of time preference for the model to be able to fit the observed risk-free rate. This is the risk-free rate puzzle pointed out first by Weil (1989).

In the representative-agent model, aggregate consumption per capita is assumed to be an adequate proxy for the consumption of capital markets investors. However, it is observed that only a small fraction of individuals in the population participates in asset markets.\footnote{Bertaut (1998), Blume and Zeldes (1993), Haliassos and Bertaut (1995), and Mankiw and Zeldes (1991) observe that only about 30-40\% of individuals in the U.S. hold stocks either directly or through defined contribution pension funds. According to the Current Population Reports, only nearly 20\% of U.S. households own publicly traded stocks and/or mutual fund shares (about 20\% of the U.S. population held such assets in 1984, 21.8\% in 1988, and 20.7\% in 1991). This level of share ownership is similar to that found by Attanasio, Banks, and Tanner (2002) for the U.K. Using data from the Family Expenditure Survey, they observe that, in keeping with the U.S., only nearly 20-25\% of U.K. households own shares directly. Agell and Edin (1990) find that 75.2\% of Swedish households hold bank checking or savings accounts and only 18.6\% hold common stocks.} Attanasio, Banks, and Tanner (2002) and Mankiw and Zeldes (1991) observe that the consumption growth of stockholders is more volatile and covaries more with excess returns to stocks than that of nonstockholders. They estimate Euler equations for stockholders and nonstockholders separately and find large differences in RRA estimates between the two groups with a larger value of risk aversion for the group of individuals classified as nonstockholders. A similar result is obtained by Vissing-Jorgensen (1998, 2002). This suggests limited asset market participation as a plausible explanation of the empirical failure of the standard CCAPM.

To allow for limited participation of agents in asset markets, the consumption of capital markets investors only must be involved in Euler equations. Correct identification of capital markets investors is, however, often impossible due to imperfection of available information on assetholding status.\footnote{See Attanasio, Banks, and Tanner (2002) and Vissing-Jorgensen (1998, 2002).} One plausible solution to the problem of sample classification when available information is an imperfect sample separation indicator is to calculate the probability of participation of a consumer in asset markets which is conditional on this...
information and then to split the sample according to whether the estimated probability of asset market participation of an agent exceeds some determined a priori trigger level. Attanasio, Banks, and Tanner (2002) follow this approach and classify households as shareholders and nonshareholders on the basis of the predicted probabilities of share ownership estimated as a probit function. Zeldes (1989) estimates the probabilities of being liquidity constrained from a logit instead of a probit equation. In the both cases, some determined a priori threshold probability is used to split the sample.\(^3\) Since the choice of a cutoff point is usually arbitrary, this method may create a misclassification problem. Another problem is that when using this approach, we suppose that the errors in a binary response model for asset ownership and the errors in Euler equations are not correlated and, therefore, implicitly assume that the decision to own assets in each time period is exogenous with respect to the consumption and savings decisions. However, as pointed out by Attanasio, Banks, and Tanner (2002) and Bertaut (1998), the decision to acquire assets is likely to be endogenous with respect to the consumption and savings decisions.\(^4\) This suggests that the errors in the binary response model for asset ownership might be correlated with the errors in Euler equations and, hence, the binary response model for asset ownership and the Euler equations for the equity premium and the risk-free rate must be estimated jointly.

Another argument in favor of the joint estimation of the binary response model for asset ownership and the Euler equations for the equity premium and the risk-free rate is that this allows to mitigate the mentioned above problem of sample misclassification arising when a consumer is classified as an assetholder only if the probability of his participation in the market of the asset in question exceeds some determined a priori trigger level. Lee and Porter (1984) consider an exogenous switching regression model when some imperfect sample separation information is available and show that employing the probabilities conditional on this additional information to classify the regimes allows to minimize the total probability of sample misclassification. Garcia, Lusardi, and Ng (1997) use this approach when studying whether the presence of liquidity constraints induces excess sensitivity of changes in consumption to lagged income and anticipated changes in income. They apply the probability weights associated with the logit function to the densities for the Euler equations for each consumer in two states and estimate both exogenous and endogenous switching regression models. Since the mentioned above method of sample classification based on comparing the probability of asset market participation to some cutoff point attaches a weight of either 0 or 1 to the Euler equation of each consumer, it is suboptimal relative to the method of sample classification based on applying the probability weights to

\(^3\) Attanasio, Banks, and Tanner (2002) define as likely shareholders individuals whose predicted probability of owning shares exceeds 0.5. A cutoff point in Zeldes (1989) is 0.6.

\(^4\) Bertaut (1998) shows that factors such as increased risk aversion and income risk can reduce the utility gains from market participation and, therefore, contribute negatively to the probability of stock ownership.
the Euler equations for each consumer in different states.

Using the 1983 and 1989 panels of the U.S. Survey of Consumer Finances, Bertaut (1998) shows that household portfolios are likely to display persistent behavior. It follows that when estimating the probability of the ownership of a particular asset, the possibility that the residuals in the binary response model may exhibit serial correlation must be taken into account.

Based on the mentioned above reasonings, as a plausible solution to the problem of sample classification when available information is an imperfect sample separation indicator, we propose the CCAPM with the stochastic discount factor (SDF), or pricing kernel, calculated as the weighted average of individual households’ marginal rate of substitution, with the weights being the probabilities of holding the asset in question. In this model, the probabilities of asset ownership are conditional on available imperfect sample separation information and estimated from the binary response model as a function of demographic and family characteristics of consumers simultaneously with the parameters of Euler equations.

According to the result in Lee and Porter (1984), weighting households by the asset holding probabilities conditional on imperfect sample separation information makes this model less susceptible to sample misclassification compared to when this information is regarded as a perfect sample separation indicator. Another advantage of this model is that it uses individual consumption growth rates rather than the aggregate consumption per capita growth rate for the group of individuals defined as asset holders according to some sample classification criterion as in Attanasio, Banks, and Tanner (2002), Mankiw and Zeldes (1991), and Vissing-Jorgensen (1998, 2002). It allows to take into account both limited asset market participation and incomplete consumption insurance when estimating Euler equations.\textsuperscript{5} Mankiw and Zeldes (1991) provide evidence that limited stock market participation alone cannot explain the equity premium puzzle. They find that although the consumption of stockholders is more highly correlated with the stock market than the consumption of nonstockholders, this covariance is not large enough to explain the equity premium puzzle with realistic values of RRA. However, Brav, Constantinides, and Géczy (2002) and Semenov (2004) show an important role played by the hypothesis of incomplete consumption insurance in explaining asset returns. Another argument in favor of considering incomplete consumption insurance is the result in Bertaut (1998) who demonstrates the importance of taking into account income risk for explaining the probability of asset market

\textsuperscript{5}Complete consumption insurance implies that consumers can use financial markets to diversify away any idiosyncratic differences in their consumption streams. It follows that under the assumption of complete consumption insurance, aggregate consumption per capita can be used in place of individual consumption and, hence, the pricing implications of a complete consumption insurance model are similar to those of the representative-consumer economy. With incomplete consumption insurance, individuals are not able to self-insure against uninsurable risks such as idiosyncratic shocks to the households’ income or divorce and, therefore, are heterogeneous.
participation.\textsuperscript{6}

To estimate both the binary response model for asset ownership and the Euler equations for the equity premium and the risk-free rate, we use the Generalized Method of Moments (GMM) (Hansen (1982)) estimation technique. Application of GMM to binary response models is straightforward when the disturbances of a given individual are appropriately assumed to be serially correlated. Under the assumption that the disturbances of a given individual are independent over time, the orthogonality conditions implied by GMM are analogous to the maximum likelihood ones. When the characteristics of consumers are assumed to be strictly exogenous, we get additional orthogonality conditions involving correction of the estimators for serial dependence of disturbances and, hence, may test the validity of the assumption of serially dependent disturbances using Hansen’s test of the overidentifying restrictions. The use of GMM allows to estimate the binary response model for asset ownership and the Euler equations for the equity premium and the risk-free rate jointly under the assumption that the residuals in the binary response model may exhibit serial correlation.\textsuperscript{7}

Micro data from the CEX are used to test the CCAPM with asset ownership probability weighted agents. Because of inability of the reported in the CEX information on asset holding status to perfectly identify households whose consumption must be involved in the Euler equations for the equity premium and the risk-free rate, we treat this information as an imperfect sample separation indicator.\textsuperscript{8} Specifying the binary response model for asset ownership as a multiperiod bivariate probit, we estimate this model and the Euler equations for the equity premium and the risk-free rate simultaneously by GMM. Our result is that, in contrast to when the reported in the CEX financial information is regarded as a perfect sample separation indicator, the CCAPM with asset ownership probability weighted agents is not rejected statistically both under conventional normal and weak-identification asymptotics and yields precise and economically realistic estimates of the RRA coefficient. The hypothesis that the households’ market participation behavior exhibits considerable persistence is not rejected statistically. The found large differences in the point estimates of the RRA coefficient when the CCAPM with asset ownership probability weighted agents is estimated under the assumptions that the disturbances in the bivariate probit model for asset ownership and the errors in the Euler equations for the equity premium and the risk-free rate are uncorrelated and when this assumption is relaxed suggest that the decision to

\textsuperscript{6}Bertaut (1998) shows that for any degree of risk aversion, the introduction of income risk lowers the probability of holding stocks.

\textsuperscript{7}When the binary response model for asset ownership is estimated jointly with Euler equations using the maximum likelihood method, as in Garcia, Lusardi, and Ng (1997), for example, the error terms of a binary response model are assumed to be serially independent.

\textsuperscript{8}See Vissing-Jorgensen (1998, 2002) for the discussion about inability to perfectly identify stockholders and bondholders when using the CEX.
own assets is likely to be endogenous with respect to the consumption and savings decisions and that allowing for this fact is important in estimating risk aversion.

The rest of the paper is as follows. The CCAPM with asset ownership probability weighted agents as well as the multiperiod binary response model for asset ownership are presented in Section 2. In Section 2, we also present the GMM estimation technique used to estimate the binary response model under the assumption that the disturbances may be serially correlated. Section 3 presents the GMM results from the estimation of the bivariate probit model for asset ownership and the Euler equations for the equity premium and the risk-free rate. Conclusions are presented in Section 4.

2 The CCAPM with Asset Ownership Probability Weighted Agents

In this section, we first derive the CCAPM in which the SDF is calculated as the sum of the individuals’ marginal rates of substitution weighted by the normalized predicted probabilities of asset holding. Then, we present the binary response model for asset ownership which allows to calculate the predicted probabilities of participation in the market of a particular asset as a function of demographic and family characteristics of consumers conditional on available imperfect information on asset holding status and show how the GMM approach can be used to estimate this model under the assumption that the error terms may exhibit serial correlation.

2.1 The SDF

Consider an economy in which consumers maximize expected lifetime discounted utility and are assumed to have homogeneous CRRA preferences:

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau \frac{C_{ht,\tau+1}^{1-\gamma}}{1-\gamma} = 1 \quad h = 1, 2, ..., H_t,$$

where \(C_{ht}\) is the agent \(h\)’s consumption at date \(t\), \(\gamma > 0\) is the Arrow-Pratt measure of RRA, and \(\delta\) is the constant time discount factor.

The investor \(h\)’s optimal consumption profile must satisfy the following Euler equation:

$$E_t \left[ \delta \left( \frac{C_{ht,t+1}}{C_{ht}} \right)^{-\gamma} R_{i,t+1} \right] = 1 \quad h = 1, 2, ..., H_t, \ i = 1, ..., I,$$

where \(R_{i,t+1}\) is the gross return of asset (or portfolio of assets) \(i\) between \(t\) and \(t + 1\) and \(I\) is the number of traded assets. Expectation in (2) is taken conditionally on the date \(t\) information set that is common to all agents.
Denote \( p_{ihlt} \) the probability of participation of agent \( h \) in asset market \( i \) at date \( t \).\(^9\)

Weighting each individual Euler equation by the probability of holding the asset in question and summing over the total sample of households, we obtain

\[
\sum_{h=1}^{H_t} p_{ihlt} E_t \left[ \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} R_{i,t+1} \right] = \sum_{h=1}^{H_t} p_{ihlt}, \quad i = 1, \ldots, I \tag{3}
\]

or, equivalently,

\[
E_t \left[ \sum_{h=1}^{H_t} w_{ihlt} \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} R_{i,t+1} \right] = 1, \quad i = 1, \ldots, I, \tag{4}
\]

where \( w_{ihlt} = \frac{p_{ihlt}}{\sum_{h=1}^{H_t} p_{ihlt}} \) is the normalized probability of asset holding, \( \sum_{h=1}^{H_t} w_{ihlt} = 1, \quad i = 1, \ldots, I \).\(^10\) It follows that for any number of households in the cross-section, \( \sum_{h=1}^{H_t} w_{ihlt} \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \) is identical to the cross-sectional expectation of the intertemporal marginal rate of substitution, \( E_h \left[ \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right] \).\(^11\)

(4) may, hence, be rewritten as

\[
E_t \left[ E_h \left[ \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right] R_{i,t+1} \right] = 1, \quad i = 1, \ldots, I. \tag{5}
\]

The set of SDFs is, therefore,

\[
M_{i,t+1} = E_h \left[ \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right], \quad i = 1, \ldots, I. \tag{6}
\]

In the special case of complete consumption insurance, the intertemporal marginal rate of substitution is identical across individuals and SDF (6) reduces to that in the representative-agent framework, \( M_{i,t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \), \( i = 1, \ldots, I \), where \( C_t \) is aggregate consumption per capita in period \( t \).

Another special case of (6) is the Constantinides-Duffie (1996) pricing kernel. To see that, assume that individual consumption growth is lognormal. Therefore, for any \( i \)

\[
E_h \left[ \delta \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right] = exp \left( \ln \delta - \gamma E_h [\Delta c_{h,t+1}] + \frac{\gamma^2}{2} var_h (\Delta c_{h,t+1}) \right), \tag{7}
\]
where $\Delta c_{h,t+1} \equiv \ln \left( \frac{C_{h,t+1}}{C_{h,t}} \right)$.

The pricing kernel is, thus, given by

$$M_{t+1} = \delta \cdot \exp \left( -\gamma E_t [\Delta c_{h,t+1}] + \frac{\gamma^2}{2} \operatorname{var}_t (\Delta c_{h,t+1}) \right).$$  \hfill (8)

Assume as in Constantinides and Duffie (1996)

$$\ln \left( \frac{C_{h,t+1}}{C_{t+1}} \right) \sim N \left( -\frac{y_{t+1}^2}{2}, y_{t+1} \right).$$  \hfill (9)

Under this assumption, $\operatorname{var}_t (\Delta c_{h,t+1}) = y_{t+1}^2$ and $E_t [\Delta c_{h,t+1}] = \Delta c_{t+1} - \frac{y_{t+1}^2}{2}$, and, therefore, (8) simplifies to the Constantinides-Duffie (1996) SDF,

$$M_{t+1} = \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \exp \left( \frac{\gamma^2 + \gamma y_{t+1}^2}{2} \right).$$  \hfill (10)

### 2.2 Measurement Error Issue

Well documented potential problem with using household level data is the existence of large measurement errors in individual consumption.\(^{12}\) Assume, as in Brav, Constantinides, and Géczy (2002) and Vissing-Jorgensen (2002), there are multiplicative observation errors in the consumption level. Since individual consumption is assumed to be misreported by some stochastic fraction $\varepsilon_{ht}$, observed consumption growth is given by $\frac{C_{h,t+1}}{C_{h,t}} \varepsilon_{ht} \varepsilon_{ht}$, where $C^*_h$ is the true consumption of individual $h$ in period $t$. From (5), it follows that

$$E_t \left[ \delta E^i_h \left( \frac{C_{h,t+1} \varepsilon_{ht+1}}{C^*_h \varepsilon_{ht}} \right)^{-\gamma} R_{i,t+1} \right] = 1, \ i = 1, ..., I.$$  \hfill (11)

If $\varepsilon_{h,t+1}$ and $\varepsilon_{ht}$ are conditionally independent of all other variables in the Euler equation, (11) implies

$$E_t \left[ E^i_h \left( \frac{\varepsilon_{h,t+1}}{\varepsilon_{ht}} \right)^{-\gamma} \right] E_t \left[ \delta E^i_h \left( \frac{C^*_h}{C^*_h} \right)^{-\gamma} \right] R_{i,t+1} = 1, \ i = 1, ..., I.$$  \hfill (12)

Assuming that in the cross-section $\ln (\varepsilon_{ht}) \sim N (\mu_{\varepsilon}, \sigma^2_{\varepsilon})$, we get

$$E^i_h \left( \frac{\varepsilon_{h,t+1}}{\varepsilon_{ht}} \right)^{-\gamma} = \exp \left( \gamma^2 \left( \sigma^2_{\varepsilon} - \sigma_{\varepsilon_{t+1}, \varepsilon_t} \right) \right), \ i = 1, ..., I,$$  \hfill (13)

where $\sigma_{\varepsilon_{t+1}, \varepsilon_t}$ denotes the covariance of $\varepsilon_{h,t+1}$ and $\varepsilon_{ht}$.

SDF (6) is, then,

$$M_{i,t+1} = \delta \cdot E^i_h \left( \frac{C^*_h}{C^*_h} \right)^{-\gamma}, \ i = 1, ..., I.$$  \hfill (14)

where $\tilde{\gamma} \equiv \delta \cdot \exp \left( \gamma^2 \left( \sigma_e^2 - \sigma_{e_{t+1}}^2 \right) \right)$. It follows that with measurement error of the above type, the parameter $\gamma$ will be consistently estimated. The fact that $\sum_{h=1}^{H_t} w_{iht} \delta \left( \frac{c_{h,t+1}}{c_{ht}} \right)^{-\gamma} = E_h^i \left[ \delta \left( \frac{c_{h,t+1}}{c_{ht}} \right)^{-\gamma} \right], i = 1, \ldots, I$, for any number of households in the cross-section establishes that this conclusion is robust to small numbers of observations in the cross-sections. Unlike the estimate of the RRA coefficient, the estimate of $\delta$ will be inconsistent by the factor $\exp \left( \gamma^2 \left( \sigma_e^2 - \sigma_{e_{t+1}}^2 \right) \right)$.

2.3 The Binary Response Model for Asset Ownership

Assume that the household $h$’s indirect utility function can be written as a linear function of household characteristics $x_{ht}$ plus an error term $\epsilon_{ht}$:

$$y_{ht}^* = \beta' x_{ht} + \epsilon_{ht}, \quad h = 1, 2, \ldots, H, \quad t = 1, 2, \ldots, T,$$

(15)

where $x_{ht}$ is a vector of observable social and economic variables thought to affect the wish to participate in capital markets.$^{13}$ and $\beta$ is the vector of parameters. The error term $\epsilon_{ht}$ includes unobserved household-specific factors that may influence the capital market participation decisions. Assume $\epsilon_{ht}$ to be identically distributed with a zero mean and variance $\sigma_{\epsilon_{ht}}^2$, $\epsilon_{ht} \sim ID (0, \sigma_{\epsilon_{ht}}^2)$.

The indirect utility function is not observable, but we can observe a dummy variable of participation or nonparticipation in capital markets, $y_{ht}$. A household participates in capital markets if his indirect utility function is positive ($y_{ht} = 1$ if $y_{ht}^* > 0$) and does not participate otherwise ($y_{ht} = 0$ if $y_{ht}^* \leq 0$). It follows that the probability of participation in an asset market is

$$P \left( y_{ht}^* = \beta' x_{ht} + \epsilon_{ht} > 0 \right) = P \left( \epsilon_{ht} > -\beta' x_{ht} \right) = P \left( u_{ht} > -\frac{\beta' x_{ht}}{\sigma_{ht}} \right) = 1 - F \left( -\frac{\beta' x_{ht}}{\sigma_{ht}} \right),$$

(16)

where $u_{ht} \sim ID (0, 1)$ and $F (\cdot)$ is the cumulative density function of $u_{ht}$. The probability of nonparticipation is, then, $F \left( -\frac{\beta' x_{ht}}{\sigma_{ht}} \right)$. Assume heteroskedasticity to be multiplicative, $\sigma_{ht} = exp \left( \lambda' z_{ht} \right)$.

With error terms $u_{ht}$ independently distributed across individuals, for the complete panel data$^{15}$ the sample likelihood function can be written in general form as

$$L = \prod_{h=1}^{H} \int_{u_{h1}}^{u_{HT}} \cdots \int_{u_{hT}}^{u_{HT}} \frac{f \left( u_{h1}, \ldots, u_{hT}; \mu \right)}{\exp \left( \lambda' z_{ht} \right)} du_{h1} \cdots du_{hT},$$

(17)

$^{13}$Arrondel and Masson (1990), Bertaut (1998), Blume and Zeldes (1994), Haliassos and Bertaut (1995), and Mankiw and Zeldes (1991), for example, find that the probability of stock market participation increases in wealth, income, and education.

$^{14}$In the model with homoskedasticity ($\sigma_{ht} = \sigma$ for all $h$ and $t$), the parameters $\beta$ and $\sigma$ are estimable only up to a scale factor (what is estimated is $\alpha \equiv \frac{\sigma}{\beta}$ and $\beta$).

$^{15}$When there are the same $H$ cross-sectional units over $T$ time periods.
where \( u_{ht} \geq -\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})} \) means that \( u_{ht} > -\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})} \) when \( y_{ht} = 1 \) and \( u_{ht} < -\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})} \) when \( y_{ht} = 0 \). \( \mu \) is the full parameter vector, \( \mu = (\beta', \lambda)' \), and \( f (u_{h1}, \ldots, u_{hT}; \mu) \) denotes the \( T \)-variate standardized density function.

Denote \( F \left(-\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})}\right) \) and \( f \left(-\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})}\right) \) as \( F_{ht}(\mu) \) and \( f_{ht}(\mu) \), respectively. Under the assumption that \( u_{ht} \) are independent over time for all \( h \), the sample likelihood function takes the form:

\[
L = \prod_{h=1}^{H} \prod_{t=1}^{T} \left[ (1 - F_{ht}(\mu))^{y_{ht}} \cdot (F_{ht}(\mu))^{1-y_{ht}} \right].
\]  

(18)

The log likelihood is, then,

\[
l = \sum_{h=1}^{H} \sum_{t=1}^{T} \left[ y_{ht} \cdot \log (1 - F_{ht}(\mu)) + (1 - y_{ht}) \cdot \log (F_{ht}(\mu)) \right].
\]  

(19)

The maximum likelihood estimate of \( \mu, \hat{\mu} \), gets as a solution to \( \max_{\mu} l \). The first-order conditions for this problem are

\[
\frac{\partial l}{\partial \beta} = \sum_{h=1}^{H} \sum_{t=1}^{T} \frac{(y_{ht} - 1 + F_{ht}(\hat{\mu})) f_{ht}(\hat{\mu}) x_{ht}}{(1 - F_{ht}(\hat{\mu})) F_{ht}(\hat{\mu}) \cdot \exp(\lambda' z_{ht})} = 0,
\]  

(20)

\[
\frac{\partial l}{\partial \lambda} = \sum_{h=1}^{H} \sum_{t=1}^{T} \frac{(y_{ht} - 1 + F_{ht}(\hat{\mu})) f_{ht}(\hat{\mu}) \beta' x_{ht} \cdot z_{ht}}{(1 - F_{ht}(\hat{\mu})) F_{ht}(\hat{\mu}) \cdot \exp(\lambda' z_{ht})} = 0
\]  

(21)

and, thus, are analogous to orthogonality conditions in GMM analysis for a just-identified model.

In a rotating sample, we do not have complete time-series observations on cross-sectional units. Let \( n_t \) (\( 0 \leq n_t \leq H \)) denote the number of individuals in the sample replaced in period \( t \). Hence, for \( T \) periods the total number of individuals observed is \( H' = H + \sum_{t=1}^{T-1} n_t \). Denote the first and last periods during that the \( h \)th individual is observed by \( t_h \) and \( T_h \), respectively. Under the assumption that error terms \( u_{ht} \) are independently distributed across individuals, the sample likelihood function is

\[
L = \prod_{h=1}^{H'} \int_{u_{ht} \geq -\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})}} \cdots \int_{u_{ht} \geq -\frac{\beta' x_{ht}}{\exp(\chi' z_{ht})}} f (u_{ht}, \ldots, u_{ht}; \mu) \, du_{ht} \ldots du_{ht} 16
\]  

(22)

If \( u_{ht} \) are serially independent for all \( h \), the sample likelihood function for our model is

\[
L = \prod_{h=1}^{H'} \prod_{t=t_h}^{T_h} \left[ (1 - F_{ht}(\mu))^{y_{ht}} \cdot (F_{ht}(\mu))^{1-y_{ht}} \right].
\]  

(23)

\[^{16}\text{See Hsiao (1986).}\]
The log likelihood can, then, be written as
\[ l = \sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} [y_{ht} \cdot \log (1 - F_{ht}(\mu)) + (1 - y_{ht}) \cdot \log (F_{ht}(\mu))] \] (24)
and the first-order conditions are basically of the same form as those for the complete panel data:
\[ \frac{\partial l}{\partial \beta} = \sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\hat{\mu})) f_{ht}(\hat{\mu}) x_{ht}}{(1 - F_{ht}(\hat{\mu})) F_{ht}(\hat{\mu}) \cdot \exp(\lambda z_{ht})} = 0 \] (25)
and
\[ \frac{\partial l}{\partial \lambda} = \sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\hat{\mu})) f_{ht}(\hat{\mu}) \gamma' x_{ht} \cdot z_{ht}}{(1 - F_{ht}(\hat{\mu})) F_{ht}(\hat{\mu}) \cdot \exp(\lambda z_{ht})} = 0. \] (26)

Avery, Hansen, and Hotz (1983) take a different approach to derive moment conditions used in estimation of \( \beta \) and \( \lambda \). Following their approach, assume that \( \epsilon_{ht} \), conditional on \( x_{ht} \) and \( z_{ht} \), are identically distributed with a zero mean and variance \( \sigma^2_{ht} = \exp(\lambda z_{ht})^2 \), \( \epsilon_{ht} | x_{ht}, z_{ht} \sim ID(0, \exp(\lambda z_{ht}))^2 \), \( h = 1, 2, ..., H, t = 1, 2, ..., T. \)\(^{17}\) Under this assumption, \( E[y_{ht} | x_{ht}, z_{ht}] = 1 - F_{ht}(\mu) \) and, therefore, the following regression equation may be estimated:
\[ y_{ht} = 1 - F_{ht}(\mu) + v_{ht}. \] (27)

The requirement \( E[\epsilon_{ht} | x_{ht}, z_{ht}] = 0 \) implies
\[ E[v_{ht} | x_{ht}, z_{ht}] = E[y_{ht} | x_{ht}, z_{ht}] - (1 - F_{ht}(\mu)) = 0, \] (28)
what means that \( v_{ht} \) is orthogonal to any arbitrary function of current \( x \)’s and \( z \)’s, namely \( g_{0,h,t} \).

The conditional heteroskedasticity of \( v_{ht} \) makes GMM particularly attractive for estimating (27).\(^{18}\) The requirement that \( v_{ht} \) is orthogonal to a suitably chosen function \( g_{0,h,t} \) produces a set of orthogonality conditions that can be used to estimate \( \beta \) and \( \lambda \):
\[ E[v_{ht} \cdot g_{0,h,t}] = 0, \ h = 1, 2, ..., H. \] (29)

Since the function \( F_{ht}(\mu) \) is nonlinear in \( \beta \) and \( \lambda \), we can choose as instruments
\[ g_{0,h,t}(\mu) = \frac{F'_{ht}(\mu)}{\text{var}(v_{ht} | x_{ht}, z_{ht})}. \] (30)

\(^{17}\) Avery, Hansen, and Hotz (1983) assume \( \epsilon_{ht} | x_{ht}, z_{ht} \sim N(0, 1), \ h = 1, 2, ..., H, t = 1, 2, ..., T. \)

\(^{18}\) \( \text{var}(v_{ht} | x_{ht}, z_{ht}) = (1 - F_{ht}(\mu)) \cdot F_{ht}(\mu). \)
With this particular choice of $g_{0,h_t}$, orthogonality conditions (29) can be rewritten as

$$\sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\mu_0)) \cdot f_{ht}(\mu_0) \cdot x_{ht}}{(1 - F_{ht}(\mu_0)) \cdot F_{ht}(\mu_0) \cdot \exp(\lambda_0 z_{ht})} = 0 \tag{31}$$

and

$$\sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\mu_0)) \cdot f_{ht}(\mu_0) \cdot \beta_0 x_{ht} \cdot z_{ht}}{(1 - F_{ht}(\mu_0)) \cdot F_{ht}(\mu_0) \cdot \exp(\lambda_0 z_{ht})} = 0. \tag{32}$$

These moment conditions exploit only the orthogonality of the regression disturbances and functions of contemporaneous $x$'s and $z$'s and are analogous to the maximum likelihood ones obtained under the assumption that the disturbances of a given individual, $u_{ht}$, be independent over time.

To relax the assumption that $u_{ht}$ are serially independent, following Avery, Hansen, and Hotz (1983), assume that $x$'s and $z$'s are strictly exogenous. It follows that the regression disturbances are also orthogonal to functions of past and future $x$'s and $z$'s, $g_{j,h_t}$:

$$E[v_{ht} \cdot g_{j,h_t}] = 0, \ h = 1, 2, \ldots, H, \ j = 1, 2, \ldots, p, \tag{33}$$

where $p \leq T_h - t_h$ is the number of lags and leads used in instruments.

With lags and leads, we get the following additional orthogonality conditions involving correction of the estimators for serial dependence of disturbances:

$$\sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\mu_0)) \cdot f_{h,t+j}(\mu_0) \cdot x_{h,t+j} \cdot \exp(-\lambda_0 z_{h,t+j})}{((1 - F_{ht}(\mu_0)) \cdot F_{ht}(\mu_0))^{1/2} \cdot [(1 - F_{h,t+j}(\mu_0)) \cdot F_{h,t+j}(\mu_0)]^{1/2}} = 0 \tag{34}$$

and

$$\sum_{h=1}^{H'} \sum_{t=t_h}^{T_h} \frac{(y_{ht} - 1 + F_{ht}(\mu_0)) \cdot f_{h,t+j}(\mu_0) \cdot \beta_0^2 x_{h,t+j} \cdot z_{h,t+j} \cdot \exp(-\lambda_0 z_{h,t+j})}{[(1 - F_{ht}(\mu_0)) \cdot F_{ht}(\mu_0)]^{1/2} \cdot [(1 - F_{h,t+j}(\mu_0)) \cdot F_{h,t+j}(\mu_0)]^{1/2}} = 0, \tag{35}$$

$j = 1, 2, \ldots, p$, where $x_{h,t+j}$ and $z_{h,t+j}$ are variables that change over time.\(^{19}\)

### 3 Empirical Results

In this section, we assess the contribution of the CCAPM with asset ownership probability weighted agents towards explaining the U.S. monthly asset returns using micro data from the CEX. In the CEX, households are asked to report their holdings of (i) stocks, bonds, mutual funds, and other such securities, (ii) U.S. Savings Bonds, (iii) savings accounts at banks, savings and loans, credit unions, etc., and (iv) checking accounts, brokerage accounts,

\(^{19}\)See Avery, Hansen, and Hotz (1983).
and other similar accounts. This information does not allow to perfectly identify households whose consumption must be involved in the Euler equations for the equity premium and the risk-free rate. Therefore, we treat this information on asset holding status as an imperfect sample separation indicator.

Specifying the binary response model for asset ownership as a multiperiod bivariate probit, we first assume that the decision to own assets is not related to the consumption and savings decisions and, hence, the disturbances in the bivariate probit model and the errors in the Euler equations for the equity premium and the risk-free rate are not correlated. Under this assumption, we apply the following two-stage estimation procedure. In a first stage, we estimate the bivariate probit model for asset ownership. In a second stage, we estimate the Euler equation (4) for the equity premium and the risk-free rate with the normalized predicted probabilities of asset holding obtained in the first stage. The estimates of the parameters of the Euler equations are then compared to those when (i) as in the basic CCAPM, all consumers are assumed to participate in asset markets and (ii) the available in the CEX information on asset holding status is regarded as a perfect sample classification indicator and the Euler equations are estimated for the subset of households classified as asset holders. Since the decision to own assets in each time period is likely to be endogenous with respect to the consumption and savings decisions and, therefore, the errors in the bivariate probit model for asset ownership and the errors in the Euler equations may be correlated, our second approach is to estimate the bivariate probit and the Euler equations for the equity premium and the risk-free rate of return simultaneously (a one-stage estimation procedure).

3.1 The Data

Households. The data used for estimation are drawn from the monthly CEX. The CEX data available cover the period from 1979:10 to 1996:3. It is a collection of data on approximately 5000 households per quarter in the U.S. The CEX is a rotating sample. As households complete their participation, they are dropped and new households move into the sample. Thus, each quarter about 20% of the consumer units are new. Each household in the sample is interviewed every three months over five consecutive quarters.\textsuperscript{21} The second through fifth interviews use uniform questionnaires to collect demographic and family characteristics as well as data on monthly consumption expenditures for the previous three months made by households in the survey.\textsuperscript{22} Various income information and information

\textsuperscript{20}The Euler equation for the equity premium involves consumption of households that hold a position in both the risky and risk-free assets and the Euler equation for the risk-free rate holds for agents that own the risk-free asset but not necessarily the risky asset.

\textsuperscript{21}The first interview is practice and is not included in the published data set.

\textsuperscript{22}Demographic variables are based upon heads of households.
on the employment of each household member is collected in the second and fifth interviews.

For the consumer units completing their participation in the first through third quarters of 1986, the Bureau of Labor Statistics has changed, beginning the first quarter of 1986, the consumer unit identification numbers so that the identification numbers for the same household in 1985 (when this household has been interviewed for the first time) and in 1986 (when it has completed its participation) are not the same. To match consumer units between the 1985 and 1986 data tapes, we use household characteristics which allow us to identify consumer units uniquely. The detailed description of the procedure used to match consumers units can be found in Semenov (2004).

In the fifth (final) interview, the household is asked to report end-of-period estimated market value of all securities (market value of all stocks, bonds, mutual funds, and other such securities) held by the consumer unit on the last day of the previous month as well as the difference between this estimated market value and the value of all securities held a year ago last month. Using these two values, we calculate each household’s asset holdings at the beginning of a 12-month recall period.

The Consumption Data. As opposed to the Panel Study of Income Dynamics (PSID), which offers only food consumption data on an annual basis, the CEX contains highly detailed data on monthly consumption expenditures. The CEX attempts to account for an estimated 70% of total household consumption expenditures. Since the CEX is designed with the purpose of collecting consumption data, measurement error in consumption is likely to be smaller for the CEX consumption data compared to the PSID consumption data. Our aggregate measure of consumption is total consumption of nondurables and services. For each household, we calculate monthly consumption expenditures for all the disaggregate consumption categories offered by the CEX. Then, we deflate obtained values in 1982-84 dollars with the CPI’s (not seasonally adjusted, urban consumers) for appropriate consumption categories. Aggregating the household’s monthly consumption across these categories is made according to the National Income and Product Account definitions of consumption aggregates. In order to transform our consumption data to a per capita basis, we normalize the consumption of each household by dividing it by the number of family members in the household. The per capita consumption growth between two periods \( t \) and \( t + 1 \) is defined as the ratio of the per capita consumption in periods \( t + 1 \) and \( t \).

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23Food consumption is likely to be one of the most stable consumption components. Furthermore, as is pointed out by Carroll (1994), 95% of the measured food consumption in the PSID is noise due to the absence of interview training.

24The CPI series are obtained from the Bureau of Labor Statistics through CITIBASE.

25The monthly consumption growth between two periods \( t \) and \( t + 1 \) is calculated if for both months consumption is not equal to zero (missing information is counted as zero consumption).
The Returns Data. The measure of the nominal market return is the value-weighted return (capital gain plus dividends) on all stocks listed on the NYSE and AMEX obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The nominal monthly risk-free rate of interest is the 1-month Treasury bill return from CRSP. The real monthly returns are calculated as the nominal returns divided by the 1-month inflation rate based on the deflator defined for consumption of nondurables and services. Market premium is calculated as the difference between the real market return and the real risk-free rate.

Data Selection Criteria. We drop from the sample nonurban households, households residing in student housing, households with incomplete income responses, and households who do not have a fifth interview. Following Brav, Constantinides, and Gécz (2002), in any given month we drop from the sample households that report in that month as zero either their food consumption or their consumption of nondurables and services, or their total consumption, as well as households with missing information on the above items. Additionally, we keep in the sample only households whose head is between 19 and 75 years of age. Following Vissing-Jorgensen (1998), we drop from our sample the bottom and the top percent of consumption growth observations for each month. The final sample consists of 463112 monthly consumption growth observations ranging from 0.16 to 6.81 within each month.

3.2 The Two-Stage Estimation Procedure
3.2.1 The Bivariate Probit Model for Asset Ownership
In our empirical investigation, we assume for each household the probability of holding both the risky and risk-free assets to be the same as the probability of owning the risk-free asset. The dummy variable of participation in asset markets, \( y_{ht} \), is equal to 1 if a household reported the positive estimated market value of all securities held a year ago last month and equal to 0 otherwise. The set of respondent characteristics thought to be relevant for the decision to acquire assets includes dummy variables for age, education, marital status, origin or ancestry, race, sex of reference person, housing tenure, composition of earners, and family type. To capture the influence of income risk, we use as the respondent characteristic the logarithm of the total amount of family income after taxes in past 12 months (converted to 1999 dollar amount). Other household characteristics used in

\[\text{We consider these extreme values to be due to reporting or coding errors.}\]
\[\text{In other words, we assume the same SDF in the Euler equations for the excess market return and the risk-free rate.}\]
\[\text{Details of all the dummy variables are included in Appendix A.}\]
our analysis are number of members in family, number of children less than 18, number of person over 64, total number of autos, and number of earners. Table I provides descriptive statistics for the data set used in the estimation.29

Specifying the binary response model for asset ownership as a multiperiod bivariate probit, we first estimate this model by GMM under the assumption that the disturbances for the same household are serially independent. Under this assumption, the parameter estimates are obtained by exploiting the following set of contemporaneous orthogonality conditions (a just-identified model):

\[
\sum_{t=1}^{T} \sum_{h=1}^{H_t} \frac{(y_{ht} - \Phi_{ht}(\mu_0)) \cdot \varphi_{ht}(\mu_0) \cdot x_{ht}}{(1 - \Phi_{ht}(\mu_0)) \cdot \Phi_{ht}(\mu_0) \cdot \exp(\lambda_0 z_{ht})} = 0
\] (36)

and

\[
\sum_{t=1}^{T} \sum_{h=1}^{H_t} \frac{(y_{ht} - \Phi_{ht}(\mu_0)) \cdot \varphi_{ht}(\mu_0) \cdot \beta_0^t x_{ht} \cdot z_{ht}}{(1 - \Phi_{ht}(\mu_0)) \cdot \Phi_{ht}(\mu_0) \cdot \exp(\lambda_0 z_{ht})} = 0,
\] (37)

where \( \Phi_{ht}(\mu_0) \equiv \Phi\left( \frac{\beta_0^t x_{ht}}{\exp(\lambda_0 z_{ht})} \right) \) is the cumulative distribution function and \( \varphi_{ht}(\mu_0) \equiv \varphi\left( \frac{\beta_0^t x_{ht}}{\exp(\lambda_0 z_{ht})} \right) \) is the density function of the standard normal.

Then, we relax the assumption of serially independent disturbances and estimate the bivariate probit model for asset ownership under the assumption that the disturbance terms may exhibit serial correlation. When the assumption of serially independent disturbances is relaxed, the probability of being assetholder is estimated exploiting (36) and (37) jointly with the following extra orthogonality conditions between disturbances and the right-hand-side variables from other time periods:

\[
\sum_{t=1}^{T} \sum_{h=1}^{H_t} \frac{(y_{ht} - \Phi_{ht}(\mu_0)) \cdot \varphi_{h,t \pm j}(\mu_0) \cdot x^*_{h,t \pm j} \cdot \exp(-\lambda_0^j z_{h,t \pm j})}{((1 - \Phi_{ht}(\mu_0)) \cdot \Phi_{ht}(\mu_0))^{1/2} \cdot (1 - \Phi_{h,t \pm j}(\mu_0)) \cdot \Phi_{h,t \pm j}(\mu_0))^{1/2}} = 0.
\] (38)

Because for each household in our data set observations on market value of all securities are available for two periods in time (at the beginning and the end of a 12-month recall period) only, we set \( j = 12 \), so as \( x^*_{h,t \pm j} \) is a vector with elements \( x^*_{h,t+12} \) for households who completed the second interview and \( x^*_{h,t-12} \) for those who completed the fifth interview.

The estimation and test results for the bivariate probit model are presented in Table II. The estimates reported in column SID (Serially Independent Disturbances) are obtained by exploiting the contemporaneous orthogonality conditions (36) and (37) (the instrument

29When estimating the probabilities of asset ownership, we restrict our sample to households who completed both the second and fifth interviews. After dropping households reported nonpositive value of total amount of family income after taxes in past 12 months, our resulting sample contains 37996 households (75992 observations).
set INSP1). Column SDD (Serially Dependent Disturbances) contains the coefficient estimates obtained by imposing the contemporaneous orthogonality conditions (36) and (37) together with the orthogonality condition (38) for the variable “number of persons over 64” in the vector $x_{h,t,i}^k$ (the instrument set INSP2). Hansen’s $J$ statistic is used to test the overidentifying restrictions and, hence, the assumption of serially dependent disturbances. Our measure of goodness of fit of the bivariate probit model is the overall percent correctly predicted.\footnote{For each household, we calculate the predicted probability of asset ownership, $\hat{p}_{ht} = \Phi_{ht}(\hat{\mu}_0)$. If $\hat{p}_{ht} \geq 0.5$, we predict the household to participate in asset markets. If $\hat{p}_{ht} < 0.5$, the household is predicted not to hold assets. For each outcome, we then calculate the percent correctly predicted. Their weighted average, with the weights being the fractions of the outcomes, is the overall percent correctly predicted.}

Numerous results demonstrate that the sampling distributions of GMM estimators and test statistics can exhibit substantial size distortions from asymptotic normality.\footnote{See Fuhrer, Moore, and Schuh (1995), Hansen, Heaton, and Yaron (1996), Kocherlakota (1990), and Tauchen (1986).} Stock and Wright (2000) investigate weak identification as one possible source of this problem and develop methods for the construction of asymptotically valid hypothesis tests and confidence sets when some or all of the parameters are weakly identified. As suggested by Stock and Wright (2000), we use the $J$ statistic, $J = TS_{cT}(\hat{\mu}_0) \xrightarrow{d} \chi^2_{IK,1-r}$, to test the validity of the bivariate probit model under the assumption of weak instruments.\footnote{IK is the number of moment conditions. The null hypothesis $H_0 : \mu = \mu_0$ is rejected statistically at the r\% significance level if the $J$ statistic exceeds the appropriate $\chi^2_{IK,1-r}$ critical value.} Here, $S_{cT}(\hat{\mu}_0)$ is the continuous updating objective function evaluated at $\hat{\mu}_0$. Under the assumption of serially independent disturbances,

$$S_{cT}(\hat{\mu}_0) = \overline{m}_{1T}(\hat{\mu}_0)^\prime V_{1T}^{-1}(\hat{\mu}_0) \overline{m}_{1T}(\hat{\mu}_0),$$

where

$$V_{1T}(\hat{\mu}_0) = T^{-1} [m_{1T}(\hat{\mu}_0) - \overline{m}_{1T}(\hat{\mu}_0)]' [m_{1T}(\hat{\mu}_0) - \overline{m}_{1T}(\hat{\mu}_0)]$$

is the heteroskedasticity robust covariance matrix of moment conditions, $V_{1T}^{-1}(\hat{\mu}_0)$ is the positive definite $IK \times IK$ efficient weight matrix,

$$\overline{m}_{1T}(\hat{\mu}_0) = T^{-1} \sum_{t=1}^{T} m_{1T}(\hat{\mu}_0),$$

$$m_{1T}(\hat{\mu}_0) = \sum_{h=1}^{H_t} \frac{(y_{ht} - \Phi_{ht}(\hat{\mu}_0)) \cdot \varphi_{ht}(\hat{\mu}_0) \cdot x_{ht}}{(1 - \Phi_{ht}(\hat{\mu}_0)) \cdot \Phi_{ht}(\hat{\mu}_0) \cdot \exp(\lambda_0 z_{ht})},$$

and

$$m_{2T}(\hat{\mu}_0) = \sum_{h=1}^{H_t} \frac{(y_{ht} - \Phi_{ht}(\hat{\mu}_0)) \cdot \varphi_{ht}(\hat{\mu}_0) \cdot \tilde{\beta}_0' x_{ht} \cdot z_{ht}}{(1 - \Phi_{ht}(\hat{\mu}_0)) \cdot \Phi_{ht}(\hat{\mu}_0) \cdot \exp(\lambda_0 z_{ht})}.$$
When disturbances are assumed to be serially correlated,

\[ S_{cT} (\tilde{\mu}_0) = \overline{m}_{2t} (\tilde{\mu}_0)' V_{2T}^{-1} (\tilde{\mu}_0) \overline{m}_{2t} (\tilde{\mu}_0), \tag{44} \]

where

\[ V_{2T} (\tilde{\mu}_0) = T^{-1} [m_{2t} (\tilde{\mu}_0) - \overline{m}_{2t} (\tilde{\mu}_0)]' [m_{2t} (\tilde{\mu}_0) - \overline{m}_{2t} (\tilde{\mu}_0)], \tag{45} \]

\[ \overline{m}_{2t} (\tilde{\mu}_0) = T^{-1} \sum_{t=1}^{T} m_{2t} (\tilde{\mu}_0), \tag{46} \]

\[ m^1_{2t} (\tilde{\mu}_0) = \sum_{h=1}^{H_t} \frac{(y \cdot \Phi (\tilde{\mu}_0)) \cdot \varphi (\tilde{\mu}_0) \cdot x \cdot exp (\lambda \cdot z)}{(1 - \Phi (\tilde{\mu}_0)) \cdot \Phi (\tilde{\mu}_0) \cdot exp (\lambda \cdot z)} \tag{47} \]

\[ m^2_{2t} (\tilde{\mu}_0) = \sum_{h=1}^{H_t} \frac{(y \cdot \Phi (\tilde{\mu}_0)) \cdot \varphi (\tilde{\mu}_0) \cdot \tilde{x} \cdot z \cdot exp (-\lambda \cdot z)}{(1 - \Phi (\tilde{\mu}_0)) \cdot \Phi (\tilde{\mu}_0) \cdot exp (\lambda \cdot z)}, \tag{48} \]

and

\[ m^3_{2t} (\tilde{\mu}_0) = \sum_{h=1}^{H_t} \frac{(y \cdot \Phi (\tilde{\mu}_0)) \cdot \varphi (\tilde{\mu}_0) \cdot \tilde{x} \cdot z \cdot exp (-\lambda \cdot z)}{[(1 - \Phi (\tilde{\mu}_0)) \cdot \Phi (\tilde{\mu}_0)]^{1/2} \cdot [(1 - \Phi (\tilde{\mu}_0)) \cdot \Phi (\tilde{\mu}_0)]^{1/2}} \tag{49} \]

We find that the model with serially independent disturbances is rejected at the 5% significance level under weak-identification asymptotics. The model with serially correlated disturbances is not rejected statistically both under normal and weak-identification asymptotics.\textsuperscript{33} This result provides some evidence that the households’ market participation behavior exhibits considerable persistence and is in line with the result in Bertaut (1998).

Under the assumption that disturbances exhibit serial correlation, the predicted probability of asset ownership increases with age (all of the coefficients on the age-range dummies are positive and significantly different from the omitted dummy (under 35) at the 5% level). This result is consistent with that obtained in Bertaut (1998). However, in opposite to his result, having age greater than or equal to 65 decreases the predicted probability of being an assetholder, suggesting that older households may be deterred from asset holding by shorter investment horizons. As expected, higher education leads to the greater predicted probability of asset market participation. The coefficients for the dummy variables for education are both positive with only the coefficient on having at least one year of college significantly different at the 5% level from the omitted dummy (never attended school). In opposite to

\textsuperscript{33}According to Hansen’s test of the overidentifying restrictions, the null hypothesis of serially dependent disturbances is not rejected statistically at the 5% level.
Bertaut (1998), we find that sex, race, and household income are relevant for the decision to acquire assets. The predicted probability of asset ownership decreases with number of members in family (the coefficient is significant at the 5% level) and increases with number of children less than 18, number of persons over 64, and number of autos (the coefficients on these variables are significant at least at the 10% level). Results for origin, housing tenure, and composition of earners provide evidence that these variables can influence asset market participation. Neither marital status (as in Bertaut (1998)), number of earners in family, nor family type is significant, suggesting no special role for these variables in predicting asset ownership. The coefficients on both variables (composition of earners and number of autos) in the variance term are significant at the 5% level.

3.2.2 Euler Equation Estimation and Test Results

In this stage, we estimate the Euler equation

\[
E_t \left[ \delta \left( \sum_{h=1}^{H_t} \bar{w}_{ht} \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right) R_{t,t+1} \right] = 1, \quad i = 1, \ldots, I
\]  

for the excess market return as

\[
E_t \left[ \left( \sum_{h=1}^{H_t} \bar{w}_{ht} \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right) (R_{M,t+1} - R_{F,t+1}) \right] = 0
\]  

and for the real risk-free interest rate as

\[
E_t \left[ \delta \left( \sum_{h=1}^{H_t} \bar{w}_{ht} \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right) R_{F,t+1} \right] = 1
\]

jointly using the two-step GMM estimation technique.

Three models are considered. In the first model, we assume that there is no limited asset market participation and all agents participate in asset markets with equal probabilities ($\bar{w}_{ht} = \frac{1}{H_t}$, where $H_t$ is the number of agents in the whole sample of households, 463112 monthly consumption growth observations). This is our benchmark model. The results for this model are summarized in Panel A of Table III. Then, we use two different approaches to allow for the limited participation of households in the capital markets. First, as in Brav, Constantinides, and Géczy (2002) and Vissing-Jorgensen (2002), we treat the available in the CEX information on asset holding status as a perfect sample classification indicator, classify as assetholders individuals that reported positive asset holdings at the beginning of a 12-month recall period, and estimate the Euler equations (51) and (52) jointly for the subset of equally-weighted assetholders ($\bar{w}_{ht} = \frac{1}{H_t}$, where $H_t$ is the number of households classified as assetholders, 70934 monthly consumption growth observations). The estimation
and test results for this model are reported in Panel B of Table III. Our second approach consists in allowing for the fact that the available in the CEX information on asset holding status does not allow to perfectly identify households whose consumption must be involved in the Euler equations for the equity premium and the risk-free rate. As in our benchmark model, we consider the whole sample of households but in this model households are no longer weighted equally. The weight applied to the household \( h \)'s intertemporal marginal rate of substitution is given by the normalized probability of his participation in asset markets estimated from the bivariate probit with serially dependent disturbances, \( \bar{w}_{ht} = \hat{p}_{ht} / \sum_{h=1}^{H_t} \hat{p}_{ht} = \Phi_{ht} (\hat{p}_0) / \sum_{h=1}^{H_t} \Phi_{ht} (\hat{p}_0) \). This is the model with probability-weighted households, the estimation and test results for which are presented in Panel C of Table III.

Since in our model the date \( t \) information set is common across the households, we may assume it to include information on lagged real market returns, risk-free rates, and consumption growth. Empirical evidence is that lagged asset returns and consumption growth have a low correlation with current returns and consumption growth. It suggests that the problem of weak identification might arise in GMM estimation of the Euler equation (50) when lagged real market returns, risk-free rates, and consumption growth are used as instruments and leads us to test the Euler equation (50) for the equity premium and the risk-free rate under the assumption that the instruments are only weakly correlated with the relevant first-order conditions.\(^{34}\)

Rewrite equation (50) as

\[
E_t \left[ \sum_{h=1}^{H_t} \bar{w}_{ht} \left( \delta \left( \frac{C_{ht+1}}{C_{ht}} \right)^{-\gamma} R_{i,t+1} - 1 \right) \right] = 0, \ i = 1, \ldots, I \tag{53}
\]

and denote \( e_{h,t+1} (\theta) = (e_{1h,t+1} (\theta), \ldots, e_{Ih,t+1} (\theta)) \) the vector of the errors associated with the Euler equation (2), where \( e_{ih,t+1} (\theta) = \delta \left( \frac{C_{ht+1}}{C_{ht}} \right)^{-\gamma} R_{i,t+1} - 1, \ i = 1, \ldots, I, \) and \( \theta = (\gamma, \delta)' \) is a parameter vector with true value \( \theta_0 = (\gamma_0, \delta_0)' \).

Let \( d_t \) be a \( K \)-dimensional vector of the common to all agents instruments contained in their information set at time \( t \). Theory, therefore, implies the \( IK \) orthogonality conditions

\[
E [m_{3t} (\theta_0)] = 0, \tag{54}
\]

where \( m_{3t}^k (\theta) = \sum_{h=1}^{H_t} \bar{w}_{ht} e_{h,t+1} (\theta) d_t^k, \ k = 1, \ldots, K \). Under the assumption that \( \hat{p}_{ht} = 1 \) for all \( h \) and \( t \), \( \bar{w}_{ht} = 1 / H_t \) and, hence, \( m_{3t}^k (\theta) = H_t^{-1} \sum_{h=1}^{H_t} e_{h,t+1} (\theta) d_t^k, \ k = 1, \ldots, K \). In that case, orthogonality conditions (54) are equivalent to those used in Jacobs (1999) when instruments are assumed to be common to all agents.

\(^{34}\)Stock and Wright (2000) find that for the intertemporal CCAPM, the weak-identification asymptotic approximations to the distributions of GMM estimators and test statistics generally match closely the finite sample distributions in opposite to the usual normal approximations.
Following Stock and Wright (2000), we treat $\delta$ as strongly identified.\textsuperscript{35} Because of concerns about weak identification of $\gamma$, we compute confidence sets for $\gamma$ immune to weak identification ($S$-sets).\textsuperscript{36} As in Stock and Wright (2000), we form an asymptotic 95\% $S$-set for $\gamma$ in which $\delta$ is concentrated out:

$$
\left\{ \gamma_0 : TS_{cT}(\gamma_0, \hat{\delta}(\gamma_0)) \leq \chi_{1.0,95}^2 \right\},
$$

where

$$
S_{cT}(\gamma_0, \hat{\delta}(\gamma_0)) = m_{3T}(\gamma_0, \hat{\delta}(\gamma_0))V_{3T}^{-1}(\gamma_0, \hat{\delta}(\gamma_0))m_{3T}(\gamma_0, \hat{\delta}(\gamma_0)),
$$

$$
V_{3T}(\gamma_0, \hat{\delta}(\gamma_0)) = T^{-1}\left[ m_{3T}(\gamma_0, \hat{\delta}(\gamma_0)) - m_{3T}(\gamma_0, \hat{\delta}(\gamma_0)) \right]'
\left[ m_{3T}(\gamma_0, \hat{\delta}(\gamma_0)) - m_{3T}(\gamma_0, \hat{\delta}(\gamma_0)) \right],
$$

$$
\hat{\delta}(\gamma_0) = T \left[ \sum_{t=1}^{T} \sum_{h=1}^{H} \hat{w}_{ht} \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma_0} R_{F,t+1} \right]^{-1}.
$$

For each of the three considered models, the Euler equations for the excess market return and the real risk-free interest rate are estimated jointly exploiting four sets of instruments. The first instrument set (INSE1) has a constant, the real market return and consumption growth rate lagged once. The second set (INSE2) is the instrument set INSE1 extended with the real risk-free rate lagged once. The third set of instruments (INSE3) has a constant, the real market return and consumption growth rate lagged twice. The fourth instrument set (INSE4) consists of the same variables as INSE3 plus the real risk-free rate lagged two periods. $TS_{cT}$ concentrated with respect to $\delta$ are graphed in Figures 1 to 4.

For all the models, moderate positive values of $\gamma$ are estimated. The parameter $\delta$ is in the conventional range, except for model M2a, for which the point estimate of the time discount factor is greater than 1. The two-step $J$ statistic fails to reject at the 5\% level the three models under conventional normal asymptotics. Under the assumption of weak

\textsuperscript{35}Given $\gamma$, the parameter $\delta$ can be estimated precisely from the Euler equation for the risk-free rate of return as $\hat{\delta}(\gamma) = T \left[ \sum_{t=1}^{T} \sum_{h=1}^{H} \hat{w}_{ht} \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma_0} R_{F,t+1} \right]^{-1}$ and, therefore, is well identified by the constant term (a constant is a strong instrument).

\textsuperscript{36}The $r\%$ $S$-set for $\gamma$ consists of the values of $\gamma$ at which the null hypothesis $H_0 : \gamma = \gamma_0$ and the overidentifying conditions are not rejected jointly at the $r\%$ significance level.
identification, all the models are rejected at the 5% significance level when the real risk-free rate is used as an instrument. When the models with equally-weighted agents are not rejected statistically under weak-identification asymptotics (models M1a and M3b), they yield the point estimates of the RRA coefficient which are insignificantly different from 0 at the 5% level. In contrast to the models with equally-weighted households, the model with probability-weighted agents allows to precisely estimate the RRA coefficient (models M1c and M3c). The asymptotic concentrated S-sets and conventional GMM confidence sets for $\gamma$ agree closely both for model M1c and model M3c. Empirical evidence is that when the predicted probabilities of asset market participation are used as weights, the two-step GMM point estimates of $\gamma$ and $\delta$ are less sensitive to instrument choice compared to when households are equally weighted.

### 3.3 The One-Stage Estimation Procedure

The one-stage procedure consists in joint estimation of the bivariate probit model and the Euler equations for the equity premium and the risk-free rate. Given that the hypothesis of persistence in the households’ market participation behavior is not rejected statistically, we estimate the parameters of interest using the orthogonality conditions (36), (37), and (38) jointly with the orthogonality conditions implied by the Euler equations for the excess market portfolio return,

$$
E_t \left[ \frac{1}{\sum_{h=1}^{H_t} \Phi_{ht} (\mu)} \left( \sum_{h=1}^{H_t} \Phi_{ht} (\mu) \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right) (R_{M,t+1} - R_{F,t+1}) \right] = 0,
$$

and the real risk-free interest rate,

$$
E_t \left[ \frac{\delta}{\sum_{h=1}^{H_t} \Phi_{ht} (\mu)} \left( \sum_{h=1}^{H_t} \Phi_{ht} (\mu) \left( \frac{C_{h,t+1}}{C_{ht}} \right)^{-\gamma} \right) R_{F,t+1} \right] = 1.
$$

Four sets of instruments are used. The first set of instruments (INSPE1) is the instrument set INSE1, we used to estimate the Euler equations in the second stage of the two-stage estimation procedure, plus the set of instruments INSP2, we used to estimate the probit model for asset ownership in the first stage of the two-stage estimation procedure. The second set (INSPE2) is the instrument set INSE2 extended with the set of instruments INSP2. The third set (INSPE3) is the instrument set INSE3 plus the set of instruments INSP2. The fourth set (INSPE4) is the instrument set INSE4 extended with the set of instruments INSP2. The two-step GMM parameter estimates and test statistics are presented in Table IV.

The obtained point estimates of the RRA coefficient and the time discount factor are in the conventional range for all the sets of instruments. According to Hansen’s test of the
overidentifying restrictions, we cannot reject the model at the 5% significance level for any instrument set under normal asymptotics. Under weak identification asymptotics, as in the case of using the two-stage estimation procedure, the model is rejected statistically at the 5% level when the real risk-free rate is used as an instrument (sets INSPE2 and INSPE4).

Comparing the results for the set of instruments including a constant and the real market return and consumption growth rate lagged twice (the only instrument set for which the model with equally-weighted assetholders is not rejected statistically under the assumption of weak identification) reveals that, as for the two-stage estimation procedure, the model with probability-weighted consumers allows to estimate the RRA coefficient more precisely than when the asset holding status is regarded as a perfect sample classification indicator (model M3b). In opposite to the point estimates from the bivariate probit model for asset ownership, the point estimates of the RRA coefficient are quite sensitive to the chosen set of instruments. The point estimates from the bivariate probit model are close to those obtained using the two-stage procedure, but are estimated more precisely. By contrast, there are evidently important differences in the point estimates of the RRA coefficient. This suggests that the errors in the bivariate probit model for asset ownership are correlated with the errors in the Euler equations for the equity premium and the risk-free rate and, hence, the decision to acquire assets is likely to be endogenous with respect to the consumption and savings decisions. This result is consistent with the results in Attanasio, Banks, and Tanner (2002) and Bertaut (1998).

4 Conclusions

To test the CCAPM with probability weighted agents, we use data from the CEX. Since the available in the CEX information on asset holdings does not provide a perfect sample classification, we treat this information as an imperfect sample separation indicator. Specifying the binary response model for asset ownership as a multiperiod bivariate probit, we use two approaches to estimate the parameters of interest.

For a start, we assume that an agent makes the decision whether or not he wants to participate in asset markets independently of the consumption and savings decisions. The following two-stage estimation approach is involved to estimate the parameters. In the first stage, we estimate the bivariate probit model for asset ownership. In the second stage, we take the estimated in the first stage probabilities of asset ownership as given and estimate the Euler equations for the equity premium and the risk-free rate. The hypothesis that the households’ market participation behavior exhibits persistence is not rejected statistically at the 5% significance level both under normal and weak-identification asymptotics. This finding is in line with the result in Bertaut (1998). The Euler equations with agents weighted
by the predicted probabilities of asset market participation, obtained from the bivariate probit model with serially dependent disturbances, are not rejected statistically both under normal and weak-identification asymptotics when the real market return and consumption growth lagged once or twice are used as instruments. For these sets of instruments, we find that weighting households by the probabilities of asset ownership allows to estimate the RRA coefficient more precisely compared to when the whole sample of equally-weighted consumers is considered or when the reported in the CEX financial information is regarded as a perfect sample separation indicator.

An intuitively appealing assumption is that households make the decision about asset market participation simultaneously with the consumption and savings decisions. It suggests that the parameters of the Euler equations must be estimated jointly with the parameters of the bivariate probit model for asset ownership. When the bivariate probit model and the Euler equations for the equity premium and the risk-free rate are estimated jointly, the CCAPM with asset ownership probability weighted agents is not rejected statistically at the 5% level both under normal and weak-identification asymptotics for the same sets of instruments for the Euler equations as when the disturbances in the bivariate probit and the errors in the Euler equations are assumed to be uncorrelated. We find that the point estimates of the RRA coefficient differ significantly between the two estimation procedures. This result provides some evidence that the decision to own assets is likely to be endogenous with respect to the consumption and savings decisions and that allowing for this fact is important for estimating risk aversion. This finding is consistent with the results in Attanasio, Banks, and Tanner (2002) and Bertaut (1998).
Appendix A: Description of the Dummy Variables

1. Asset market participation
   - 1  Assetholder
   - 0  Otherwise

2. Age of reference person:
   - Age 35-44
     - 1  Age of reference person is \( \geq 35 \) and \( \leq 44 \)
     - 0  Otherwise
   - Age 45-54
     - 1  Age of reference person is \( \geq 45 \) and \( \leq 54 \)
     - 0  Otherwise
   - Age 55-64
     - 1  Age of reference person is \( \geq 55 \) and \( \leq 64 \)
     - 0  Otherwise
   - Age 65 and over
     - 1  Age of reference person is \( \geq 65 \)
     - 0  Otherwise
   - Omitted dummy - Age of reference person is less than 35

3. Education of reference person:
   - Education: High school degree
     - 1  Elementary (1-8 years)
     - High school (1-4 years), less than High school graduate
     - High school graduate (4 years)
     - 0  Otherwise
   - Education: College degree
     - 1  College (1-4 years), less than college graduate
     - College graduate (4 years)
     - More than 4 years of college
     - 0  Otherwise
   - Omitted dummy - Never attended school

4. Marital status of reference person:
   - Married
     - 1  Married
     - 0  Otherwise
   - Omitted dummy - Widowed, Divorced, Separated, Never married

5. Origin or ancestry of reference person:
Origin: European
1  German
   Italian
   Irish
   French
   Polish
   Russian
   English
   Scottish
   Dutch
   Swedish
   Hungarian
   Welsh
0  Otherwise

Origin: Spanish
1  Mexican American
   Chicano
   Mexican
   Puerto Rican
   Cuban
   Central or South American
   Other Spanish
0  Otherwise

Origin: Afro-American
1  Afro-American
0  Otherwise

Omitted dummy - Other and Don’t Know

6. Race of reference person:

Race: White
1  White
0  Otherwise

Race: Black
1  Black
0  Otherwise

Omitted dummy - American Indian, Aleut, Eskimo, Asian, Pacific Islander, Other

7. Sex of reference person:

Male
1  Male
0  Otherwise

Omitted dummy - Female

8. Housing tenure:
House owned
1 Owned with mortgage
   Owned without mortgage
   Owned mortgage not reported
0 Otherwise
Omitted dummy - Rented, Occupied without cash payment

9. Composition of earners:

Composition of earners
1 Reference person only
   Reference person and spouse
   Reference person, spouse and others
   Reference person and others
   Spouse only
   Spouse and others
   Others only
0 Otherwise
Omitted dummy - No earners

10. Family type:

Husband and wife families
1 Husband and wife only
   Husband and wife, own children, oldest child < 6
   Husband and wife, own children only, oldest child > 5, <= 17
   Husband and wife, own children only, oldest child > 17
   All other husband and wife families
0 Otherwise
Omitted dummy - One parent, male, own children at least one age < 18; One
parent, female, own children, at least one age < 18; Single consumers; Other families

\[37\] "Own" children include blood sons and daughters, step children, and adopted children.
Appendix B: Tables

Table I. Summary Statistics

Data from the CEX between 1979:10 and 1996:3. Sample of 75992 observations.

Panel A: Moments and Quintiles

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SE</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of reference person</td>
<td>46.12</td>
<td>14.76</td>
<td>0.26</td>
<td>-1.08</td>
<td>34</td>
<td>44</td>
<td>59</td>
</tr>
<tr>
<td>Number of members in family</td>
<td>2.79</td>
<td>1.56</td>
<td>1.14</td>
<td>2.40</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Ln family income</td>
<td>10.35</td>
<td>0.86</td>
<td>-1.13</td>
<td>3.74</td>
<td>9.88</td>
<td>10.48</td>
<td>10.94</td>
</tr>
<tr>
<td>Number of earners</td>
<td>1.51</td>
<td>1.02</td>
<td>0.74</td>
<td>1.58</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Number of children less than 18</td>
<td>0.81</td>
<td>1.17</td>
<td>1.59</td>
<td>3.05</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Number of persons over 64</td>
<td>0.24</td>
<td>0.55</td>
<td>2.26</td>
<td>4.36</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of autos</td>
<td>1.38</td>
<td>1.00</td>
<td>1.21</td>
<td>5.31</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
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</table>

Panel B: Frequency Distributions

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</thead>
<tbody>
<tr>
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<td>Frequency</td>
<td>Percent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset market participation</td>
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<td>81.49</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>57743</td>
<td>75.99</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>62584</td>
<td>82.36</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>63997</td>
<td>84.22</td>
</tr>
<tr>
<td>Age 65 and over</td>
<td>64147</td>
<td>84.41</td>
</tr>
<tr>
<td>Education: High school degree</td>
<td>37049</td>
<td>48.75</td>
</tr>
<tr>
<td>Education: College degree</td>
<td>39222</td>
<td>51.61</td>
</tr>
<tr>
<td>Married</td>
<td>29075</td>
<td>38.26</td>
</tr>
<tr>
<td>Origin: European</td>
<td>49311</td>
<td>64.89</td>
</tr>
<tr>
<td>Origin: Spanish</td>
<td>70903</td>
<td>93.30</td>
</tr>
<tr>
<td>Origin: Afro-American</td>
<td>68399</td>
<td>90.01</td>
</tr>
<tr>
<td>Race: White</td>
<td>11129</td>
<td>14.64</td>
</tr>
<tr>
<td>Race: Black</td>
<td>67647</td>
<td>89.02</td>
</tr>
<tr>
<td>Male</td>
<td>24745</td>
<td>32.56</td>
</tr>
<tr>
<td>House owned</td>
<td>23932</td>
<td>31.49</td>
</tr>
<tr>
<td>Composition of earners</td>
<td>11450</td>
<td>15.07</td>
</tr>
<tr>
<td>Family type</td>
<td>30127</td>
<td>39.64</td>
</tr>
</tbody>
</table>
Table II. Estimation and Test Results for the Bivariate Probit Model

Data from the CEX between 1979:10 and 1996:3. The estimates reported in column SID (Serially Independent Disturbances) are obtained by exploiting the contemporaneous orthogonality conditions (36) and (37) only (the instrument set INSP1). Column SDD (Serially Dependent Disturbances) contains coefficient estimates using GMM imposing contemporaneous orthogonality conditions (36) and (37) and orthogonality condition (38) for the variable “number of persons over 64” in the vector $x_{h,t+j}$ (the instrument set INSP2). Parameters are significant at *10% and †5% significance levels under conventional normal asymptotics. The $J$ statistic is Hansen’s test of the overidentifying restrictions. The $J$ statistics in square brackets are computed as $T S_{cT} (\hat{\mu}_0)$. The $P$ value is the marginal significance level associated with the $J$ statistic.

<table>
<thead>
<tr>
<th>Variables</th>
<th>SID</th>
<th>SDD</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Param.</td>
<td>SE</td>
</tr>
<tr>
<td>Intercept</td>
<td>-7.7178†</td>
<td>0.4355</td>
</tr>
<tr>
<td>Age 35-44</td>
<td>0.2045†</td>
<td>0.0217†</td>
</tr>
<tr>
<td>Age 45-54</td>
<td>0.2428†</td>
<td>0.0201†</td>
</tr>
<tr>
<td>Age 55-64</td>
<td>0.4352†</td>
<td>0.0246†</td>
</tr>
<tr>
<td>Age 65 and over</td>
<td>0.3992†</td>
<td>0.0560</td>
</tr>
<tr>
<td>Education: High school degree</td>
<td>0.4178†</td>
<td>0.2097</td>
</tr>
<tr>
<td>Education: College degree</td>
<td>0.9505†</td>
<td>0.2086</td>
</tr>
<tr>
<td>Number of members in family</td>
<td>-0.2177†</td>
<td>0.0175†</td>
</tr>
<tr>
<td>Ln family income</td>
<td>0.5552†</td>
<td>0.0362</td>
</tr>
<tr>
<td>Married</td>
<td>-0.0170</td>
<td>0.0701</td>
</tr>
<tr>
<td>Number of earners</td>
<td>-0.0041</td>
<td>0.0169</td>
</tr>
<tr>
<td>Origin: European</td>
<td>0.0769†</td>
<td>0.0239</td>
</tr>
<tr>
<td>Origin: Spanish</td>
<td>-0.4785†</td>
<td>0.0366‡</td>
</tr>
<tr>
<td>Origin: Afro-American</td>
<td>0.0007</td>
<td>0.1209</td>
</tr>
<tr>
<td>Number of children less than 18</td>
<td>0.0942†</td>
<td>0.0199</td>
</tr>
<tr>
<td>Number of persons over 64</td>
<td>0.0349</td>
<td>0.0408</td>
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<td>Race: White</td>
<td>0.1726*</td>
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<td>Race: Black</td>
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</tr>
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<td>Male</td>
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<td>0.0278</td>
</tr>
<tr>
<td>House owned</td>
<td>0.4402†</td>
<td>0.0401</td>
</tr>
<tr>
<td>Composition of earners</td>
<td>-0.2825†</td>
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<td>Family type</td>
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</tr>
<tr>
<td>Number of autos</td>
<td>0.0158</td>
<td>0.0100</td>
</tr>
</tbody>
</table>

$\sigma_{ht} = \exp (\lambda' z_{ht})$:

| Composition of earners           | 0.1665†  | 0.0345  | 0.1172†  | 0.0332  |
| Number of autos                  | 0.0559†  | 0.0119  | 0.0525†  | 0.0102  |
| $J$ statistic                    | [1112.1543] | 3.1465 | [17.4807] | 0.0935  |
| $P$ value                        | [0.0000] | 0.0761 | [0.8938] | 0.0000  |
| Percent correctly predicted      | 81.66    | 81.62   |
Data from the CEX between 1979:10 and 1996:3. The Euler equations for the excess market return (51) and the real risk-free interest rate (52) are estimated jointly using a two-step GMM approach. Four sets of instruments are exploited. The first instrument set (INSE1) has a constant, the real market return and consumption growth rate lagged once. The second set (INSE2) is the instrument set INSE1 extended with the real risk-free rate lagged once. The third set of instruments (INSE3) has a constant, the real market return and consumption growth rate lagged twice. The fourth instrument set (INSE4) consists of the same variables as INSE3 plus the real risk-free rate lagged two periods. *IK* is the number of moment conditions. Standard errors in parentheses. The *J* statistic is Hansen’s test of the overidentifying restrictions. The *J* statistics in square brackets are computed as $TS_{cT}(\hat{\gamma}_0, \hat{\delta}_0)$. Parameters are significant at *$10\%$ and †$5\%$ levels under conventional normal asymptotics. *J* statistics are significant at †$5\%$ level. The 95% $S$-set for $\gamma$ is based on $S_{cT}(\gamma_0, \hat{\delta}(\gamma_0))$ (only nonnegative values of $\gamma$ are considered). $\emptyset$ denotes an empty $S$-set (there is no parameter value consistent with the overidentifying conditions).

<table>
<thead>
<tr>
<th>IK</th>
<th>Model</th>
<th>Instruments</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$J$ statistic</th>
<th>95% $S$-set for $\gamma$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.006) ⋃ (0.09, 9.50)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>INSE1</td>
<td>0.4404</td>
<td>0.9940†</td>
<td>1.74</td>
<td>(0.3343) (0.0113) [9.95]</td>
</tr>
<tr>
<td>6</td>
<td>M1a</td>
<td>INSE1</td>
<td>0.2294</td>
<td>1.0020†</td>
<td>2.03</td>
<td>(0.1631) (0.0017) [38.37†]</td>
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<tr>
<td>8</td>
<td>M2a</td>
<td>INSE2</td>
<td>0.6842†</td>
<td>0.9869†</td>
<td>2.07</td>
<td>(0.2867) (0.0180) [17.40†]</td>
</tr>
<tr>
<td>6</td>
<td>M3a</td>
<td>INSE3</td>
<td>0.9533†</td>
<td>0.9765†</td>
<td>2.64</td>
<td>(0.2281) (0.0189) [50.74†]</td>
</tr>
<tr>
<td>8</td>
<td>M4a</td>
<td>INSE4</td>
<td>0.6044</td>
<td>0.9889†</td>
<td>1.67</td>
<td>(0.5118) (0.0312) [12.82†]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>INSE1</td>
<td>0.4691</td>
<td>0.9997†</td>
<td>2.13</td>
<td>(0.3102) (0.0131) [32.85†]</td>
</tr>
<tr>
<td>6</td>
<td>M3b</td>
<td>INSE3</td>
<td>0.9065</td>
<td>0.9643†</td>
<td>2.36</td>
<td>(0.7021) (0.0702) [10.14]</td>
</tr>
<tr>
<td>8</td>
<td>M4b</td>
<td>INSE4</td>
<td>1.0429†</td>
<td>0.9599†</td>
<td>3.24</td>
<td>(0.3141) (0.0346) [31.05†]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>INSE1</td>
<td>0.6328*</td>
<td>0.9846†</td>
<td>1.78</td>
<td>(0.3641) (0.0215) [10.70]</td>
</tr>
<tr>
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<td>M2c</td>
<td>INSE2</td>
<td>0.4369*</td>
<td>0.9975†</td>
<td>2.03</td>
<td>(0.2419) (0.0084) [19.32†]</td>
</tr>
<tr>
<td>6</td>
<td>M3c</td>
<td>INSE3</td>
<td>0.7538†</td>
<td>0.9781†</td>
<td>2.01</td>
<td>(0.3223) (0.0245) [10.58]</td>
</tr>
<tr>
<td>8</td>
<td>M4c</td>
<td>INSE4</td>
<td>0.8771†</td>
<td>0.9809†</td>
<td>2.57</td>
<td>(0.2111) (0.0175) [49.33†]</td>
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</tbody>
</table>
Table IV. Results from the Joint Estimation of the Bivariate Probit Model and the Euler Equations

Data from the CEX between 1979:10 and 1996:3. The bivariate probit model (under the assumption of serially correlated disturbances) and the Euler equations for the excess market return (60) and the real risk-free interest rate (61) are estimated jointly using a two-step GMM approach. Four sets of instruments are exploited. The first set of instruments (INSPE1) is the instrument set INSE1 plus the set of instruments INSP2. The second set (INSPE2) is the instrument set INSE2 extended with the set of instruments INSP2. The third set (INSPE3) is the instrument set INSE3 plus the set of instruments INSP2. The fourth set (INSPE4) is the instrument set INSE4 extended with the set of instruments INSP2. Parameters are significant at *10% and †5% significance levels under conventional normal asymptotics. The $J$ statistic is Hansen’s test of the overidentifying restrictions. The $J$ statistics in square brackets are computed under the assumption of weak identification. The $P$ value is the marginal significance level associated with the $J$ statistic.

<table>
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<tr>
<th>Variables</th>
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<th>Param.</th>
<th>SE</th>
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<td>-6.4941†</td>
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<td>0.1949†</td>
<td>0.0142</td>
<td>0.1972†</td>
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</tr>
<tr>
<td>Age 45-54</td>
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<td>0.0180</td>
<td>0.2253†</td>
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</tr>
<tr>
<td>Age 55-64</td>
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<td>0.0186</td>
<td>0.3962†</td>
<td>0.0180</td>
</tr>
<tr>
<td>Age 65 and over</td>
<td>0.3249†</td>
<td>0.0399</td>
<td>0.3205†</td>
<td>0.0380</td>
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<tr>
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<td>-0.0452</td>
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<td>-0.2122†</td>
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<td>0.4885†</td>
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<td>0.0561</td>
<td>0.0057</td>
<td>0.0520</td>
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<td>0.0105</td>
<td>0.0097</td>
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<td>0.0194</td>
<td>0.0803†</td>
<td>0.0188</td>
</tr>
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<td>0.0966†</td>
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<td>0.0271</td>
<td>0.0680†</td>
<td>0.0264</td>
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<tr>
<td>Race: White</td>
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<td>0.0522</td>
<td>0.1412†</td>
<td>0.0496</td>
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<td>0.0693†</td>
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<td>0.4056†</td>
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<td>0.0503</td>
<td>-0.2024†</td>
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<td>0.0767*</td>
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<td>0.0249†</td>
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<td>$\sigma_h = \exp(\lambda'z_h)$ :</td>
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<td>Percent correctly predicted</td>
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Table IV (continued)

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<td>Param.</td>
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<td>0.0242†</td>
<td>0.0063</td>
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\[ \sigma_{ht} = \exp (\lambda' z_{ht}) \]

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<th>INSPE4</th>
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<tr>
<td>Percent correctly predicted</td>
<td>81.62</td>
<td></td>
<td>81.60</td>
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</tr>
</tbody>
</table>
References


[29] Zeldes, S., 1989, Consumption and Liquidity Constraints: An Empirical Investigation, 
Figure 1: $T_{SCF}$ concentrated with respect to $\delta$ (Models M1a, M1b, and M1c).

Figure 2: $T_{SCF}$ concentrated with respect to $\delta$ (Models M2a, M2b, and M2c).
Figure 3: $T_{CF}$ concentrated with respect to $\delta$ (Models M3a, M3b, and M3c).

Figure 4: $T_{CF}$ concentrated with respect to $\delta$ (Models M4a, M4b, and M4c).