Partial Ambiguity

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Abstract

This paper investigates attitude towards partial ambiguity in a laboratory setting using different decks of 100 cards. We assess certainty equivalents on three symmetric variants of the fully ambiguous urn in the classical Ellsberg 2-urn paradox by limiting the range of the possible number of red cards with the rest black. In the interval variants, the possible number of red cards ranges from n to $100 - n$. In the disjoint variants, the possible number of red cards can range from 0 to n and from $100 - n$ to 100. In the third symmetric variant called two-point ambiguity, the number of red cards is limited to either n or $100-n$. For both interval and disjoint ambiguity, subjects tend to value betting on a deck with a smaller set of ambiguous states more. For two-point ambiguity, subjects exhibit greater aversion as n goes from 50 (no ambiguity) to 10 in four steps. Paradoxically, there is a reversal from $n = 10$ to $n = 0$ which turns out to be valued similarly as the no-ambiguity lottery with $n = 50$. We further examine attitude towards skewed partial ambiguity by eliciting subjects' preference between betting on a deck with a known number of n red cards versus betting on a deck whose possible number of red cards can range from 0 to $2n$. Here, subjects tend to go from being averse to ambiguity at $n = 50$ and $n = 40$ to being ambiguity seeking for n less than 30. We also discuss the implications of our findings for existing models of decision making under uncertainty in the literature.

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1 Introduction

The classical 2-urn thought experiment of Keynes (1921, p.75) and Ellsberg (1961) suggests that people generally favor betting on an urn with a known composition of 50 black and 50 white balls over betting on another urn with an unknown composition of black or white balls which add to 100. Ellsberg (1961) further suggests a 3-color experiment in which subjects would rather bet on red than on black and to bet on not red than not black in an urn with 30 red balls and 60 balls with unknown composition of yellow and black balls. Such preference, dubbed ambiguity aversion, casts doubt on the descriptive validity of subjective expected utility and has given rise to a sizable theoretical and experimental literature (see Camerer and Weber, 1992; Al-Najjar and Weinstein, 2009). Notice that the nature of ambiguity in the three-color paradox with drawing red having a known of chance of $1/3$ versus the chance of drawing blue being anywhere between 0 and $2/3$ is skewed relative to that in the 2-urn paradox. While experimental evidence corroborating ambiguity aversion for the 2-urn paradox has been pervasive, the corresponding evidence for the 3-color paradox appears mixed. In their 1985 paper, Curley and Yates examine different comparisons involving skewed ambiguity, e.g., an unambiguous bet of p chance of winning versus an ambiguous bet in which the chance of winning can be anywhere between 0 and $2p$ and observe ambiguity neutrality when the p is less than 0.4. This is corroborated by the finding of ambiguity neutrality in the 3-color urn in two recent papers (Charness, Karni and Levin, 2012; Binmore, Stewart, and Voorhoeve, 2011). By contrast, ambiguity affinity for more moderate levels of skewed ambiguity have been observed in Kahn and Sarin (1988) and more recently in Abdellaoui et al. (2011) and Abdellaoui, Klibanoff and Placido (2011).

In their 1964 paper, Becker and Brownson introduce a refinement of the 2-urn paradox to the case of symmetric partial ambiguity with the number of red balls (or black balls) in the unknown urn being constrained to be in a symmetric interval, e.g., $[0.4, 0.6]$ or $[.25, .75]$ in relation to a fully ambiguous urn of $[0, 100]$ and the $50 - 50$ urn denoted by $\{50\}$. They find that subjects tend to be more averse to bets involving larger intervals of ambiguity.

This motivates us to examine two additional kinds of symmetric ambiguous lotteries. One involving two possible compositions – $\{n\}$ and $\{100-n\}$ – shares the same end points as an interval ambiguity lottery $[n, 100-n]$. This property has implications for maxmin expected utility (MEU henceforth) of Gilboa and Schmeidler's (1989) and its derivatives including Ghirardato, Maccheroni and Marinacci (2004) and Maccheroni, Marinacci and Rustichini (2006).

Another kind of symmetric partial ambiguity consisting of union of two disjoint intervals $[0, n] \cup [100-n, 100]$ is complementary to $[n, 100-n]$ since full ambiguity can be viewed as a convex combination of $[n, 100-n]$ and $[0, n] \cup [100-n, 100]$. As we shall discuss in the penultimate section of our paper, this latter property has strong implications for models that view ambiguity as the second stage distribution of possible states occurring at an ex ante first stage (Segal, 1987; Klibanoff, Marinacci, and Mukerji, 2005; Nau, 2006; Seo, 2009; Ergin and Gul, 2009). The proposed disjoint ambiguity also possesses a property that is akin to a form of discontinuity: for $n < 50$, drawing a red card from $[0, n]$ precludes drawing a red card from $[100-n, 100]$, but this constraint vanishes when n equals 50. This discontinuity seems incompatible with the intuition of the value of a disjoint ambiguous lottery being determined largely by the size of its set of ambiguous states, which changes continuously as n goes to 50.

In Part I of our study, the observed patterns of behavior towards symmetric partial ambiguity are summarized as follows:

1. For both interval and disjoint partial ambiguity, we observe aversion to increasing size of ambiguity.

2. The certainty equivalents (CE) of the two-point ambiguity decreases from $\{50\}$ to $\{40, 60\}$, from $\{40, 60\}$ to $\{30, 70\}$, from $\{30, 70\}$ to $\{20, 80\}$, and from $\{20, 80\}$ to $\{10, 90\}$ except for the last comparison when its CE increases significantly from $\{10, 90\}$ to $\{0, 100\}$. Notably, CE of $\{0, 100\}$ is not significantly different from that of $\{50\}$.

3. Mean CE of the 2-point ambiguity lotteries exceeds the mean CE of the interval ambiguity

lotteries which in turn exceeds the mean CE of the disjoint ambiguity lotteries.

In Part II of our study, we examine subjects' attitude towards different levels of skewed ambiguity in a design that relates to what is used in Curley and Yates (1985). We find that subjects tend to exhibit a switch in ambiguity attitude from aversion to affinity at around 30% for the known probability. This provides a rationale for the mixed evidence for ambiguity aversion in the 3-color urn. Our finding also echoes a further suggestion of Ellsberg described in footnote 4 of Becker and Brownson (1964). "*Consider two urns with 1000 balls each. In Urn 1, each ball is numbered from 1 to 1000, and in Urn 2 there are an unknown number of balls bearing any number. If you draw a specific number say 687, you win a prize. There is an intuition that many subjects would prefer the draw from Urn 2 over Urn 1, that is, ambiguity seeking when probability is small".* This intuition has been tested by Einhorn and Hogarth (1985, 1986) in a hypothetical choice study involving 274 MBA students. They find that 19% of their subjects are ambiguity averse with respect to the classical Ellsberg paradox while 35% choose the ambiguous urn when 0.002 is the interval of ambiguity rather than the unambiguous urn with an unambiguous winning probability of 0.001.

The rest of this paper is organized as follows. Section 2 presents details of our experimental design. Section 3 reports our experimental findings. Section 4 discusses the implications of our experimental findings for a number of decision making models in the literature, including Choquet expected utility (Schmeidler, 1989; Gilboa, 1987), models based on multiple priors (Gilboa and Schmeidler 1989; Ghirardato, Maccheroni and Marinacci, 2004; Maccheroni, Marinacci and Rustichini 2006), models adopting a 2-stage approach (Segal, 1987; Klibanoff, Marinacci and Mukerji, 2005; Nau, 2006; Ergin and Gul, 2009; Seo, 2009), and models based on source preference (Chew and Sagi, 2008; Abdellaoui et al., 2011). Section 5 discusses the related literature and concludes.

2 Experimental Design

We adopt the following notation. We use $\{n\}$ to denote an unambiguous lottery based on a deck of n red cards and $100-n$ black cards (betting on red). An ambiguous lottery is denoted by the set of ambiguous states $\mathcal{A}$. Specifically, $\mathcal{A} = \{n, 100-n\}$ for a symmetric two-point ambiguity, $\mathcal{A} = [n, 100-n]$ for symmetric interval ambiguity, and $\mathcal{A} = [0, n] \cup [100-n, 100]$ for symmetric disjoint ambiguity.[1]

We further define three benchmark lotteries, B_0 for $\mathcal{A} = \{50\}$, B_1 for $\mathcal{A} = \{0, 100\}$ and B_2 for $\mathcal{A} = [0, 100]$. B_1 appears to admit some ambiguity in interpretation. Being either all red or all black may give it a semblance of a $50-50$ lottery in parallel with its intended interpretation as being 2-point ambiguous. Interestingly, B_2 admits an alternative description as follows. It can first be described as comprising 50 cards which are either all red or all black while the composition of the other 50 cards remains unknown. This process can be applied to the latter 50 cards to arrive at a further division into 25 cards which are either all red or all black while the composition of the remaining 25 cards remains unknown. Doing ad infinitum gives rise to dyadic decomposition of $[0, 100]$ into subintervals which are individually either all red or all black.

Part I of our study is based on the following 3 groups of six lotteries.

Two-point ambiguity. This involves 6 lotteries with symmetric two-point ambiguity:

$$B_0 = \{50\},\ P_1 = \{40, 60\},\ P_2 = \{30, 70\},\ P_3 = \{20, 80\},\ P_4 = \{10, 90\},\ B_1 = \{0, 100\}.$$

Interval ambiguity. This comprises 6 lotteries with symmetric interval ambiguity:

$$B_0 = \{50\},\ S_1 = [40, 60],\ S_2 = [30, 70],\ S_3 = [20, 80],\ S_4 = [10, 90],\ B_2 = [0, 100].$$

Disjoint ambiguity. This involves 6 lotteries with symmetric disjoint ambiguity:

$$B_1 = \{0, 100\},\ D_1 = [0, 10] \cup [90, 100],\ D_2 = [0, 20] \cup [80, 100],\ D_3 = [0, 30] \cup [70, 100],\ D_4 = [0, 40] \cup [60, 100],\ B_2 = [0, 100].$$

[1] For symmetric ambiguity lotteries, notice that $\mathcal{A}$ remains the same whether one bets on red or on black. Thus, a symmetric ambiguity lottery can be denoted by its associated $\mathcal{A}$. For skewed ambiguity, the lottery $[0, 2n]$ is based on betting on red.

The ambiguity structure of the 15 lotteries in Part I are illustrated below.

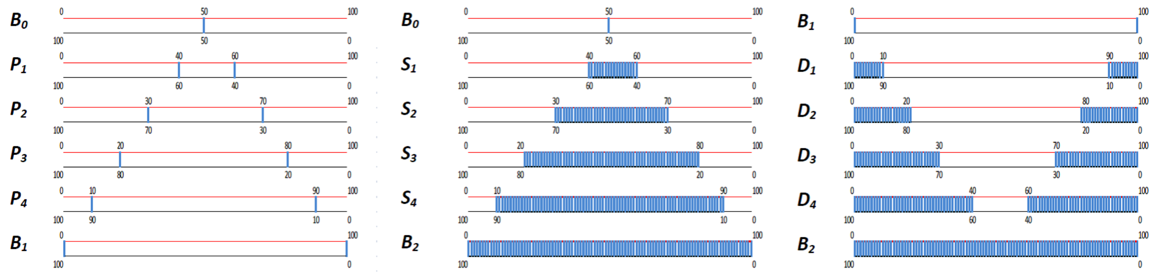

Figure 1. Illustration of 15 treatments in 3 groups.[2]

Part II of our study concerns attitude towards skewed partial ambiguity. It comprises 6 comparisons between two skewed lotteries: $r_n = \{n\}$ and $u_n = [0, 2n]$ where $n = 5, 10, 20, 30, 40$ and 50.

For both Part I and II lotteries, betting correctly on the color of a card (subject to their choice in Part I while fixed to be red in Part II) drawn delivers S$40 (about US$30) for the subject while betting incorrectly delivers nothing. To elicit the CE of a lottery in Part I, we used a price list design (e.g., Miller, Meyer, and Lanzetta, 1969; Holt and Laury, 2002), where subjects are asked to choose between betting on the color of the card drawn and getting some certain amount of money. For each lottery, subjects have 10 binary choices corresponding to 10 certain amounts ranging from S$6 to S$23. The order of appearance of the 15 lotteries in Part I is randomized for each subject who each makes 150 choices in all. Subsequent to Part I, we conduct Part II of our experiment consisting of 6 binary choices with the order of appearance randomized.

At the end of the experiment, in addition to a S$5 show-up fee, each subjects is paid based on his/her randomly selected decisions in the experiment. For Part I, one out of 150 choices is randomly chosen using dice. For Part II, one subject is randomly chosen to receive the payment based on one random choice out of his/her 6 binary choices. (see Appendix A

[2]Interpretation of the figures is the following: the upper red line represents the number for red cards and the lower black line for black cards, while one vertical blue line represents one possible combination of the deck. Also note that $\{50\}$, $\{0, 100\}$ and $[0, 100]$ are limit cases for different groups.

for experimental instruction).

We are aware that our adoption of a *random incentive mechanism* (RIM) could be subject to violation of the *reduction of compound lottery axiom* (ROCLA) or the independence axiom (e.g., Holt, 1986). In Starmer and Sugden's (1991) study of RIM, they find that their subjects' behavior are inconsistent with ROCLA. More recently, Harrison, Martinez-Correa and Swarthout (2011) test ROCLA specifically and their finding is mixed. While the analysis of choice patterns suggests violations of ROCLA, their econometric estimation suggests otherwise. The use of RIM has become prevalent in part because it offers an efficient way to elicit subjects' preference besides being cognitively simple (see Harrison and Rutstrom 2008 for a review).

We recruited 56 undergraduate students from National University of Singapore (NUS) as participants using advertisement posted in its Integrated Virtual Learning Environment. The experiment consisted of 2 sessions with 20 to 30 subjects for each session. It was conducted by one of the authors with two research assistants. After arriving at the experimental venue, subjects were given the consent form approved by at NUS' institutional review board. Subsequently, general instructions were read to the subjects followed by our demonstration of several example of possible composition of the deck before subjects began making decisions. After finishing Part I, subjects were given the instructions and decision sheets for Part II. Most subjects completed the decision making tasks in both parts within 40 minutes. At the end of the experiment, subjects received payment based on a randomly selected decision made in addition to a S$5 show-up fee. The payment stage took up about 40 minutes.

3 Observed Choice Behavior

This section presents the observed choice behavior at both aggregate and individual levels and a number of statistical findings.

Part I. Summary statistics ares presented in Figure 2.[3] We apply the Friedman test to check whether the CEs of the 15 decks come from a single distribution. We reject the null hypothesis that the CEs come from the same distribution ($p < 0.001$). Besides replicating the standard finding – CE of $\{50\}$ is significantly higher than that of $[0, 100]$ (paired Wilcoxon Signed-rank test, $p < 0.001$), our subjects have distinct attitudes towards different types of partial ambiguity. Specifically, for the comparison between $\{50\}$ and $[0, 100]$, 62% of the subjects exhibit ambiguity aversion, 33% of the subjects exhibit ambiguity neutrality, and 5% of the subjects exhibit ambiguity affinity.

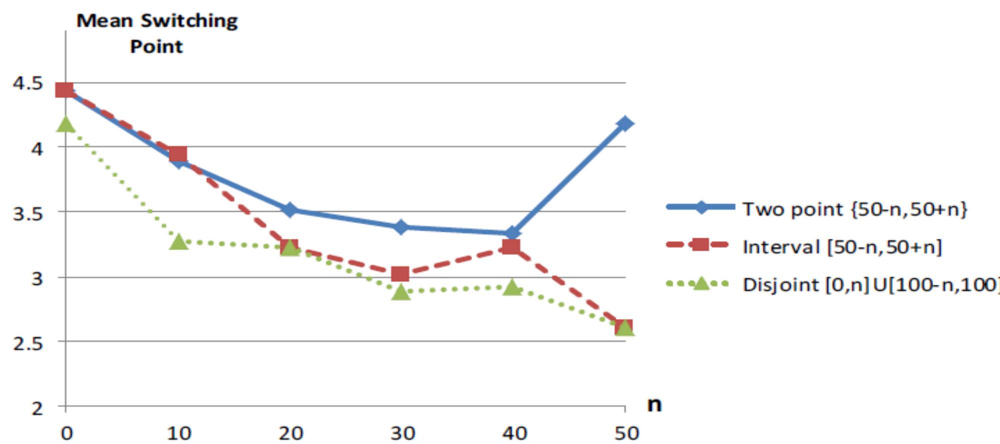

Figure 2. Switching points for lotteries in Part I.

The CEs for the 15 lotteries are highly and positively correlated in ranging from 58.8% to 91.6% (see Table S3 in Appendix B for pair-wise Spearman correlations). The correlations between risk attitude measure by the CEs for $B_0 = \{50\}$ and ambiguity attitude, measured by the difference in CEs between that of B_0 and those 14 ambiguous bets are generally highly correlated, between 36.7% and 63.8%, except for $B_1 = \{0, 100\}$ with a correlation of 9.8% (see Table S4 in Appendix B). The pairwise correlations for the ambiguity attitude towards the 14 ambiguous bets are also highly positive, ranging from 55.1% to 87.3%, except for the

[3] Out of 15 Part I tasks, one subject exhibits multiple switching in one task and another exhibit multiple switching in three tasks. Their data for these 4 tasks are exclude from our analysis.

[3] Data are coded in terms of the number of times each S chooses the lottery over a sure amount in the 10 binary choices. Out of 57 subjects, 2 exhibited multiple switching for a total of 4 decision sheets. For details, please refer to Table S1 in Appendix B.

correlations with B_1 which range from 9.6% to 49.2% (see Table S5 in Appendix B). The correlations identified here are similar to those reported in Halevy (2007), and suggest a common link between risk attitude and ambiguity attitude except for B_1 which corroborates the earlier observation that it may admit an additional interpretation as being almost a 50-50 lottery.

Using the Trend test, we check subsequently whether there is a significant trend for each group. This yields the following two observations.

Observation 1 (*Interval and disjoint ambiguity*): For lotteries related to interval ambiguity, B_0, S_1, S_2, S_3, S_4 and B_2, there is a statistically significant decreasing trend in CE as size of $\mathcal{A}_S$ increases ($p < 0.001$). For lotteries related to disjoint ambiguity, B_1, D_1, D_2, D_3, D_4 and B_2, there is also a statistically significant decrease in CE as the size of $\mathcal{A}_D$ increases ($p < 0.001$).

Moreover, we count the number of individuals exhibiting specific patterns in Observation 1. For the 6 interval ambiguity lotteries, 24.1% of the subjects have the *same CEs*, 25.9% of the subjects have *non-increasing CEs*, while none of the subjects has *non-decreasing CEs*. For the 6 lotteries in the disjoint ambiguity, 24.1% of the subjects have the same CEs, 20.3% of the subjects have non-increasing CEs, and 5.5% of the subjects have non-decreasing CEs.

As discussed in the Introduction, given that the size of ambiguity of $[0, n] \cup [100 - n, 100]$ would double in the limit as n approaches 50, one may expect to observe an abrupt drop in CE as n increases from 40 to 50. This does not appear to be case. The relatively smooth change of CE in the overall data may suggest that subjects see the size of ambiguous states in $[0, n] \cup [1 - n, 1]$ as being $2n$ rather than n.

Observation 2 (*Two-point ambiguity*): For lotteries related to two-point ambiguity, B_0, P_1, P_2, P_3, P_4, and B_1, there is a significant *nonincreasing trend in the CEs* from $B_0 = \{50\}$ to $P_4 = \{10, 90\}$ ($p < 0.001$). Interestingly, the CE of B_1 reverses this trend and is significantly higher than the CE of P_4 (paired Wilcoxon Signed-rank test, $p < 0.005$). Moreover, the CE of B_1 is not significantly different from that of B_0 (paired Wilcoxon Signed-rank test,

$p > 0.323$).

At the individual level, for the 6 two-point ambiguity lotteries, 25.9% of the subjects have *the same CEs*, 16.6% of the subjects have *non-increasing CEs*, 22.2% of the subjects have *non-increasing CEs until* $\{10, 90\}$ *with an increase at* B_1, and 5.5% of the subjects have *non-decreasing CEs*. Between B_0 and B_1, 44.5% of the subjects have the same CEs, 35.2% of the subjects display a higher CE for B_0 than that for B_1, and 23.2% of the subjects exhibit the reverse. Between B_1 and $\{10, 90\}$, 44.6% of the subjects have the same CEs, 41.1% of the subjects have a higher CE for B_1 than for $\{10, 90\}$, and 14.3% of the subjects exhibit the reverse, again corroborating the potentially ambiguous nature of B_1.

Observation 3 (*Across group*): The mean CE of the two-point ambiguity lotteries, P_1, P_2, P_3, P_4 and B_1, exceeds that of the corresponding interval ambiguity lotteries, S_1, S_2, S_3, S_4 and B_2 ($p < 0.006$). The mean CE of the interval ambiguity lotteries, B_0, S_1, S_2, S_3 and S_4, exceeds that of the corresponding disjoint ambiguity lotteries, B_1, D_1, D_2, D_3, D_4, even though each pair of S_i and D_i have the same size of ambiguity ($p < 0.017$).[4]

At the individual level, between two-point ambiguity and interval ambiguity, 25.9% of the subjects have the same mean CEs across the five pairs of lotteries, $\{P_1, S_1\}$, $\{P_2, S_2\}$, $\{P_3, S_3\}$, $\{P_4, S_4\}$ and $\{B_1, B_2\}$, 55.6% of the subjects have higher CE for two-point than for the corresponding interval ambiguity. The rest of 18.5% exhibit the reverse. Between interval ambiguity and disjoint ambiguity, 29.6% of the subjects have the same mean CEs across the five pairs, $\{S_1, D_1\}, \{S_2, D_2\}, \{S_3, D_3\}, \{S_4, D_4\}$ and $\{B_0, B_2\}$, 48.1% of the subjects have higher mean CEs for interval ambiguity than that for the corresponding disjoint ambiguity,

[4] Pairwise comparisons of CEs between two-point ambiguity lotteries and their corresponding interval ambiguity lottery with the same best and worst priors are not significantly different. In addition, pairwise comparisons between interval ambiguity lotteries and their corresponding disjoint ambiguity lotteries maintaining the same size of ambiguity are also not significant.

and the rest 22.3% of the subjects have the reverse preference.

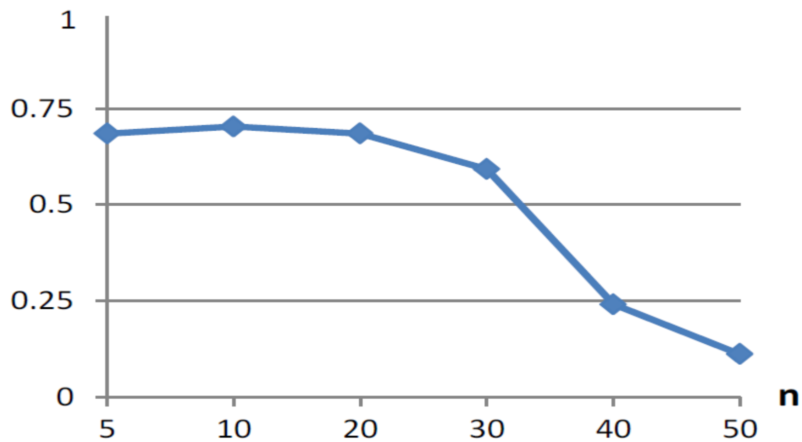

Figure 3. Proportion of subjects choosing the ambiguous lottery $[0, 2n]$.[5]

Part II. Figure 3 summarizes the proportion of subjects choosing the ambiguous deck. As anticipated, between $[0, 100]$ and $\{50\}$, a small proportion of 12.5% choose the latter. When the proportion of subjects choosing the ambiguous lottery is significantly lower (higher) than the chance frequency of 0.5, we take the pattern to be ambiguity averse (seeking). Using a simple t-test of difference in proportions, we arrive at the following observation.

Observation 4 (*Skewed ambiguity*): Subjects are significantly averse to moderate ambiguity $[0, 80]$ and $[0, 100]$ ($p < 0.001$ for both cases) and significantly tolerant of skewed ambiguity for $[0, 10]$, $[0, 20]$ and $[0, 40]$ ($p < 0.002$ in each case). There appears to be a switch towards becoming ambiguity seeking at around $[0, 60]$ (marginally significant at $p < 0.105$).

Analyzing the behavior across all 6 choices, 14.3% of the subjects are consistently ambiguity averse, 5.4% are consistently ambiguity seeking, but 39.3% are ambiguity averse towards $[0, 80]$ and $[0, 100]$ and ambiguity seeking towards $[0, 10]$, $[0, 20]$ and $[0, 40]$.

One issue in the experimental studies of ambiguity is that subjects may feel suspicious that somehow the deck is stacked against them. Such a sentiment may be a confounding factor when eliciting ambiguity attitude. In general, a minimal requirement to control for

[5] For details, please refer to Table S2 in Appendix B.

suspicion would appear to be to let subjects choose which ambiguous event to bet on, e.g., subjects can choose whether to bet on red or black in the 2-color urn. (Einhorn and Hogarth, 1985, 1986; Kahn and Sarin, 1988, Abdellaoui et al., 2011; Abdellaoui, Klibanoff and Placido 2011). For symmetric partial ambiguity in Part I, we control for the effect of suspicion by allowing the subjects to choose which color to bet on, yet we consistently observe ambiguity aversion. The effect of suspicion is expected to be more pronounced for skewed partial ambiguity in Part II when subjects only win on drawing a red card. It is noteworthy that despite the possibility of suspicion, we observe ambiguity affinity for three of the skewed ambiguous lotteries $[0, 5]$, $[0, 10]$, and $[0, 20]$ and that we do not observe a greater level of ambiguity aversion for $[0, 100]$.

Table S6 in Appendix B displays the Spearman correlations in ambiguity attitude of all 6 decisions. We find the correlation between $[0, 100]$ and $[0, 80]$ to be highly positive and that the correlation between $[0, 20]$ and $[0, 10]$ is also highly positive. By contrast, the correlation between $[0, 100]$ and $[0, 10]$ is marginally significantly negative ($p < 0.103$) which is compatible with a good proportion of subjects switching from ambiguity averse towards the moderate ambiguity of $[0, 80]$ and $[0, 100]$ to ambiguity seeking for $[0, 10]$, $[0, 20]$, and $[0, 40]$.

4 Theoretical Implications

This section discusses the implications of the observed choice behavior for a number of models of attitude toward ambiguity in the literature. One approach involves using a nonadditive capacity in place of a subjective probability measure in part to differentiate among complementary events that are revealed to be equally likely. A related approach involves the use of multiple priors to model the presence of ambiguity as in MEU (Wald, 1950; Hurwicz, 1951; Gilboa and Schmeidler, 1989) and developed further in its derivatives – α-MEU (Ghirardato, Maccheroni and Marinacci 2004) and variational preference (Maccheroni, Marinacci and Rustichini 2006). A different approach involves evaluating a lottery with ambiguous states in a two-stage manner. Models adopting this approach include Segal (1987), Klibanoff, Marinacci and Mukerji (2005), Nau (2006), Seo (2009) and Ergin and Gul (2009). Building on the idea of source dependence in Tversky and Kahneman (1992), Fox and Tversky (1995) and Heath and Tversky (1991), Chew and Sagi (2008) model source preferences that exhibit limited probabilistic sophistication and distinguish between risks from unambiguous states and those from ambiguous states.

To facilitate our analysis, we impose the following behavioral assumptions:

Symmetry (Part I): For treatment $i \in \{B_0, ..., P_1, ..., S_1, ..., D_1, ...\}$, the decision maker is indifferent between betting on red and black.

Conditional Symmetry (Part II): For treatment $u_n = [0, 2n]$ with $2n$ cards of unknown color, the decision maker is indifferent between betting on red and black conditional on not having drawn among the $100 - 2n$ black cards.

For the benchmark SEU model or more generally probabilistic sophistication, the probabilities of the events R_i and B_i always equal 0.5 given symmetry where R_i and B_i denote the respective events in treatment i. In particular,

$$SEU_i = v(w)/2,$$

where w denotes the payment should subjects guess correctly. Thus, SEU predicts that all lotteries in Part I have the same CEs. For Part II, a similar argument based on conditional symmetry implies that $r_n \sim u_n$ for each n. Both implications are incompatible with the observed behavior.

4.1 Non-additive Capacity Approach

One alternative to SEU, dubbed Choquet expected utility (CEU), is to formulate a non-additive generalization by using a capacity in place of a probability measure (Gilboa, 1987; Schmeidler, 1989). Under CEU, the utility for lottery i is given by:

$$\nu(R_i)v(w) + (1 - \nu(R_i))v(0) = \nu(B_i)v(w) + (1 - \nu(B_i))v(0),$$

with $\nu(R_i) = \nu(B_i)$ from symmetry. In relaxing additivity, the capacities or decision weights assigned to red (or black) for different Part I lotteries need not be the same. At the same time, for unambiguous lotteries, we typically assume that ν is additive over unambiguous events so that $\nu(R_{\{n\}}) = \widehat{n}$, where $\widehat{n}$ refers to the probability $n/100$. It follows that CEU can generate the pattern of behavior in Part I and Part II if $\nu(\cdot)$ preserves the observed ordering. In particular, for symmetric partial ambiguity lotteries, $\nu(R_i) = \nu(B_i) < 0.5$ for $i \neq B_0$, while $\nu(R_{u_n}) > \widehat{n}$ for n less than 30 and $\nu(R_{u_n}) < \widehat{n}$ for n greater than 30.

4.2 Multiple Priors Approach

Gilboa and Schmeidler (1989) offer the first axiomatization of the MEU specification in which an ambiguity averse decision maker behaves 'as if' there were an opponent who could influence the occurrence of specific states to his/her disadvantage. This intuition is captured by equating the utility of an ambiguous lottery with the expected utility corresponding to the worst prior in a convex set of priors Π. It is straightforward to see that this model can account for the classical 2-urn Ellsberg paradox. As the set of ambiguous states for lottery i is symmetric around $n = 50$, the corresponding set of priors Π_i is also symmetric.[6] Thus, the MEU of lottery i is given by:[7]

$$\min_{\mu \in \Pi_i} \mu(R_i) v(w) = \min_{\mu \in \Pi_i} \mu(B_i) v(w).$$

It follows that $B_0 \succ P_1 \succ P_2 \succ P_3 \succ P_4 \succ B_1$, $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$ and $B_1 \sim D_1 \sim D_2 \sim D_3 \sim D_4 \sim B_2$ if we require Π_i to depend only on the convexification of the set of priors. This contradicts our Observations 1, 2 and 3. Without any restriction on the sets of priors, MEU can account for most of the observed behavior with a judicious choice of the worst prior for each ambiguous lottery.

[6] Notice that we need to convexify the set of priors in disjoint ambiguous lotteries since the set of ambiguous states is not convex.

[7] Note that the utility is the same with $\mu(R)$ or $\mu(B)$ due to symmetry, thus we use only $\mu(R)$ for subsequent exposition. We also normalize $v(0) = 0$.

For Part II, MEU implies that $r_n \succ u_n$ under Conditional Symmetry, which is incompatible with the observed affinity for sufficiently skewed ambiguity (Observation 4).

Ghirardato, Maccheroni and Marinacci (2004) axiomatize the α-MEU model (α-M) as a linear combination of maxmin EU and maxmax EU. Their representation, adapted to our setting, is as follows:

$$\alpha_i \min_{\mu \in \Pi_i} \mu(R_i) v(w) + (1 - \alpha_i) \max_{\mu' \in \Pi_i} \mu'(R_i) v(w).$$

Besides inheriting most of the properties of MEU, the implications of α-MEU model depend on the value of α_i. Suppose α_i is the same for all i and the assumed set of priors coincide with actual set possible priors,[8] MEU and α-MEU have the same implications for Part I and Part II as long as $\alpha > 0.5$. When allowing the freedom of choosing α, α-MEU can explain all observed patterns. Notably, the observed pattern will impose some monotonicity property of α regarding the size of ambiguity states. For example, to explain the aversion to increasing size of ambiguity in the disjoint group $[0, n] \cup [100 - n, 100]$, α needs to be an increasing function of n, while for skewed ambiguous lottery u_n in Part II, a needs again to be increasing of n to accommodate the observed switch from ambiguity affinity to aversion. At last, Gajdos et al. (2008) propose a "contraction" model, which permits a weighted combination between SEU and MEU, and the implications would be similar to those of MEU and α-MEU.

Maccheroni, Marinacci and Rustichini (2006) propose an alternative generalization of MEU called Variational Preference (VP) as follows:

$$\min_{\mu \in \Delta} \{\mu(R_i) v(w) + c_i(\mu)\},$$

where $c_i(\mu) : \Delta(S) \to [0, \infty)$ is an index of ambiguity aversion. They show that VP could be reduced to MEU if c_i is an indicator function for Π_i. If we restrict c_i to be the same for all lotteries, then it will imply all lotteries in Part I are the same, which is obviously implausible, while there will be no testable predictions if there are no restrictions on c_i. One approach is to make c_i and c_j the same conditioning on the priors that are common to i and

[8] Ghirardato, Maccheroni, and Marinacci (2004) also axiomatize this representation where α is constant.

j while c_i and c_j each becomes unbounded when the underlying prior does not belong to the respective sets of priors. In this case, we have:

$$\min\{VP([n,100-n]),VP([0,n]\cup[100-n,100])\}\text{ is constant for all }n,$$

which is rejected by our Observation 1. The other approach permits size-dependent c functions, if c_i becomes smaller when the size of ambiguous states becomes larger, then VP is able to accommodate the observed aversion to increasing size of ambiguity in Part I. But, for Part II treatments, we have $r_n \succ u_n$ for all n under Conditional Symmetry, which does not fit Observation 4.

4.3 Two Stage Approach

The idea of linking ambiguity aversion to aversion to two stage risks coupled with a failure of the reduction of compound lottery axiom (ROCLA) is evident in the works of Becker and Brownson (1964) and Gardenfors (1979). This is formalized in Segal (1987) who proposes a two-stage model of ambiguity aversion with a common rank-dependent utility for both first and second stage risks. Maintaining a two-stage setting without requiring ROCLA, several subsequent papers (Klibanoff et. al., 2005; Nau, 2006; Ergin and Gul, 2009; Seo 2009) provide axiomatizations of a decision maker possessing distinct preferences across the two stages to model ambiguity aversion. We shall discuss successively here the implications of our data on adopting a two-stage approach using both identical and distinct preference specifications for the two stages.

To facilitate our analysis, we impose the following assumption.

Belief Consistency: Stage-1 beliefs π_i for all i in Part I are updated using Bayesian rule from π_{B_2} which has the maximal support (set of ambiguous states).

In the sequel, we assume uniform stage-1 beliefs $\pi_{\mathcal{A}}$ on the set of ambiguity states $\mathcal{A}$ for each ambiguous lottery, and discuss successively the implications of adopting a two-stage approach using both identical and distinct utility functions. The assumption of uniform stage-1 beliefs follows from symmetry, conditional symmetry together belief consistency. We offer an induction based argument as follows. Consider a skewed ambiguity in which only one ball has unknown color. Given conditional symmetry, a decision maker is indifferent between red and black conditioning on this unknown ball. This implies that Stage-1 belief $\pi_{[0,1]}$ takes the same value for each state $\{0\}$ and $\{1\}$. Similarly, conditional symmetry and belief consistency implies that $\pi_{[0,2]}(\{0\})$ equals $\pi_{[0,2]}(\{1\})$ which in turn equals $\pi_{[0,2]}(\{2\})$. This argument can be extended to show that stage-1 belief $\pi_{[0,100]}$ assumes the same value for all states, i.e., stage-1 beliefs are uniform.

Same Utility in Both Stages

ROCLA has been invoked in evaluating compound lotteries using different NEU models. Another approach, called *compound independence*, is to replace each branch of a compound lottery by its CE and evaluate the resulting simple lottery. Both approaches are equivalent under EU, but this is generally not the case for NEU models. Segal (1987, 1990) applies rank-dependent utility (Quiggin 1982) with compound independence but not ROCLA and showed that the decision maker can exhibit ambiguity aversion under certain restrictions on the probability weighting function. Segal's representation is as follows:

$$\int v(c_\mu)\, df(M_i), \text{ where } c_\mu = v^{-1}(v(w) f(\mu(R_i))),$$

where f is an increasing probability weighting function, c_μ is the CE for a stage-2 risk μ, and M_i is the cumulative distribution function of π_i, which is linear due to uniform belief. Assuming a convex probability weighting function f, we have the following implications: $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$, $B_1 \succ D_1 \succ D_2 \succ D_3 \succ D_4 \succ B_2$ and $B_0 = B_1 \succ P_j$.

The intuition for these implications is the following: for two-point group $\{n, 100-n\}$, as n deviates from 50, the decision weight on stage-2 risk $\{100-n\}$ becomes $f(0.5)$, which is less than 0.5 due to the convexity of f, thus the evaluation drops at first since the value changes of $\{n\}$ and $\{100-n\}$ relative to $\{50\}$ are the same when n is close to 50. As n decreases, this effect is offset by the effect that the value of $\{100-n\}$ $(f(1-\widehat{n})\, v(w))$ increases faster than the value of $\{n\}$ $(f(\widehat{n})\, v(w))$ drops, which is again due to the convexity of f, thus creating a reversal at last. The minimum point occurs at $n^\star$ such that

$f'(1-\widehat{n}^{\star})/f'(\widehat{n}^{\star}) = (1-f(0.5))/f(0.5)$, which could be 10 as in Observation 2.

For the interval group $[n, 100-n]$, the intuition is a bit more complicated: as n deviates from 50, the decision weight on the best stage-2 risk $\{100-n\}$ is $f(1/(2n+1))$, which becomes disproportionately smaller compared to that on the other stage-2 risks. To the opposite, the decision weight on the worst stage-2 risk $\{n\}$ is $1-f(2n/(2n+1))$, which becomes disproportionately larger. This effect of changes in decision weights offsets the effect of increasing value of $\{100-n\}$, thus we do not have the reversal when n approach 0 as in point group. The intuition for the disjoint group is similar.

With the same restrictions on f, we can have $r_n \prec u_n$ for n small while $r_n \succ u_n$ for n large.[9] Next, we show by an intuitive example that the implications for cross group comparisons under the same restrictions may fail. Consider the lotteries $[49, 51]$ and $\{49, 51\}$, the difference between these two is that the decision weight on $\{50\}$ in lottery $[49, 51]$ is transferred to $\{49\}$ and $\{51\}$ in lottery $\{49, 51\}$, and the transferred weight to $\{51\}: f(1/2)-f(1/3)$, is less than that to $\{49\}: f(2/3)-f(1/2)$, due to the convexity of f. Thus, similar intuition as that for two-point group suggests that $[49, 51] \succ \{49, 51\}$, contradicting Observation 3.

Besides RDU, one may also consider other NEU models, including (semi) weighted utility (Chew, 1983; Chew, 1989) and disappointment aversion utility (Gul 1991) in the class of betweenness models. Observation 1 shows that the valuation of $[n, 100-n]$ is increasing in n while the valuation of $[0, n] \cup [100-n, 100]$ is decreasing in n. Moreover, interval group starts from $[0, 100]$ and ends at $\{50\}$ while disjoint group starts from $\{0, 100\}$ and ends at $[0, 100]$. Due to the insignificant difference between $\{50\}$ and $\{0, 100\}$ in Observation 2, there must exist some $n^{\star}$ such that $[n^{\star}, 100-n^{\star}] \sim [0, n^{\star}] \cup [100-n^{\star}, 100]$, then stage-1 betweenness would imply $[n^{\star}, 100-n^{\star}] \sim [0, n^{\star}] \cup [100-n^{\star}, 100] \sim [0, 100]$, which is incompatible with Observation 1.

Distinct Utilities across Two Stages

One may argue that stage-1 risk and stage-2 risk are distinct hence we need to depart from

[9] See Segal (1987) problem 3 for an example

having two identical utility functions as in Segal (1987). Several papers, e.g., Klibanoff, Marinacci and Mukerji (2005), Nau (2006), and Seo (2009), provide a two-stage model with distinct EU's, labeled DEU, as follows:

$$\int \phi(c_\mu)\, d\pi_i, \text{ where } c_\mu = v^{-1}(\mu(R_i)\, v(w)),$$

where ϕ is stage-1 utility function and v stage-2 utility function, and ϕ being more concave than v corresponds to "smooth ambiguity aversion" in their model. Given that $\phi \circ v^{-1}$ is concave, we have the following implications.

1. $[0, 100]$ is between $[n, 100 - n]$ and $[0, n] \cup [100 - n]$.

2. Within group comparison. $B_0 \succ P_1 \succ P_2 \succ P_3 \succ P_4 \succ B_1$ for two-point ambiguity, $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$ for interval ambiguity, and $B_1 \prec D_1 \prec D_2 \prec D_3 \prec D_4 \prec B_2$ for disjoint ambiguity.

3. Cross group comparison. $[n, 100 - n] \succ \{n, 100 - n\} \succ [0, n] \cup [100 - n]$.

Implication 1 is due to stage-1 independence[10] while implications 2 and 3 are straightforward as the decision maker is averse to stage-1 mean-preserving spread in terms of the distribution of CEs. In particular, Implication 1 holds with only uniform belief. But, Implication 1 is incompatible with Observation 1 while Implication 2 and 3 is incompatible with Observation 1, 2 and 3. For Part II treatments, a concave $\phi \circ v^{-1}$ implies that $r_n \succ u_n$ which is rejected by Observation 4. Thus we need to abandon both uniform belief and concavity of $\phi \circ v^{-1}$ for DEU to accommodate the observed behavior.[11]

One may consider adopting two distinct NEU models across two stages (see Ergin and Gul, 2009). As observed earlier, we can rule out independence or betweenness as candidates for Stage-1 utility and consider instead RDU. With Stage-1 RDU, it is clear that adopting EU for the second stage cannot generate a reversal for the point group since the utility for $\{n, 100 - n\}$, given by $f(0.5)(1 - \widehat{n})\, v(w) + (1 - f(0.5))\, \widehat{n} v(w)$, is monotonic in n. More generally, suppose that the stage-2 utility is similar to RDU. Recall that two-stage RDU

[10]The assumption of second order independence in Klibanoff, Marinacci and Mukerji (2005) has been further discussed in Epstein (2010) and Klibanoff, Marinacci and Mukerji (2012).

[11]Numerical examples of Segal (1987) and Klibanoff, Marinacci and Mukerji (2005) are provided in Appendix C.

with the same convex probability weighting functions fails only in across-group comparison, now consider a convex stage-1 probability weighting function f and a piecewise linear stage-2 weighting function g connecting 0 to $f(0.5)$ and $f(0.5)$ to 1. Similar intuitions as in Segal (1987) apply when one considers implications for within group comparisons. For across-group comparison, the utility for a two-point lottery $\{n, 100-n\}$ becomes $f(0.5)\, g(1-\widehat{n})\, v(w) + (1-f(0.5))\, g(\widehat{n})\, v(w)$ which is constant, and will be higher than the utility for the interval group $[n, 100-n]$, which is monotonically decreasing. Thus, we may perturb the function g to be strictly convex and obtain a reversal in two-point group, hence deriving implications of a two-stage distinct-RDU model which are compatible with our observations.

4.4 Source Preference Approach

The idea of source dependence and familiarity bias was suggested in several papers by Amos Tversky and his colleagues in the 1990's (Tversky and Kahneman, 1992; Heath and Tversky, 1992; Fox and Tversky, 1995). Subsequently, Chew and Sagi (2008) provide an axiomatic model of source preference in terms of limiting probabilistic sophistication to smaller families of events – decision makers behave as if they have sub-domains within which probabilistic sophistication holds. Adapted to our setting, their model endogenously delivers a one-stage representation for the benchmark lotteries, B_0, B_1, and B_2, and a two-stage representation for the various forms of partial ambiguity in which the unknown red and black cards form a conditional small world while the rest forms the other one. Notice that the source view of partial ambiguity can accommodate the two-stage approach by assuming that all states $\{n\}$ are exchangeable. In the following, we derive the implications of our data for a different application of source preference approach to the various partial ambiguity treatments.

Interval Ambiguity ($[50-n, 50+n]$): The two end-intervals whose total length is $100-2n$ are known – half red and half black – while the interval portion with length $2p_n$ is ambiguous. Assume that events of equal length within "known" and within "ambiguous" are exchangeable, the overall CE is given by:

$$v^{-1}\left(2\widehat{n}v\left(d\right)+\left(1-2\widehat{n}\right)v\left(c\right)\right),$$

where v is the stage-1 utility and $c=CE_k\left(\frac{1}{2}\delta_w+\frac{1}{2}\delta_0\right)$ and $d=CE_u\left(\frac{1}{2}\delta_w+\frac{1}{2}\delta_0\right)$ with CE_k and CE_u as the CE functionals for known and ambiguous domains respectively. Here, $c>d$ corresponds to ambiguity aversion.

Disjoint Ambiguity ($[0,n]\cup[100-n,100]$): Either of the two end intervals with length n is ambiguous, while the remainder with length $100-n$ is either all red or all black. Assume that events of equal length within "either or" are exchangeable, the CE is given by:

$$v^{-1}\left(\widehat{n}v\left(d\right)+\left(1-\widehat{n}\right)v\left(c'\right)\right),$$

where $c'=CE_e\left(\frac{1}{2}\delta_w+\frac{1}{2}\delta_0\right)$ with CE_e as the CE functional for the either all red or all black domain. Here, $c'>d$ corresponds to another form of ambiguity aversion which matches the observed pattern of $B_1\succ B_2$. Notice that the above expression for CE converges to $v^{-1}\left(0.5v\left(d\right)+0.5v\left(c'\right)\right)$ rather than d as n approaches 50. This relates to the dyadic decomposition of $[0,100]$ into subintervals which are individually either all red or all black discussed in Section 2. For the source model to deliver the same CE for B_2, we need to restrict its evaluation to undecomposed intervals of ambiguity.

The above implication of discontinuous behavior at $n=50$ does not appear to be compatible with the relatively smooth change of CE for disjoint group in the overall data. This suggests that subjects may view the size of ambiguous states in $[0,n]\cup[100-n,100]$ as being $2n$, while viewing $100-2n$ as being either all red or all black. Should subjects possess this incorrect understanding, the CE would be given by:

$$v^{-1}\left(2\widehat{n}v\left(d\right)+\left(1-2\widehat{n}\right)v\left(c'\right)\right),$$

which will converge continuously to d for the full ambiguity case.

Two-point Ambiguity ($\{50-n,50+n\}$): The two end intervals whose total length is $100-2n$ are known - half red and half black - while the interval portion $2n$ is either all red or all black, and the CE is given by:

$$v^{-1}\left(2\widehat{n}v\left(c'\right)+\left(1-2\widehat{n}\right)v\left(c\right)\right).$$

We have the following implications on the source model.

1. For interval, disjoint and two-point ambiguity, monotonicity alone of v implies $B_0 \succ S_1 \succ S_2 \succ S_3 \succ S_4 \succ B_2$, $B_1 \succ D_1 \succ D_2 \succ D_3 \succ D_4 \succ B_2$ and $B_0 \succ P_1 \succ P_2 \succ P_3 \succ P_4 \succ B_1$.

2. $P_j \succ S_j \succ D_j$ if $c \geq c' \geq d$ under the misperceived view of D_j.

Note that these two implications holds with only probabilistic sophistication on stage-1 utility, which can take a variety of forms besides EU.

3. For Part II, the CE is $v^{-1}\left(2\widehat{n}v\left(d\right)\right)$ for u_n, and $c_n = CE_k\left(\widehat{n}\delta_w + \left(1-\widehat{n}\right)\delta_0\right)$ for its risk counterpart r_n. When n is small, suppose we have $c_n > \widehat{n}w > 2\widehat{n}d$ (the first inequality corresponds to risk seeking while the second inequality corresponds to ambiguity aversion). Then $v\left(c_n\right) > v\left(2\widehat{n}d\right) > 2\widehat{n}v\left(d\right)$ if v is concave, which is incompatible with Observation 4. With stage-1 utility taking on a NEU form, the specification can exhibit the desired behavior. For example, with stage-1 WU, CE for u_n becomes $v^{-1}\left(2\widehat{n}\omega\left(d\right)v\left(d\right)/\left(2\widehat{n}\omega\left(d\right)+1-2\widehat{n}\right)\right)$, which can accommodate the observed affinity in small probability and aversion in moderate probabilities if the weighing function satisfies $\omega\left(d\right) > 1$, after normalizing $\omega\left(0\right) = \omega\left(w\right) = 1$. Alternatively, with stage-1 RDU, CE for u_n is then given by $v^{-1}\left(f\left(2\widehat{n}\right)v\left(d\right)\right)$, which can also fit Observation 4 if probability weighting function f exhibits an inverted S shape.

4.5 Summary

Without further assumptions to render more tractability, most models can explain a good range of the observed behavior. The following table summarizes the implications from various models based on additional assumptions that are specific to the different models. It is worthwhile to revisit the behavior of the 2-point ambiguous lotteries $\{n, 100-n\}$ as it approaches $B_1 = \{0, 100\}$ whose either all red or all black nature may give it some semblance of $B_0 = \{50\}$.Among subjects with nonconstant CE's for the 2-point ambiguous lotteries, 22.4% assign nonincreasing CE's as n approaches 50 while 30.0% assign nonincreasing CE's only until n equals 40 when there is a reversal with the CE being close to that of B_0. The data suggest that some subjects see B_1 as B_0 due to its all red or all black nature. To the extent this may be the case, the observed discontinuous behavior in the two-point group runs counter to several models of ambiguity reviewed in this section. One way to address this discontinuity is to posit that some subjects do view B_1 and B_0 as being similar and assign them similar values for their CE's. This accounts for the tick with an asterisk for MEU, α-MEU, DEU, and source preference in Table 1 below. A cross with an asterisk indicates a failure to account for the corresponding observed behavior under some specific auxiliary condition discussed in the preceding subsections.

Attitude towards Partial Ambiguity			CEU	Multiple Priors			Two-Stage				Source
				VP	αM	MEU	Same		Distinct		
							EU	NEU	EU	NEU	
Part I	*Obs* 1	Aversion to size of A_S and A_D	✓	x*	x*	x*	x	✓	x*	✓	✓
	Obs 2	Aversion to spread of A_P except at B_1	✓	✓	✓*	✓*	x	✓	✓*	✓	✓*
	Obs 3	2-Point > Interval > Disjoint	✓	✓	x*	x*	x	x*	x*	✓	✓
Part II	Skewed	Affinity to ambiguity	✓	x	x*	x	x	✓	x*	✓	✓
	Moderate	Aversion to ambiguity	✓	✓	✓	✓	x	✓	✓	✓	✓

**under additional auxiliary conditions*

Table 1. Summary of implications of our data for different models

5 Conclusion

Much of the research following Ellsberg (1961) has tended to focus on ambiguity aversion in an all or nothing fashion – either fully known or fully ambiguous (see review in the introduction) with few exceptions, e.g., Becker and Brownson (1964) and Curley and Yates (1985). In this paper, we introduce novel variants of partial ambiguity, namely two-point ambiguity and disjoint ambiguity, study attitude towards partial ambiguity experimentally, and discuss the implications of the observed behavior on a number of models of ambiguity attitude. Our results contribute to a growing experimental literature on testing various models of decision making under uncertainty. Hayashi and Wada (2011) make use of a 'snakes and ladder' game and find evidence against the descriptive validity of MEU. Using a design involving the two-color urn being drawn twice with replacement, Yang and Yao (2011) show that both MEU and DEU inherit specific implications which are incompatible with observed behavior. L'Haridon and Placido (2010) test Machina's (2009) examples of several Ellsbergian variants which are shown in Baillon, L'Haridon, and Placido (2011) to violate the implications of MEU, DEU, VP in addition to CEU. Machina (2012) offers further challenge to ambiguity models with additional Ellsbergian variants, which along with his 2009 examples turn out to be compatible with the representation in Segal (1987) according to Dillenberger and Segal (2012).[12]

Partial ambiguity offers a potentially fruitful avenue to extend existing models to situations where the information possibilities are incomplete or conflicting. Consider an example due to Gardenfors and Sahlin (1982):

> *Consider Miss Julie who is invited to bet on the outcome of three different tennis matches. As regards match A, she is very well-informed about the two players. Miss Julie predicts that it will be a very even match and a mere chance will determine the winner. In match B, she knows nothing whatsoever about the relative strength of the contestants, and has no other information that is relevant for predicting the winner of the match. Match C is similar to match B except that Miss Julie has happened to hear that one of the contestants is an excellent tennis player, although she does not know anything about which player it is, and that the second player is indeed an amateur so that everybody considers the outcome of the match a foregone conclusion.*

The kind of risks illustrated in this example – match A for known risk, match B for interval ambiguity, and match C for disjoint ambiguity – seem representative of what we observe including the setting of entrepreneurial risks as suggested by Knight (1921). In addition, attitude towards skewed ambiguity, especially the extreme ones, is of particular interest when one concerns designing lottery tickets (Quiggin, 1991) such as whether consumers with

[12] Some experimental studies of behavior relating to ambiguity attitude includes those linking it to compound lotteries (Yates and Zukowski, 1976; Chow and Sarin, 2002; Halevy, 2007; Abdellaoui, Klibanoff, and Placido, 2011; Miao and Zhong, 2012) and those linking it to familiarity bias (Tversky and Kahneman 1992; Chew et al, 2008; Abdelloui et al, 2011; Chew, Ebstein, and Zhong, 2012).

skewed ambiguity affinity may prefer pari-mutuel bets over fixed odd bets. Finally, we note that the notion of partial ambiguity can be used in domains where ambiguity aversion has been successfully applied, including finance (Epstein and Wang, 1994; Epstein and Schneider, 2008; Mukerji and Tallon, 2001), contract theory (Mukerji, 1998), and game theory (Marinacci, 2000).

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A Instructions

DECISION MAKING STUDY

GENERAL INSTRUCTIONS

Welcome to our study on decision making. The descriptions of the study contained in this instrument will be implemented **fully and faithfully**.

Each participant will receive on average $20 for the study. The overall compensation includes a $5 show up fee in addition to earnings based on how you make decisions.

All information provided will be kept CONFIDENTIAL. Information in the study will be used for research purposes only. Please refrain from discussing any aspect of the specific tasks of the study with any one.

1. The set of decision making tasks and the instructions for each task are the same for all participants
2. It is important to read the instructions CAREFULLY so that you understand the tasks in making your decisions.
3. At ANY TIME, if you have questions, please raise your hand.
4. PLEASE DO NOT communicate with others during the experiment.
5. Do take the time to go through the instructions carefully in making your decisions.
6. Cell phones and other electronic communication devices are not allowed.

INSTRUCTION FOR PART I

This is the first of two parts for today's study. It is made up of 15 decision sheets. Each decision sheet is of the form illustrated below.

	Option A	Option B	Decision
1	A	B1	A B
2	A	B2	A B
3	A	B3	A B
4	A	B4	A B
5	A	B5	A B
6	A	B6	A B
7	A	B7	A B
8	A	B8	A B
9	A	B9	A B
10	A	B10	A B

Each such table lists 10 choices to be made between a fixed Option A and 10 different Option B's.

Option A involves a lottery, guessing the color of a card randomly drawn from a deck of 100 cards with different compositions of red and black. If you guess correctly, you receive $40; otherwise you receive nothing. Different tasks will have different compositions of red and black cards as described for each task.

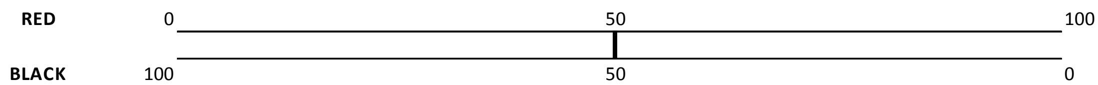
The Option B's refer to receiving the specific amounts of money for sure, and are arranged in an ascending manner in the amount of money.

For each row, you are asked to indicate your choice in the final "Decision" column – A or B – with a tick (√).

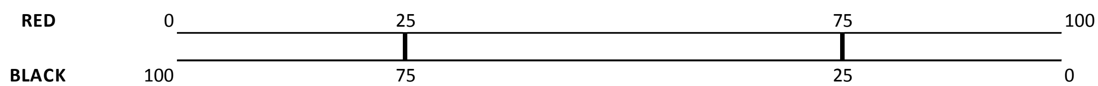
Examples of Option A

Each example involves your drawing a card randomly from a deck of 100 cards containing red and black cards.

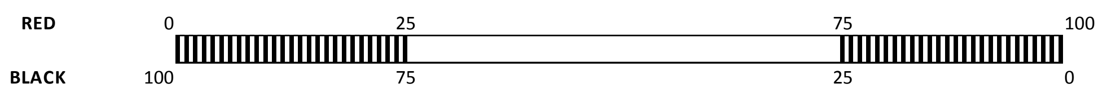
Example 1: The deck has 50 red cards and 50 black cards as illustrated below.



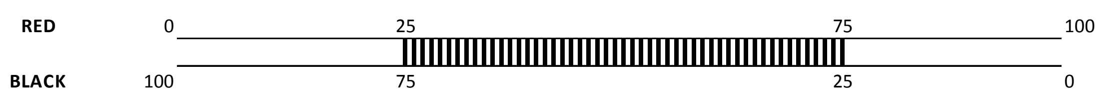
Example 2: The deck has either 25 or 75 red cards with the rest of the cards black, as illustrated below.



Example 3: The number of red cards may be anywhere between 0 and 25 or between 75 and 100 with the rest of the cards black, as illustrated below.

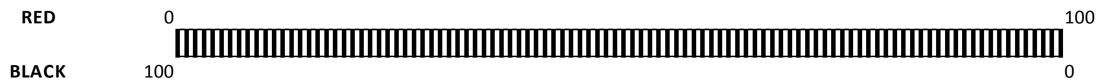


Example 4: The number of red cards may be anywhere between 25 and 75 with the rest of the cards black, as illustrated below.



Example 5: The number of red cards is anywhere between 0 and 100 with the rest of the

cards black, as illustrated below.

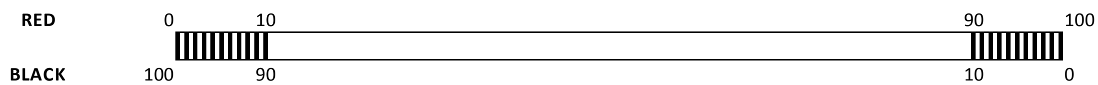


Selection of decision sheet to be implemented: One out of the 15 Decision Sheets (selected randomly by you) will be implemented. Should the sheet be chosen, one of your 10 choices will be further selected randomly and implemented.

Sample Decision Sheet of PART I

PART I DECISION SHEET - DECK [0-10]U[90-100]

This situation involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 10 or from 90 to 100 with the rest of the cards black, as illustrated below.



Option A: You guess the color first. You then draw a card from the deck of cards constructed in the above described manner. If you guess the color correctly, you receive $40. Otherwise, you receive $0.

The Option B column lists 10 amounts each corresponding to what you will receive for sure if you choose this option.

DECISION: For each of the 10 rows, please indicate your decision in the final column with a tick (√).

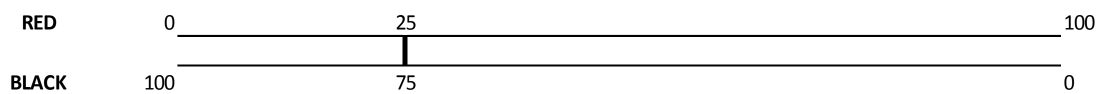
	Option A	Option B	Decision
1	Betting on the cards	Receiving $6 for sure	A B
2	Betting on the cards	Receiving $9 for sure	A B
3	Betting on the cards	Receiving $11 for sure	A B
4	Betting on the cards	Receiving $13 for sure	A B
5	Betting on the cards	Receiving $14 for sure	A B
6	Betting on the cards	Receiving $15 for sure	A B
7	Betting on the cards	Receiving $16 for sure	A B
8	Betting on the cards	Receiving $18 for sure	A B
9	Betting on the cards	Receiving $20for sure	A B
10	Betting on the cards	Receiving $23 for sure	A B

INSTRUCTION FOR PART II

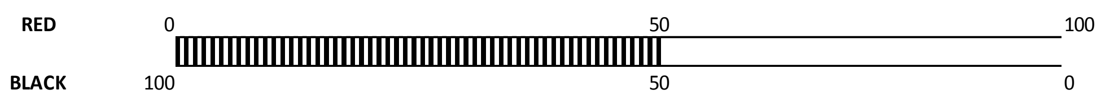
This is the second and final part for today's study. In this part, you will make 6 binary choices. At the end of this part, one of you will be randomly chosen to receive the outcome of one of his/her decisions, also randomly chosen, out of the 6 binary choices made.

Example: Consider Option A and Option B below.

Option A: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The deck has 25 red cards, and 75 black cards. If you draw the red card, you win $40, otherwise you win nothing.



Option B: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 50 with the rest of the cards black. If you draw the red card, you win $40, otherwise you win nothing.



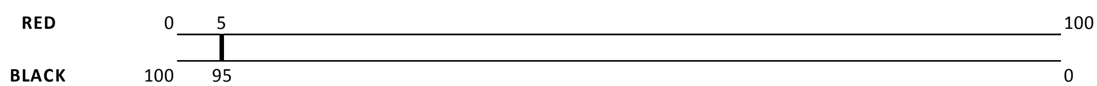
Please circle your choice: A B

Sample Decision Sheet of PART II

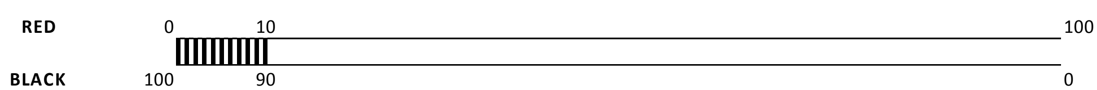
DECISION SHEET FOR PART II

Consider Option A and Option B below.

Option A: This bet involves your drawing a card randomly from a deck of 100 cards containing 5 red cards and 95 black cards. If you draw a red card, you receive $40. Otherwise, you receive $0.



Option B: This bet involves your drawing a card randomly from a deck of 100 cards containing red and black cards. The number of red cards may be anywhere from 0 to 10 with the rest of the cards black. If you draw a red card, you receive $40. Otherwise, you receive $0.



Please circle your choice: A B

B Supplementary Tables

Two-point				Interval				Disjoint			
Lottery	*Mean*	*S.E*	*N*	*Lottery*	*Mean*	*S.E*	*N*	*Lottery*	*Mean*	*S.E*	*N*
B_0	**4.51**	0.42	55	B_0	**4.51**	0.42	55	B_1	**4.16**	0.46	56
P_1	**3.96**	0.40	56	S_1	**3.96**	0.38	56	D_1	**3.31**	0.37	56
P_2	**3.55**	0.37	56	S_2	**3.34**	0.35	56	D_2	**3.29**	0.35	56
P_3	**3.40**	0.37	55	S_3	**3.04**	0.34	55	D_3	**2.98**	0.32	55
P_4	**3.39**	0.38	56	S_4	**3.36**	0.40	56	D_4	**3.04**	0.35	56
B_1	**4.16**	0.46	56	B_2	**2.71**	0.37	56	B_2	**2.71**	0.37	56

Table S1. Summary statistics of switching point for the lotteries in Part I.

Ambiguity	*Mean*	*S.E.*	*N*
[0,100]	0.125	0.044	56
[0, 80]	0.250	0.057	56
[0, 60]	0.607	0.065	56
[0, 40]	0.696	0.061	56
[0, 20]	0.696	0.061	56
[0, 10]	0.696	0.061	56

Table S2. Proportion of subjects choosing the ambiguous lottery.

	B0	P1	P2	P3	P4	B1	S1	S2	S3	S4	B2	D1	D2	D3	D4
B0	1														
P1	0.835	1													
P2	0.851	0.913	1												
P3	0.675	0.783	0.838	1											
P4	0.668	0.756	0.744	0.886	1										
B1	0.802	0.780	0.760	0.679	0.777	1									
S1	0.850	0.878	0.881	0.796	0.785	0.796	1								
S2	0.789	0.882	0.876	0.752	0.696	0.665	0.860	1							
S3	0.719	0.852	0.862	0.740	0.648	0.588	0.836	0.912	1						
S4	0.624	0.769	0.782	0.857	0.791	0.608	0.783	0.848	0.780	1					
B2	0.634	0.772	0.781	0.712	0.682	0.560	0.721	0.795	0.888	0.768	1				
D1	0.761	0.780	0.839	0.828	0.805	0.709	0.822	0.777	0.706	0.816	0.712	1			
D2	0.725	0.784	0.769	0.793	0.785	0.659	0.817	0.871	0.787	0.878	0.759	0.821	1		
D3	0.761	0.791	0.822	0.758	0.757	0.667	0.798	0.842	0.793	0.829	0.777	0.788	0.912	1	
D4	0.692	0.781	0.785	0.749	0.685	0.612	0.786	0.844	0.806	0.830	0.832	0.807	0.916	0.876	1

Table S3. Spearman correlation of CEs for lotteries in Part I.

Ambiguity attitude	P1	P2	P3	P4	***B1***	S1	S2	S3	S4	B2	D1	D2	D3	D4
Risk attitude	0.364	0.439	0.484	0.515	***0.098***	0.411	0.608	0.634	0.471	0.567	0.619	0.638	0.580	0.504

Table S4. Spearman correlation of risk attitude and ambiguity attitudes in Part I.

	P1	P2	P3	P4	B1	S1	S2	S3	S4	B2	D1	D2	D3	D4
P1	1.000													
P2	0.752	1.000												
P3	0.725	0.873	1.000											
P4	0.709	0.725	0.794	1.000										
B1	*0.301*	*0.219*	*0.285*	*0.492*	1.000									
S1	0.733	0.712	0.642	0.696	*0.313*	1.000								
S2	0.652	0.667	0.656	0.644	*0.277*	0.697	1.000							
S3	0.681	0.722	0.661	0.621	*0.096*	0.719	0.830	1.000						
S4	0.720	0.737	0.693	0.617	*0.288*	0.685	0.830	0.782	1.000					
B2	0.650	0.755	0.721	0.716	*0.126*	0.640	0.754	0.866	0.736	1.000				
D1	0.551	0.620	0.624	0.610	*0.111*	0.647	0.764	0.851	0.720	0.871	1.000			
D2	0.655	0.761	0.699	0.714	*0.168*	0.676	0.836	0.865	0.801	0.856	0.790	1.000		
D3	0.653	0.647	0.655	0.703	*0.241*	0.718	0.857	0.840	0.832	0.814	0.832	0.870	1.000	
D4	0.556	0.805	0.784	0.686	*0.289*	0.600	0.607	0.636	0.674	0.714	0.663	0.591	0.615	1.000

Table S5. Spearman correlation of ambiguity attitudes in Part I.

	[0,100]	**[0,80]**	**[0,60]**	**[0,40]**	**[0,20]**	**[0,10]**
[0,100]	1					
[0,80]	0.530	1				
[0,60]	0.194	0.380	1			
[0,40]	0.132	0.112	0.423	1		
[0,20]	0.015	0.112	0.344	0.578	1	
[0,10]	-0.220	-0.067	0.344	0.324	0.578	1

Table S6. Spearman correlation of ambiguity attitudes in Part II.

C Numerical Examples

The following are numerical examples of two-stage RDU (Segal 1987) and two-stage distinct EU (Klibanoff, Marinacci, and Mukerji, 2005), with $v(x) = \sqrt{x}$ (normalize $v(w) = 1$), $f(p) = p^2$ and $\phi(x) = x^{1/3}$.

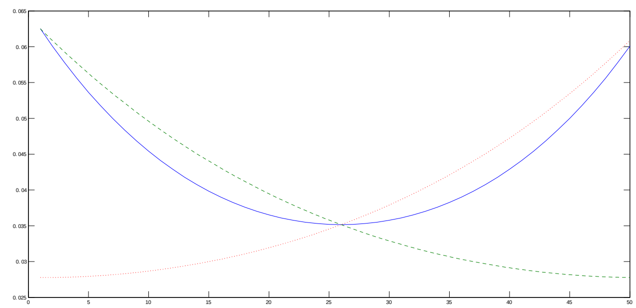
Solid Line: Two-point Group $\{50-n, 50+n\}$.

Dashed Line: Interval Group $[50-n, 50+n]$.

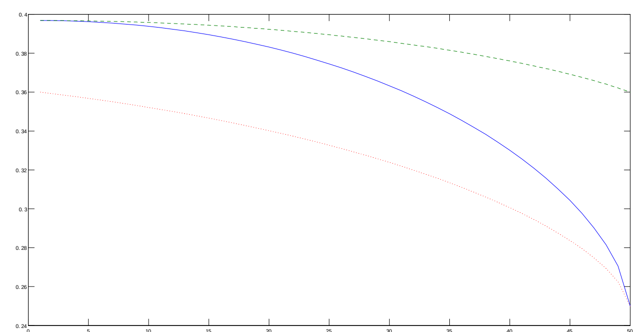
Dotted Line: Disjoint Group $[0, 50-n] \cup [50+n, 100]$.

$x - axis$: n.

$y - axis$: CE for different treatments.



Two-stage Identical RDU (Segal 1987)



Two-stage Distinct EU (Klibanoff, Marinacci, and Mukerji, 2005)