Collateralized Borrowing and Risk Taking at Low Interest Rates*†

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Abstract

A view advanced in the aftermath of the late-2000s financial crisis is that lower than optimal interest rates lead to excessive risk taking by financial intermediaries. We evaluate this view in a quantitative dynamic model where interest rate policy affects risk taking through two channels. First, policy influences the attractiveness of safe bond investments relative to riskier assets. Moreover, policy changes the amount of safe bonds available for collateralized borrowing in interbank markets. In this framework, collateral constraints provide a safeguard against increases in risk taking. Lower than optimal policy rates lead to tighter collateral constraints and reduce risk taking.

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1 Introduction

The recent financial crisis has renewed interest in the determinants of portfolio investments into safe and risky assets by financial intermediaries. A standard result in the theory of portfolio choice is that a risk averse investor's optimal investment into risky assets is decreasing in the return to safe assets. This insight suggests that low policy rates may increase the riskiness of financial intermediaries' portfolios, by altering the returns to safe assets. To the extent that increased investments in risky assets exceed the social optimum, there may be important welfare consequences.

In this paper, we examine how changes in the policy rate affect the portfolio choices of financial intermediaries, in an environment in which safe assets can be used as collateral to facilitate borrowing. Two facts motivate our decision to model collateralized borrowing: first, collateralized borrowing is a primary margin of balance sheet adjustment for intermediaries (Adrian and Shin (2010)) and, second, the cost of such borrowing is tightly linked to monetary policy rates. Our findings encompass the standard portfolio choice result and, at the same time, highlight the importance of collateral for risk taking. At low interest rates, low demand for safe assets results in a shortage of collateral, which limits borrowing in the interbank market and ultimately results in reduced risk taking by intermediaries.

We develop a dynamic model with aggregate and idiosyncratic uncertainty in which the monetary authority controls the real interest rate on safe bonds.[1] Each period financial intermediaries with limited liability receive deposits and equity from households and invest into safe bonds and risky projects. The latter are investments into the production technologies of

[1] Implicitly, we assume that the monetary authority is successful in ensuring price stability. In this context, we consider whether the monetary authority can control risk taking of intermediaries through the real interest rates on safe assets and examine the implications for the macroeconomy. Having nominal interest rates as a policy instrument would enrich the policy insights, but is beyond the scope of this paper.

small firms, and their returns are correlated with aggregate productivity.[2] After this initial portfolio decision, intermediaries find out whether they hold high-risk projects, with high variance and high expected return, or low-risk projects, with low variance and low expected return. Given this information, intermediaries reoptimize their portfolios using collateralized borrowing in the interbank market. For example, when aggregate productivity is expected to be high, intermediaries with high-risk projects—call them high-risk intermediaries—trade their bonds to invest more into their risky projects. These projects are relatively attractive from a social point of view due to their high expected return, and are even more attractive from the intermediaries' point of view because potential losses in the event of a contraction are avoided through limited liability (as in Allen and Gale (2000)). Low-risk intermediaries on the other side of the transaction accept bonds and reduce exposure to their risky projects, which have lower expected returns.

In this environment, monetary policy influences risk taking by financial intermediaries directly, through a portfolio channel, and indirectly, through a collateral channel. Changes in risk taking through the portfolio channel are similar to those discussed in Merton (1969), Samuelson (1969) and Fishburn and Porter (1976). Namely, at low interest rates, intermediaries purchase fewer safe bonds and invest more into riskier assets with a higher expected return.[3] The innovation in our paper is to consider the transmission mechanism from monetary policy to risk taking through the quantity of collateral. At low interest rates, financial intermediaries allocate few resources to safe assets and the resulting scarcity of collateral provides a safeguard against increased risk taking.

Collateralized borrowing in our model is beneficial because it facilitates reallocation of resources between intermediaries in response to new information about the riskiness of their

[2] In our model, the investment market is segmented in that households cannot invest directly in risky projects of small firms and are forced to use intermediaries. This is similar to Gale (2004). Noncorporate, nonfinancial firms are the data counterpart for the small firms in our model. For simplicity, we do not model loans between financial intermediaries and these firms, but rather assume that intermediaries operate their production technologies directly. To allow our model to be consistent with U.S. data, we also model a nonfinancial corporate sector (see Section 2).

[3] This idea is also the basis of Rajan (2006), who discusses excessive risk in the financial sector.

investments. However, borrowing against safe bonds also allows intermediaries to take advantage of their limited liability by overinvesting in risky projects. This is socially costly because financial intermediaries can go bankrupt, in which case, payments to its depositors are guaranteed by the government-funded deposit insurance.[4] The role of the monetary authority is to set interest rate policy so as to mitigate the moral hazard problem of intermediaries.[5] This is achieved by making the collateral constraint of intermediaries bind at the optimal policy.

We solve for the optimal interest rate policy and consider the implications of lower than optimal interest rates for risk taking and welfare. We say that risk taking of financial intermediaries is *excessive* if investments in high-risk projects in the decentralized economy exceed the social optimum, defined as the solution to a social planner problem. We calibrate our model's parameters to match key characteristics of economic expansions and contractions and of the financial sector in the U.S. economy. We find that, at the optimal interest rate policy, there is excessive risk taking, but welfare losses relative to the social optimum are very small. In addition, lower than optimal interest rates lead to less risk taking by financial intermediaries. This is because, quantitatively, the collateral risk taking channel dominates the portfolio risk taking channel. The intuition is that the collateral risk taking channel constrains high-risk intermediaries who have the strongest incentives to overinvest in risky projects. We conclude that the collateral channel provides a safeguard against increased risk taking, especially at low interest rates.

In the model outlined so far, collateralized borrowings can be interpreted as repurchasing agreements (repos).[6] Empirically, repos are an important margin of portfolio adjustment, as suggested by Adrian and Shin (2010), and are largely collateralized using government bonds. Consistent with this evidence, financial intermediaries in our model only

[4] In our model, deposit insurance is provided at no cost, consistent with empirical evidence in Pennacchi (2006). For details, also see footnote 16.

[5] We note that moral hazard leads to a failure of the Modigliani and Miller (1958) theorem, see Hellwig (1981) and Myers (2003).

[6] A repo transaction is a sale of a security and a simultaneous agreement to repurchase the security at a future date. Repos are secured loans in which the borrower receives money against collateral.

use government bonds as collateral. The implicit theoretical assumption is that government bonds are special because there is no information asymmetry about their value.[7]

Related Literature

Our paper contributes to the growing literature studying the risk taking channel of monetary policy, a term coined by Borio and Zhu (2008). Several papers find empirical evidence that, when interest rates are low for an extended period, banks take on more risks.[8] There are also theoretical explorations of this link, for example, Dell'Ariccia, Laeven, and Marquez (2010). Our paper complements this body of work, by evaluating the impact of lower than optimal interest rates on risk taking in a quantitative dynamic general equilibrium model calibrated to the U.S. economy. Through the lens of our model, low interest rates per se do not increase risk taking.

Our paper is closely related to Gertler and Kiyotaki (2010) and Gertler, Kiyotaki, and Queralto (2011).[9] These authors consider the effects of credit policies (e.g. discount window lending, equity injections) and macro prudential policies (e.g. subsidies to issuance of outside equity) on financial intermediation and risk taking incentives, in environments in which banks choose equity and deposits endogenously. Our work is similar to these two papers in that we build a quantitative model in which intermediaries make endogenous portfolio choices. An important difference is that we allow intermediaries to invest in safe bonds, which are later used as collateral in interbank borrowing. This allows us to highlight the role of monetary policy in affecting risk taking through the quantity and value of available collateral.

[7] It is well documented that, in the run-up to the recent financial crisis, some assets, such as asset-backed securities, used as collateral in the repo market were not truly safe (see Gorton (2010), Gorton and Metrick (2011), Krishnamurthy, Nagel, and Orlov (2011) and Hoerdahl and King (2008)). This type of collateral disappeared from the repo market as the crisis unfolded. Considering other types of collateral assets is an interesting extension of our model, that we leave for future work.

[8] For example, Gambacorta (2009), Ioannidou, Ongena, and Peydró (2009), Jiménez, Ongena, Peydró, and Saurina (2009), Delis and Kouretas (2010) and Altunbas, Gambacorta, and Marques-Ibane (2010) use data from different countries to show that banks grant riskier loans and soften lending standards when interest rates are low. de Nicolò, Dell'Ariccia, Laeven, and Valencia (2010) use U.S. commercial bank Call Reports to document a negative relationship between the real interest rate and the riskiness of banks' assets.

[9] These papers augment the existing quantitative macro models with financial amplification mechanism à la Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

Our paper is also related to the literature studying the impact of collateral constraints on the macroeconomy. For example, Kiyotaki and Moore (1997) show that shocks to credit-constrained firms are amplified and transmitted to output through changes in collateral values. Our paper makes an important contribution by highlighting that relaxing collateral constraints can result in increased risk taking with adverse effects for real activity.

There is an extensive theoretical literature that examines other related aspects of financial intermediation. For example, Dubecq, Mojon, and Ragot (2009) study the interaction between capital regulation and risk. They find that opaque capital regulation leads to uncertainty about the risk exposure of financial intermediaries, a problem which is more severe at low interest rates. Shleifer and Vishny (2010), consider a model in which financial intermediaries alter capital allocation based on investor sentiment, and volatility of this sentiment transmits to volatility in real activity. Stein (1998) examines the transmission mechanism of monetary policy in a model in which banks' portfolio choices respond to changes in the availability of financing via insured deposits. Diamond and Rajan (2009), Acharya and Naqvi (2010) and Agur and Demertzis (2010) examine the optimal policy when the monetary authority has a financial stability objective. Farhi and Tirole (2009) and Chari and Kehoe (2009) consider moral hazard consequences of government bailouts.

The paper is organized as follows. Section 2 presents the model and derives equilibrium properties. Section 3 outlines the methods we use to pin down our model's parameters. Section 4 describes the various experiments and the main results of the paper. Section 5 concludes.

2 Model Economy

The economy is populated by a measure one of identical households, a measure π_m of identical nonfinancial firms, a measure $1-\pi_m$ of financial intermediaries and a government. Financial intermediaries are initially identical and later split into high-risk or low-risk. Time is discrete and infinite. Each period, the economy is subject to an exogenous aggregate shock which affects the productivity of all firms, as outlined in section 2.2. The aggregate state $s_t \in \{\overline{s}, \underline{s}\}$ follows a first-order Markov process. The history of aggregate shocks up to t is s^t. A summary of the timing of events in our model is presented in Section A.1 of the Appendix.

2.1 Households

At the beginning of period t, the aggregate state s_t is revealed and households receive returns on their previous period investments, wage income and lump-sum taxes or transfers from the government. Households split the resulting wealth, $w(s^t)$, into current consumption, $C(s^t)$, and investments that will pay returns in period $t+1$.

Investments take the form of deposits, nonfinancial sector equity and financial sector equity. Deposits, $D_h(s^t)$, earn a fixed return, $R^d(s^t)$, which is guaranteed by deposit insurance. Equity invested in financial intermediaries, $Z(s^t)$, is a risky investment which gives households a claim to the profits of the intermediaries. The return per unit of equity is $R^z(s^{t+1})$. Similarly, the equity investment into the nonfinancial sector, $M(s^t)$, entitles the household to state contingent returns next period, $R^m(s^{t+1})$.

Households supply labour inelastically. We assume that labour markets are segmented.[10] Fraction π_m of a household's time is spent working in the nonfinancial sector, and fraction $1-\pi_m$ is spent in the financial sector. Wage rates vary by sector, the type of firm within the sector and the aggregate state of the economy: $W_m(s^t)$ is the wage rate paid by nonfinancial firms given history s^t, while $W_j(s^t)$ is the wage rate paid by a financial intermediary of type $j \in \{h, l\}$. Throughout, h denotes high-risk and l denotes low-risk intermediaries. With these assumptions, labour supplied to each firm is normalized to one unit, for any realization of the aggregate state.

[10] The assumption of a labour market segmentation is done for convenience. Relaxing this assumption to allow labour to move across firms and sectors, would reinforce the risk taking channel present in our model, as both capital and labour would flow in the same direction.

The household's problem is given by:

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \varphi\left(s^t\right) \log C\left(s^t\right)$$

$$\begin{aligned}
\text{subject to} \quad &: \\
w\left(s^t\right) &= R^m\left(s^t\right) M\left(s^{t-1}\right) + R^d\left(s^{t-1}\right) D_h\left(s^{t-1}\right) + R^z\left(s^t\right) Z\left(s^{t-1}\right) \\
&\quad + \pi_m W_m\left(s^t\right) + \left(1-\pi_m\right) \pi_l W_l\left(s^t\right) + \left(1-\pi_m\right) \pi_h W_h\left(s^t\right) + T\left(s^t\right) \\
w\left(s^t\right) &= C\left(s^t\right) + M\left(s^t\right) + D_h\left(s^t\right) + Z\left(s^t\right)
\end{aligned}$$

where β is the discount factor, $\varphi\left(s^t\right)$ is the probability of history s^t, π_j with $j \in \{h, l\}$ is the probability of working for financial intermediary of type j, where $\pi_h + \pi_l = 1$, and $T\left(s^t\right)$ are lump-sum transfers if $T\left(s^t\right) \geq 0$, or lump-sum taxes otherwise.

2.2 Firms

Financial and nonfinancial firms differ in the way they are funded, in the types of investments they make and the productivity of these investments. *Financial firms* finance their operations through equity and deposits. The main difference between these two forms of funding is that equity returns are contingent on the realization of the aggregate state in the period when they are paid, while returns to deposits are not. In addition, equity returns are bounded below by zero due to the limited liability of intermediaries, while deposit returns are guaranteed by deposit insurance. Financial intermediaries invest into safe government bonds and risky projects. The latter are investments into the production technologies of small firms and can be of two types: high-risk projects with productivity $q_h\left(s_t\right)$ and low-risk projects with productivity $q_l\left(s_t\right)$.[11] *Nonfinancial firms* are funded through household equity only.[12]

[11] We assume that financial intermediaries operate the production technologies of small firms directly. By not modeling loans between intermediaries and these firms, we abstract from information problems à la Bernanke and Gertler (1989). Also see footnote 2.

[12] In the model, the important assumption is that the nonfinancial sector is funded through state contingent claims. We use equity for simplicity, but we could also allow for state contingent corporate bonds. Our assumption is consistent with the fact that in U.S. data, corporate nonfinancial firms are mostly equity financed.

All equity raised is invested into capital whose return depends on the productivity of the production technology in the nonfinancial sector, $q_m(s_t)$. Note that, implicitly, households in our model invest directly into the risky production technology of nonfinancial firms. However, they need intermediaries to invest into the risky projects of small firms.

We assume that high-risk financial intermediaries are more productive during a good aggregate state ($s_t = \overline{s}$), and less productive during a bad aggregate state ($s_t = \underline{s}$), compared to low-risk financial intermediaries. Formally, $q_h(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_h(\underline{s})$. Moreover, we consider that the productivity of the production technology of nonfinancial firms is such that: $q_h(\overline{s}) \geq q_m(\overline{s}) > q_l(\overline{s}) \geq q_l(\underline{s}) > q_m(\underline{s}) > q_h(\underline{s})$. For details on the parameterization of these relative productivity levels, see section 3.

2.2.1 Financial Sector

There is a measure $1 - \pi_m$ of financial intermediaries. The problem of an intermediary is to choose a portfolio that maximizes the expected value of its equity. Each period, initially identical financial intermediaries receive the same amounts of deposits and equity from the households and make the same investments into government bonds and risky projects.

After the initial investment decisions, intermediaries acquire more information about the riskiness of their projects. With probability π_j, the project an intermediary previously invested into is of type $j \in \{h, l\}$ (i.e. j is i.i.d., see Section 3 for a discussion on π_j). We refer to intermediaries as being high-risk or low-risk intermediaries, based on the type j of their risky projects. The probabilities, π_h and $\pi_l = 1 - \pi_h$, are time and state invariant and known. Once $j \in \{h, l\}$ is known, but before the realization of s_t, intermediaries trade bonds in the interbank market (repo market) in order to adjust the amount of resources invested into the risky projects. This timing assumption is meant to capture the idea that information about the riskiness of projects evolves over time. As a result, financial intermediaries adjust their portfolios, but may be constrained in their choices.

Although intermediaries start out as identical each period, the funds they receive from households vary with the aggregate state, allowing the model to capture interesting dynamics such as sustained high levels of investment into high-risk projects (see Section 4 for details).

We now describe the two stages of an intermediary's problem that take place during period $t-1$. This shows how capital used for production in the financial sector in period t is determined.

Portfolio Choice in the Bond Market

After production in period $t-1$ has taken place, intermediaries receive resources from households and make investment decisions that pay off in t. Financial intermediaries don't know the type of risky projects and maximize expected profits, taking as given future trades in the repo market. Since households own all firms in the economy, firms value profits at history s^t according to the households' marginal utility of consumption weighted by the probability of history s^t. Let $\lambda(s^t) = \varphi(s^t)/C(s^t)$.

Taking as given $\lambda(s^t)$, the amount of equity issued by an intermediary, $z(s^{t-1})$, the future repo market activities and all prices, an intermediary chooses deposit demand, $d(s^{t-1})$, safe bonds, $b(s^{t-1})$, risky investments, $k(s^{t-1})$, and labour, $l(s^{t-1})$, to maximize the expected profits in $(P1)$:

$$\max \sum_{j \in \{h,l\}} \pi_j \sum_{s^t | s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P1}$$

subject to:

$$z\left(s^{t-1}\right) + d\left(s^{t-1}\right) = k\left(s^{t-1}\right) + p\left(s^{t-1}\right) b\left(s^{t-1}\right) \tag{1}$$

$$V_j\left(s^t\right) = \max \left\{ \begin{array}{l} q_j(s_t)\left[k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})\right]^{\theta}\left[l(s^{t-1})\right]^{1-\theta-\alpha} \\ +q_j(s_t)(1-\delta)\left[k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})\right] \\ +\left[b(s^{t-1}) - \tilde{b}_j(s^{t-1})\right] - R^d(s^{t-1})d(s^{t-1}) - W_j(s^t)l(s^{t-1}), 0 \end{array} \right\} \tag{2}$$

where $V_j(s^t)$ are profits for intermediary $j \in \{h,l\}$ at history s^t, $p(s^{t-1})$ is the bond price, $\tilde{p}(s^{t-1})$ is the repo market price, and $\tilde{b}_j(s^{t-1})$ is the amount of bonds traded in the repo market by intermediary j.

The production technology operated by intermediary j is $q_j(s_t)\left[k_j(s^{t-1})\right]^{\theta}\left[l(s^{t-1})\right]^{1-\theta-\alpha}$, where $q_j(s_t)$ is the productivity parameter, $k_j(s^{t-1}) \equiv k(s^{t-1}) + \tilde{p}(s^{t-1})\tilde{b}_j(s^{t-1})$ is the amount of resources invested in the risky projects and $l(s^{t-1})$ is the amount of labour employed. Recall that we abstract from labour redistribution and normalize $l(s^{t-1})$ to 1. Parameters θ and α satisfy $\alpha, \theta \in [0,1]$, $1-\alpha-\theta \geq 0$. If $\alpha > 0$ there is a fixed factor present in the production process. In the absence of bankruptcy, this factor's returns are payable to the equity holders.

In equation (2), the undepreciated capital stock of firms is adjusted by the productivity level. This allows for variation in the value of capital, similar to Merton (1973) and Gertler and Kiyotaki (2010). The idea is that while capital may not depreciate in a physical sense during contraction periods, it does so in an economic sense. In a case study of aerospace plants, Ramey and Shapiro (2001) show that the decrease in the value of installed capital at plants that discontinued operations is higher than the actual depreciation rate. In addition, Eisfeldt and Rampini (2006) provide evidence that costs of capital reallocation are strongly countercyclical.

Portfolio Adjustments via Repo Market

Once intermediaries find out their type $j \in \{h, l\}$, they adjust the riskiness of their portfolios by trading bonds, $\tilde{b}_j(s^{t-1})$, amongst themselves. Intermediaries choose $\tilde{b}_j(s^{t-1})$ to solve:

$$\max \sum_{s^t|s^{t-1}} \lambda\left(s^t\right) V_j\left(s^t\right) \tag{P2}$$

where $V_j(s^t)$ is given in equation (2) and $\tilde{b}_j(s^{t-1}) \in \left[-\frac{k\left(s^{t-1}\right)}{\tilde{p}\left(s^{t-1}\right)}, b\left(s^{t-1}\right)\right]$.

Here, $\tilde{b}_j(s^{t-1})$ can be interpreted either as sales of bonds or, alternatively, as repurchasing agreements.[13]

[13] While we model $\tilde{b}_j\left(s^{t-1}\right)$ as bond sales, incorporating explicitly the repurchase of bonds—which is

Empirically, collateralized repos are an important margin of balance sheet adjustment by intermediaries and a good indicator of financial market risk, as suggested by Adrian and Shin (2010). In our model, the redistribution of resources that takes place through the repo market allows financial intermediaries to change their risk exposure in light of new information obtained about their investments. Intermediaries who use bonds as collateral in the repo market increase the amount of resources allocated to risky investments. By the same token, intermediaries who give resources against collateral decrease their risk exposure. Our model is consistent with evidence that repo lending allows participants to "hedge against market risk exposures arising from other activities" (FSB (2012)).

Intermediaries can collateralize either a subset or all of their bonds in exchange for an equal amount of resources to be invested in risky projects.[14] That is, the intermediaries' ability to increase their risky investment is limited by their bond market activities. Higher purchases of bonds make balance sheets seem safer initially, but may lead to increased risk taking through the repo market.

2.3 Nonfinancial sector

There are π_m identical nonfinancial firms which are funded entirely through household equity. Each nonfinancial firm enters period t with equity $M\left(s^{t-1}\right)/\pi_m$ from households which is invested into capital. Hence, $M\left(s^{t-1}\right)/\pi_m = k_m\left(s^{t-1}\right)$. The problem of a nonfinancial firm is to choose capital and labour to produce output:

$$\max\left\{y_m\left(s^t\right) + q_m\left(s_t\right)(1-\delta)\,k_m\left(s^{t-1}\right) - R^m\left(s^t\right)k_m\left(s^{t-1}\right) - W_m\left(s^t\right)l_m\left(s^{t-1}\right)\right\}$$
$$\text{subject to: } y_m\left(s^t\right) = q_m\left(s_t\right)\left(k_m\left(s^{t-1}\right)\right)^{\theta}\left(l_m\left(s^{t-1}\right)\right)^{1-\theta}.$$

typical in a repo agreement—would yield identical results.

[14] A repo transaction in the data may require the borrower to pledge collateral in excess of the loan received. See, for example, Krishnamurthy, Nagel, and Orlov (2011) who document that average haircuts vary between 2 and 7 percent by type of collateral. Currently, our model abstracts from haircuts in the repo market. If we allow for a fixed haircut, we can prove that the allocation is identical, because the equilibrium repo price, $\tilde{p}\left(s^{t-1}\right)$, adjusts with the size of the haircut so that resources obtained through the repo market remain unchanged.

We introduce this sector in order to bring our model closer to U.S. data. Specifically, this allows our model to be consistent with a high equity to deposit ratio observed for U.S. households, a low equity to deposit ratio in the U.S. financial sector and the relative importance of the two sectors in U.S. production.

2.4 Government

The government issues bonds that financial intermediaries can use either as an asset or as a medium of exchange on the repo market. At the end of period $t-1$, the government sells bonds, $B\left(s^{t-1}\right)$, at price, $p\left(s^{t-1}\right)$. These bonds pay off during period t. The proceeds from the bond sales are deposited with financial intermediaries.[15] Each financial intermediary receives $D_g\left(s^{t-1}\right)/\left(1-\pi_m\right)$ of government deposits, where

$$D_g\left(s^{t-1}\right) = p\left(s^{t-1}\right) B\left(s^{t-1}\right)$$

To guarantee the fixed return on deposits the government provides deposit insurance at zero price which is financed through household taxation.[16] The government balances its budget after the production takes place at the beginning of period t :[17]

$$T\left(s^{t}\right) + B\left(s^{t-1}\right) + \Delta\left(s^{t}\right) = R^{d}\left(s^{t-1}\right) D_g\left(s^{t-1}\right)$$

Here, $\Delta\left(s^{t}\right)$ is the amount of deposit insurance necessary to guarantee the fixed return on deposits, $R^{d}\left(s^{t-1}\right)$. Given the limited liability of intermediaries, if they are unable to pay $R^{d}\left(s^{t-1}\right)$ on deposits, they pay a smaller return on deposits which ensures they break-even.

[15] Alternatively, the proceeds from the bond sales could be transferred to households. Our results would not change.

[16] See Pennacchi (2006, pg. 14), who documents that since 1996, deposit insurance has been essentially free for U.S. banks. In our model, the assumption of a zero price of deposit insurance is not crucial. What matters is that the insurance is not priced in a way that eliminates moral hazard. This means, for example, that the deposit insurance can not be made contingent on the portfolio decisions of the intermediaries.

[17] We concentrate on new issuance of bonds only and abstract from outstanding bonds for computational reasons. Considering the valuation effects of current policy in the presence of outstanding bonds may be an interesting extension of the model.

The rest is covered by deposit insurance.

The main policy instrument is the price of government bonds, $p\left(s^{t-1}\right)$. The government satisfies any demand for bonds given this price. The interpretation is that the monetary authority uses open market operations (i.e. purchases or sales of government bonds) to control interest rates. The key decision from the government's perspective is to choose the bond return $1/p^*\left(s^{t-1}\right)$ that maximizes the welfare of the households in the decentralized economy:

$$p^*\left(s^{t-1}\right) = \arg\max_{p(s^{t-1})} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \varphi\left(s^t\right) \log C\left(s^t\right) \tag{P3}$$

subject to: $C\left(s^t\right)$ is part of a competitive equilibrium given policy $p\left(s^{t-1}\right)$

2.5 Market clearing

There are eight market clearing conditions. The *labour market clearing conditions* state that labour demanded by financial intermediaries and nonfinancial firms equals labour supplied by households:

$$(1-\pi_m)\, l\left(s^{t-1}\right) = 1-\pi_m; \quad \pi_m l_m\left(s^{t-1}\right) = \pi_m$$

The *goods market clearing condition* equates total output produced with aggregate consumption and investment. Output produced by nonfinancial firms is $\pi_m q_m\left(s^t\right)\left(k_m\left(s^{t-1}\right)\right)^\theta$, while output produced by financial firms is $(1-\pi_m)\sum_{j\in\{l,h\}} \pi_j q_j\left(s^t\right)\left(k_j\left(s^{t-1}\right)\right)^\theta$, where $k_j\left(s^{t-1}\right)$ are resources allocated to the risky projects after repo market trading.

$$\begin{aligned} C\left(s^t\right) + M\left(s^t\right) + D_h\left(s^t\right) + Z\left(s^t\right) &= \pi_m q_m\left(s_t\right)\left[\left(k_m\left(s^{t-1}\right)\right)^\theta + (1-\delta)\, k_m\left(s^{t-1}\right)\right] \\ &\quad + (1-\pi_m)\sum_{j\in\{l,h\}} \pi_j q_j\left(s_t\right)\left[\left(k_j\left(s^{t-1}\right)\right)^\theta + (1-\delta)\, k_j\left(s^{t-1}\right)\right] \end{aligned}$$

Financial markets clearing conditions ensure that the deposit markets, equity markets and bond markets clear. Deposits demanded by financial intermediaries equal deposits from the households and the government:

$$D_h\left(s^{t-1}\right)+D_g\left(s^{t-1}\right)=D\left(s^{t-1}\right)=\left(1-\pi_m\right)d\left(s^{t-1}\right)$$

In the bond market, total bond sales by the government equal the bond purchases by financial intermediaries.

$$B\left(s^{t-1}\right)=\left(1-\pi_m\right)b\left(s^{t-1}\right)$$

In the repo market, trades between the different types of intermediaries must balance.

$$\sum_{j\in\{l,h\}}\pi_j\tilde{b}_j\left(s^{t-1}\right)=0 \tag{3}$$

Total equity invested by households in the financial and nonfinancial sectors are distributed over the firms.

$$\begin{aligned} M\left(s^{t-1}\right) &= \pi_m k_m\left(s^{t-1}\right) \\ Z\left(s^{t-1}\right) &= \left(1-\pi_m\right)z\left(s^{t-1}\right) \end{aligned}$$

2.6 Social Planner Problem

We consider the following social planner's problem as a reference point for our decentralized economy. For ease of comparison between the two environments, we abuse language and refer to the existence of financial and nonfinancial sectors even in the context of the social planner's problem. At the beginning of period t, the aggregate state, s_t, is revealed and production takes place using capital that the social planner has allocated to the different technologies of production: $k_m\left(s^{t-1}\right)$ for the nonfinancial sector, $k_h\left(s^{t-1}\right)$ and $k_l\left(s^{t-1}\right)$ for the high-risk and low-risk technologies of the financial sector. The resulting wealth is then split between consumption and capital to be used in production at $t+1$. At the time of this decision, the social planner does not distinguish between the high-risk and low-risk technologies of the financial sector used in production next period, and simply allocates resources, $k_b(s^t)$, to both of them. Once their type is revealed, the social planner reallocates resources between the two technologies.

The social planner solves:

$$\max E \sum_{t=0}^{\infty} \beta^t \log C\left(s^t\right)$$

subject to:

$$\begin{aligned}
C\left(s^t\right) + \pi_m k_m\left(s^t\right) + (1-\pi_m)\, k_b\left(s^t\right) & \\
&= \pi_m q_m(s_t)\left[\left(k_m\left(s^{t-1}\right)\right)^{\theta} + (1-\delta)\, k_m\left(s^{t-1}\right)\right] \\
&\quad + (1-\pi_m)\,\pi_l q_l(s_t)\left[\left(k_l\left(s^{t-1}\right)\right)^{\theta} + (1-\delta)\left(k_l\left(s^{t-1}\right)\right)\right] \\
&\quad + (1-\pi_m)\,\pi_h q_h(s_t)\left[\left(k_h\left(s^{t-1}\right)\right)^{\theta} + (1-\delta)\, k_h\left(s^{t-1}\right)\right] \\
k_l\left(s^t\right) &= k_b\left(s^t\right) - \frac{\pi_h}{\pi_l} n\left(s^t\right) \\
k_h\left(s^t\right) &= k_b\left(s^t\right) + n\left(s^t\right)
\end{aligned}$$

where $n(s^t)$ is the amount of resources given to (or taken from) each high-risk production technology. To achieve this reallocation, $\frac{\pi_h}{\pi_l} n(s^t)$ resources need to be taken away from (or given to) each low-risk technology.

From a social planner's perspective, it is optimal for resources to flow to high-risk intermediaries during expansion periods and to low-risk intermediaries during contractions. To induce these reallocation flows in the decentralized economy, bond prices, $p(s^t)$, need to be appropriately chosen by the monetary authority (see results Section 4 for details).

2.7 Competitive Equilibrium Properties

In this section, we discuss equilibrium properties of our model and present results on the relationship between equilibrium bond prices and the return to deposits. In addition, we define what we mean by risk taking behavior of financial intermediaries and provide intuition for how interest rate changes affect risk taking.

2.7.1 Constrained Repo Market

Financial intermediaries maximize expected returns to equity, but benefit from limited liability. When a bad aggregate shock has occurred, intermediaries of type j who are unable to pay the promised rate of return to depositors declare bankruptcy. Equity holders receive no return on their investments, while the returns to depositors are covered by deposit insurance. Limited liability introduces an asymmetry in that it allows the high-risk intermediary to make investment decisions that bring large profits in good times, while being shielded from losses in bad times. In our numerical experiments, only the high-risk intermediaries go bankrupt.

For a given policy, $p(s^t)$, the equilibrium can either have an unconstrained repo market or a constrained repo market. If all financial intermediaries choose to pledge only a fraction of bonds as collateral in the repo market, i.e. $\tilde{b}_j(s^t) < b(s^t)$, we refer to the equilibrium as having an *unconstrained repo market*. An equilibrium with a *constrained repo market* is one in which either high-risk or low-risk intermediaries pledge all their bond holdings as collateral. Numerically, we find that when the interest rate policy is chosen optimally, the equilibrium always has a constrained repo market. The intuition is that optimal policy aims to restrict risk taking of high-risk financial intermediaries, who otherwise may take advantage of their limited liability and overinvest in risky projects. An effective way to restrict risk taking and potential bankruptcy is to limit the amount of bonds, so that collateral for future trading in the repo market is scarce. We note that in all numerical experiments discussed in Section 4—including those in which interest rate policies deviate from the optimum—the equilibrium has a constrained repo market.

Due to the limited liability of financial intermediaries and the possibility of a constrained repo market, we need to employ non-linear techniques to solve our model. We use a collocation method with occasionally binding non-linear constraints (for details, see Appendix

A.2).

2.7.2 Bond Prices and the Return to Deposits

Proposition 1 *The equilibrium bond prices and the return to deposits satisfy:* $p\left(s^{t-1}\right) = \tilde{p}\left(s^{t-1}\right)$ *and* $R^{d}\left(s^{t-1}\right) \geq \frac{1}{p\left(s^{t-1}\right)}$. *The last inequality is strict in the case of a constrained repo market.*

Proof. These results follow from the first order conditions of the financial intermediaries' problems. Appendix A.3 outlines the proof. ■

In the model, there are no financial frictions or regulatory constraints that would make bond prices and repo prices different.[18] In addition, returns to deposits are weakly greater than returns to bonds, since otherwise there would be a profit opportunity. Namely, an intermediary would have incentives to pay a slightly higher deposit return to attract additional deposits and be able to invest more into bonds. The result $R^{d}\left(s^{t-1}\right) \geq \frac{1}{p\left(s^{t-1}\right)}$ can also be interpreted in terms of the option value provided by bonds in this economy. Bonds have value to intermediaries because they can be retraded on the repo market. Whenever some intermediaries are constrained in the amount of collateral they hold, bonds carry a discount: $R^{d}\left(s^{t-1}\right) > \frac{1}{p\left(s^{t-1}\right)}$. However, if both high-risk and low-risk intermediaries have sufficient bonds, the option value of bonds is zero: $R^{d}\left(s^{t-1}\right) = \frac{1}{p\left(s^{t-1}\right)}$.

Proposition (1) is important for two reasons. First, it shows that interest rate policy has a direct effect on the repo market. In fact, the close relationship we obtain between policy, $1/p\left(s^{t-1}\right)$, and the repo rate, $1/\tilde{p}\left(s^{t-1}\right)$, is supported by U.S. evidence, as shown in Bech, Klee, and Stebunovs (2010). Second, the return to depositors is bounded below by the interest rate on government bonds. Thus, the interest rate policy not only affects the choices financial intermediaries make, but also affects the investment choices of households. In quantitative experiments, we find the latter effect to be weaker than the former.

[18] Introducing a capital regulation constraint, for example, would generate a wedge between the equilibrium bond price and the repo price.

2.7.3 Risk Taking: Measurement and Impact of Policy

We use our model to assess whether and how interest rate policy influences risk taking of intermediaries. To this end, we make the notion of risk taking precise. We define risk taking as the percentage deviation in resources invested in the high-risk projects in a competitive equilibrium relative to the social planner. Formally,

$$r\left(s^{t-1}\right) = \frac{k_h^{CE}\left(s^{t-1}\right) - k_h^{SP}\left(s^{t-1}\right)}{k_h^{SP}\left(s^{t-1}\right)} \cdot 100 \tag{4}$$

where superscripts $\{CE, SP\}$ denote whether the variable is part of the solution to the competitive equilibrium for a given interest rate policy or part of the social planner's problem. Here, $k_h^{SP}\left(s^t\right) = k^{SP}\left(s^t\right) + n^{SP}\left(s^t\right)$ is the capital that the social planner invests in the high-risk technology and $k_h^{CE}\left(s^{t-1}\right) \equiv k^{CE}\left(s^{t-1}\right) + \tilde{p}^{CE}\left(s^{t-1}\right)\tilde{b}_h^{CE}\left(s^{t-1}\right)$ is the capital invested in the high-risk projects in the competitive equilibrium.

A positive value of $r\left(s^{t-1}\right)$ in equation (4) tells us that there is excessive risk taking in the competitive equilibrium, while a negative value indicates too little risk taking. In numerical results, we plot the cyclical behaviour of risk taking, but also report an aggregate measure defined as the average over expansions and contractions, $r \equiv E\left[r\left(s^{t-1}\right)\right]$.

In what follows, we provide some intuition on how interest rate changes affect risk taking during an expansion or a contraction. In particular, we discuss how lower returns to safe bonds affect investments in risky projects in the bond market, as well as the portfolio reallocation between high-risk and low-risk intermediaries in the interbank market.

Purchases in the bond market are positively related to bond returns, which means that *all* intermediaries invest more capital into risky projects at low interest rates. However, the amount of risk taking assumed by financial intermediaries also depends on the volume of interbank market transactions. The effect of lower bond returns on repo market activity differs depending on the aggregate shock of the economy and on whether the collateral constraint of intermediaries binds or not. Here, we focus our discussion on the numerically relevant case when collateral constraints bind. The repo market allows socially beneficial reallocation of resources towards the more productive intermediaries, who lower their holdings of bonds to invest additional resources in their risky projects. Resources flow towards the high-risk intermediaries in an expansion and towards the low-risk intermediaries in a contraction. In a *constrained repo market* equilibrium, the portfolio reallocation between intermediaries is restricted due to scarce collateral (i.e. fewer bonds purchased in the bond market at low interest rates). During an expansion, high-risk intermediaries would like to invest more in high-risk projects, but they are constrained from borrowing more. By the same token, during a contraction, fewer resources are reallocated from the high-risk to the more productive low-risk intermediaries. Numerically, whenever interest rates are sufficiently below the optimal rates, risk taking is lower than the social optimum during an expansion, and higher than the social optimum during a contraction.

Empirically, expansion periods are longer than contractions. Our calibrated model is consistent with this fact. This means that, in our model with a constrained repo market, lower interest rates lead to less risk taking, on average, relative to the social planner problem. Section 4 and Figure 3 provide additional details on changes in risk taking at low interest rates when collateral constraints bind.

3 Calibration

This section outlines our approach for determining the various parameters of the model and describes the data we use. We calibrate the following parameters: β, θ, the aggregate shock transition matrix Φ, and π_h. We determine π_m, α, δ, $q_h(\overline{s})$, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$ using a minimum distance estimator. All parameter values are summarized in Tables 1 and 2.

The utility discount factor, β, is calibrated to ensure an annual real interest rate of 4% in our quarterly model. We obtain $\beta = 0.99$. The capital income share is determined using data from the U.S. National Income and Product Account (NIPA) provided by the Bureau of Economic Analysis (BEA) for the period 1947 to 2009. We find $\theta = 0.29$ for the business sector.

To calibrate the transition matrix for the aggregate state of the economy, we use the Harding and Pagan (2002) approach of identifying peaks and troughs in the real value added of the U.S. business sector, from $1947Q1$ to $2010Q2$.[19] We find 11 contractions with an average duration of 5 quarters. Hence, the probability of switching from a bad realization of the aggregate shock at time $t-1$ to a good realization at time t is $\phi\left(s_t = \overline{s} | s_{t-1} = \underline{s}\right) = 0.20$. Moreover, the probability of switching from an expansion period to a contraction is $\phi\left(s_t = \underline{s} | s_{t-1} = \overline{s}\right) = 0.055$. The calibrated transition matrix is

$$\Phi = \begin{bmatrix} \phi\left(s_t = \overline{s} | s_{t-1} = \overline{s}\right) & \phi\left(s_t = \underline{s} | s_{t-1} = \overline{s}\right) \\ \phi\left(s_t = \overline{s} | s_{t-1} = \underline{s}\right) & \phi\left(s_t = \underline{s} | s_{t-1} = \underline{s}\right) \end{bmatrix} = \begin{bmatrix} 0.945 & 0.055 \\ 0.2 & 0.8 \end{bmatrix}.$$

The idiosyncratic shock in the economy—the type of risky projects financial intermediaries invest in—is assumed to be i.i.d. to retain tractability of the numerical solution. The motivation behind the i.i.d. assumption is that the financial sector in the U.S. economy is complex and the subset of financial intermediaries who are considered the most risky changes considerably over time.

For this reason, it is difficult to determine the share of high risk financial intermediaries in the data. We set π_h equal to 15% and perform sensitivity analysis with respect to this parameter. In the context of the recent financial crisis, one can think of brokers and dealers as a proxy for high-risk intermediaries in the U.S. Under this assumption and using U.S. Flow of Funds data from 2000 to 2007, we find that financial assets of brokers and dealers were, on average 4% of the financial assets of all financial institutions and 20% of the financial assets of depository institutions.[20] Our benchmark value of π_h is between these two esti-

[19] The business cycles we identify closely mimic those determined by the NBER.

[20] We note that the 20% average masks a large variation, from 18% in early 2000s to 28% in the eve of the recent crisis.

mates. It should be noted that, while the assumption that brokers and dealers are high-risk intermediaries seems reasonable for the recent crisis, the widespread use of off-balance sheet activities among other institutions suggests that this definition may be too narrow.

Next, we determine the following 9 parameters: the importance of the nonfinancial sector, π_m, the fixed factor in the production function of the financial sector, α, the depreciation rate, δ, and the productivity parameters, $q_h(\overline{s})$, $q_h(\underline{s})$, $q_m(\overline{s})$, $q_m(\underline{s})$, $q_l(\overline{s})$, $q_l(\underline{s})$. The absolute level of productivity is not important in our model. As a result, we normalize the productivity of the high-risk intermediary in the good aggregate state, $q_h(\overline{s}) = 1$. We estimate the remaining eight parameters using eight data moments described below. Unless otherwise noted, we use quarterly data from $1987Q1$ to $2010Q2$. We focus on this time period because U.S. inflation was low and stable.

1. The first moment we target in our estimation procedure is the share of output produced by the nonfinancial sector. This pins down the value of π_m in our model. We identify our model's total output with the U.S. business sector value added published by the BEA. In addition, we identify the nonfinancial sector in our model with the U.S. corporate nonfinancial sector.[21] We aim to match the average value added share of the corporate nonfinancial sector of 66.9% observed in the U.S. since 1987.

2. The parameter α influences the returns to equity in our model's financial sector, which, in turn, depend on the equity to total assets ratio of the intermediaries. We use the equity to asset ratio for corporate financial businesses as a second data moment to target in our estimation. Using data from the U.S. Flow of Funds from $1987Q1$ to $2010Q2$, we find this ratio to be, on average, 19.83%.

[21] Note that we treat the remainder of the U.S. business sector, namely the corporate financial businesses and the noncorporate businesses, as the model's financial intermediation sector. In U.S. data, noncorporate businesses are strongly dependent on the financial sector for funding. In the past three decades, bank loans and mortgages were 60 to 80 percent of noncorporate businesses' liabilities. For simplicity, we do not model these loans, but rather assume that the financial intermediary is endowed with the technology of production of noncorporate businesses.

3. In our model, the depreciation rate is stochastic and is given by:

$$\frac{\pi_m q_{m,t} \delta k_{m,t} + (1-\pi_m)(\pi_h q_{h,t} \delta k_{h,t} + \pi_l q_{l,t} \delta k_{l,t})}{\pi_m k_{m,t} + (1-\pi_m)(\pi_h k_{h,t} + \pi_l k_{l,t})}$$

We determine the value of δ to ensure that the average depreciation rate in the model matches the data, namely 2.5% per quarter.

4. We target the peak-to-trough decline in real output in the business sector, averaged across all contraction periods since 1947. We detrend output by a constant growth trend to make it stationary. Then, using the turning points approach in Harding and Pagan (2002), we find the average decline in output to be 6.48%.

5. We aim to match a coefficient of variation for the U.S. business sector output of 3.75%. We calculate this statistic after removing a linear trend from the logarithm of output.

6. We target a coefficient of variation for U.S. household net worth of 8.17%. To obtain this statistics, we use U.S. Flow of Funds data and detrend the logarithm of household net worth using a polynomial of order three. We focus on net-worth because it is closely related to the state variable $w(s^t)$ in our model.

7. We aim to match a ratio of household deposits to total financial assets of 17.2%, as observed in U.S. Flow of Funds data.

8. Finally, we aim to match the recovery rate during bankruptcy. We use an estimate provided by Acharya, Bharath, and Srinivasan (2003), which states that, the average recovery rate on corporate bonds in the United States during 1982 to 1999 was 42 cents on the dollar.

We determine all eight parameters jointly using a minimum distance estimator to match the target moments above. Let Ω_i be a model moment and $\tilde{\Omega}_i$ be the corresponding data moment. Our procedure solves the problem $(P4)$ below, where the optimal price p^* is the solution to problem $(P3)$ shown in Section 2.4. Notice that in $(P4)$ we impose restrictions on the ordering of productivity parameters across the different technology types, as discussed in Section 2.2.

$$
\begin{aligned}
Q^* &= \arg \min_{\substack{Q=\{q_m(\overline{s}),q_m(\underline{s}),q_l(\overline{s}),\\ q_l(\underline{s}),q_h(\underline{s}),\delta,\alpha,\pi_m\}}} \sum_{i=1}^{8} \left(\frac{\Omega_i - \tilde{\Omega}_i}{\tilde{\Omega}_i} \right)^2 \qquad \text{(P4)} \\
\text{s.t.} &: \; q_h(\underline{s}) < q_m(\underline{s}) < q_l(\underline{s}) \leq q_l(\overline{s}) < q_m(\overline{s}) \leq q_h(\overline{s}) \text{ and} \\
& \quad \Omega_i \text{ is implied in a competitive equilibrium given policy } p^*
\end{aligned}
$$

We start out with a guess Q_1^* and solve for an optimal policy p^* using $(P3)$. Next, we take this optimal policy as given and choose parameters to minimize the distance between our model moments and the corresponding data moments, as shown in $(P4)$. This step yields Q_2^*. We continue the procedure till convergence is achieved. The reason for choosing this two-step procedure is because our model is highly nonlinear and the initial guess is very important in finding a competitive equilibrium solution. The solution guess we start with is the social planner's solution.

Tables 2 presents the estimated parameters. Table 3 shows that the model matches the data moments well. Notice that despite the assumption that depreciation is stochastic, the model is able to perfectly match the average depreciation observed in the data.

4 Results

4.1 Risk Taking and Welfare in the Model

In this section, we present results from the competitive equilibrium and contrast them with the optimal social planner solution.

Our first finding is that the social planner allocation cannot be implemented as a competitive equilibrium. We aim to find prices, including the interest rate policy, that would implement the social planner allocation as a competitive equilibrium in our model with financial and nonfinancial sectors. This would require that, in a bad aggregate state, the returns to deposits and bonds satisfy: $R^d < 1/p$, which violates the competitive equilibrium result derived in Proposition 1. The intuition for our finding is as follows. In a bad aggregate state, it is optimal to shift resources from high-risk to low-risk intermediaries, which are now relatively more productive. Implementing the social planner optimal allocation has two implications for competitive equilibrium prices. First, high-risk intermediaries would need to buy a large value of bonds in the repo market, so as to shift their portfolio away from their risky projects. To provide these incentives, bond returns need to be sufficiently high implying that bond prices need to be sufficiently low in a bad aggregate state. Second, returns to deposits need to be relatively low so that intermediaries can pay back depositors. In combination, prices would have to satisfy $R^d < 1/p$, which contradicts Proposition 1. Therefore, the social planner allocation cannot be implemented. The interpretation of this result is that interest rate policy alone cannot eliminate the moral hazard problem of the high-risk financial intermediaries.

Given that the social planner allocation is not implementable, we find the optimal bond price, $p^*\left(s^{t-1}\right)$, that maximizes the welfare of the representative consumer. Numerically, we solve $(P3)$ shown in Section 2.4 by taking the function $p\left(\cdot\right)$ from the space of linear spline functions.

We use two metrics to compare competitive equilibrium results to the social planner allocation. First, we use the risk taking measure defined in Section 2.7.3 to determine whether a particular interest rate policy implies too much or too little risk taking relative to the social planner. In addition, we consider a standard welfare measure. We define the lifetime consumption equivalent (LTCE) as the percentage decrease in the optimal consumption from the social planner allocation required to give the consumer the same welfare as the consumption from the competitive equilibrium with a given interest rate policy.

Experiment 1: Optimal interest rate policy, $\left[p^*\left(s^{t-1}\right)\right]^{-1}$. We optimize over the bond price policy function numerically, as discussed in Problem $P3$. Figure 1 presents simulation results for a sequence of one hundred random draws of the aggregate shock.

We find that optimal returns to safe bonds are procyclical. The reason is as follows. Returns to bonds are linked to returns to deposits (recall Proposition 1). In addition, returns to deposits are linked to expected returns to equity through non-arbitrage conditions. Low returns to bonds in contractions allow returns to deposits to be low and ensure that potential bankruptcy costs are minimized.

In addition, whenever returns to bonds are low, the supply of government bonds is also low (see second row of subplots in Figure 1). The idea behind this result is that the monetary authority uses open market operations to control interest rates. In contractions, the monetary authority lowers returns to bonds by purchasing government bonds. As a result, the price of bonds, p, increases and the amount of bonds, B, declines. The equilibrium value of government bonds, pB, falls, which reduces the value of collateral that can be used to borrow in contractions.

The third row of subplots in Figure 1 shows that there is excessive risk taking in the competitive equilibrium, as more resources are invested in high-risk projects compared to the amount allocated by the social planner ($k_{h,t}^{CE} > k_{h,t}^{SP}$). The excessive risk taking in the competitive equilibrium is mainly driven by periods with good realizations of the aggregate state, when the value of collateral, $p_t B_t$, is high and resources in the repo market are reallocated from the low-risk to the high-risk projects. Risk taking in contractions is lower than in expansions, but is still in excess of the social planner optimum. In contractions, the value of collateral limits the reallocation of resources from high-risk to low-risk projects, leaving high-risk intermediaries with higher than optimal investment in risky projects.

Lastly, Figure 1 shows that output produced in the competitive equilibrium is higher relative to the social optimum, because more resources are invested in productive high-risk projects. However, consumption paths in the two environments track each other closely.

To obtain measures of welfare losses and risk taking in our competitive equilibrium relative to the social planner, we average over the results of 500 simulations of 750 periods each. Table 4 shows that at the optimal interest rate policy, the excessive risk taking is $r = 4.97\%$.

That is, investments in high-risk projects are about 5 percent higher, on average, in the competitive equilibrium relative to the social planner allocation. The average is taken over expansion and contraction periods. The excessive risk taking leads to a small welfare loss, amounting to 0.0166% in LTCE.

There are a couple of other interesting lessons from this experiment. First, reallocation of resources via the repo market is beneficial, as it brings the economy closer to the social optimum. Shutting down the interbank repo market in the competitive equilibrium reduces welfare relative to the social planner by an amount equivalent to lowering consumption throughout the lifetime by about 1 percent. Second, at the optimal interest rate policy, government transfers to households are positive, on average, due to net revenues from issuing bonds. This is true despite the fact that the government provides deposit insurance at no cost, and it needs to tax households to guarantee deposit returns when the high-risk intermediaries become bankrupt.

Experiment 2: Level shifts in the optimal interest rate policy. We consider uniform upward or downward shifts in interest rates relative to those under the optimal policy. Namely, the schedules of bond returns we consider are: $\left[p^*\left(s^{t-1}\right)\right]^{-1} \pm \psi$, where $p^*\left(\cdot\right)$ is the optimal bond price and ψ is a constant, say 0.1 percentage points. The model's results under these alternate policies are contrasted with the results from Experiment 1 to illustrate how lower or higher than optimal interest rates change risk taking and welfare.

Figure 2 shows risk taking and welfare results for a wide range of values of ψ. Similar to the results displayed in Table 4, the welfare and risk taking in Figure 2 are averages over 500 simulations of 750 periods each. In both subplots, the x-axis shows the deviations from the optimal equilibrium policy, $\left[p^*\left(s^{t-1}\right)\right]^{-1}$ which we consider, ranging from -2 to $+2$ percentage points at annual rates. The optimal policy is the zero mark on the x-axis. We find that small deviations from the optimal policy, say 50 basis points, entail relatively small welfare losses, but sizable changes in risk taking. Moreover, higher than optimal bond returns lead to more risk taking, while lower than optimal bond returns lead to less risk taking (also see Table 4).

As conjectured in Section 2.7, the equilibrium with optimal interest rate policy (the square dot in Figure 2) features a constrained repo market. That is, in good times, when the aggregate shock is high ($\overline{s}$), high-risk financial intermediaries hold no bonds on their balance sheet after the repo market transactions take place. Similarly, in bad times, low-risk financial intermediaries sell all their bonds on the repo market. In fact, even when deviations in the optimal policy are considered, the equilibrium also features a constrained repo market.

Figure 2 shows that reductions in the interest rate for bonds below the optimum not only lower risk taking, but also eliminate excessive risk in our model compared to the social optimum. Here is the intuition for this result. When the aggregate productivity is expected to be high, high-risk intermediaries have insufficient funds allocated to risky investments and need to trade bonds in the repo market to adjust their portfolios. However, a low quantity and a low value of collateral constrain their choices. Thus, in good times, investments in high-risk projects in the competitive equilibrium are lower than the social planner optimum (see simulation "optimal policy minus 50 basis points at an annual rate" shown in Figure 3). As a result, aggregate risk taking, defined as an average over expansions and contractions in our simulations, is lower than the social optimum, whenever policy rates are sufficiently below the optimum.

The final observation from Figure 2 is that large reductions in bond returns result in a shutdown of the repo market in good times. In our numerical experiments, deviations of at least 170 basis points below the optimal policy lead intermediaries to demand no bonds in good times. Even though high-risk intermediaries invest all resources in risky assets in good times, they are still underinvesting relative to the social planner. As the bond market shuts down in good times, the households give slightly more resources to financial intermediaries. This result generates the kink in the subplots of Figure 2. To the left of the kink, risk taking is still lower compared to the social planner, but less so.

Sensitivity analysis: Share of High-Risk Intermediaries In the numerical results presented so far, high-risk financial intermediaries represented 15 percent of all intermediaries (or 4.35 percent of all firms in the economy). We examine how our results on welfare and risk taking change when high-risk intermediaries are a smaller or bigger fraction of all intermediaries, i.e. $\pi_h = 13\%$ or $\pi_h = 17\%$. In both of these cases, we re-optimize over the policy function.

The results from the sensitivity analysis are reported in Table 5. The quantitative results change with π_h. Higher π_h leads to slightly higher risk taking and slightly larger welfare losses at the optimal interest rate policy. However, the qualitative result remains the same: lower than optimal interest rates lead to lower risk taking relative to the social planner.

5 Conclusion

The recent financial crisis has stirred interest in the relationship between lower than optimal interest rates and the risk taking behavior of financial institutions. We examine this relationship in a dynamic general equilibrium model that features deposit insurance, limited liability of financial intermediaries, as well as heterogeneity in the riskiness of intermediaries' portfolios.

There are two channels through which interest rate policy influences risk taking in our model. The *portfolio channel* illustrates the idea that lower than optimal policy rates reduce the returns to safe assets and lead intermediaries to shift investments towards riskier assets. In turn, given fewer bond purchases in the bond market, intermediaries have less collateral available for repo market transactions. Hence, *the collateral channel* constrains the ability of intermediaries to take on more risk through the repo market, after they receive further information regarding the riskiness of their projects. We calibrate our model to U.S. data, and show that, our decentralized economy with optimal interest rate policy features excessive risk taking and has welfare that is very close, though below, the social optimum. While both risk taking channels lead to important changes in the intermediaries' portfolios, we find that, for reasonably large variations around the optimal policy, the collateral channel dominates quantitatively. Thus, lower than optimal interest rates lead to less risk taking.

There are different potential extensions to our work. First, our paper focuses on a time consistent interest rate policy. It may be worthwhile to evaluate the consequences of a departure from this assumption. In addition, our model can be easily extended to allow for financial regulations. For example, a simple Basel II type regulation, would require financial intermediaries to hold at least 8 percent equity against risky capital investments. This regulation would constrain portfolio choices of intermediaries in the bond market, and would also introduce a wedge between the bond price and the repo price, and thus have an impact on collateralized borrowing of intermediaries. We conjecture that Basel II type capital regulation would be successful in reducing the excessive risk taking of intermediaries at the optimal interest rate policy, but may also reduce welfare relative to the social optimum. To minimize welfare losses, it may be necessary to analyze state dependent optimal capital regulation or to consider alternative financial regulation instruments.

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A Appendix

A.1 Timing of Model Events

Let $s_t \in \{\overline{s}, \underline{s}\}$ be the aggregate shock at time t. Let $s^t = (s_1, s_2, ...s_t)$ be the history of the aggregate shock up to time period t. Note that $s^t = (s^{t-1}, s_t)$. The timing of the economy is as follows.

- Each nonfinancial firm enters period t with equity $M(s^{t-1})/\pi_m$, capital $k_m(s^{t-1})$ and labour $l_m(s^{t-1})$. Each financial intermediary enters period t with $\tilde{b}_j(s^{t-1})$ safe assets, $k_j(s^{t-1})$ risky assets, $d(s^{t-1})$ deposits, equity $z(s^{t-1})$ and labour $l(s^{t-1})$.
- At the beginning of period t, the aggregate shock s_t realizes and financial and non-financial firms find out their current productivity shocks: $q_j(s_t)$ for intermediaries of type $j \in \{h, l\}$ and $q_m(s_t)$ for nonfinancial firms. All firms produce output using the capital that has been allocated to production at the end of period $t-1$.
- Nonfinancial firms pay wage income $W_m(s^t)\, l_m(s^{t-1})$ and equity returns $R^m(s^t)\, k_m(s^{t-1})$.
- Financial intermediaries pay state contingent returns to labour and may declare bankruptcy, if they are unable to pay the return on deposits, $R^d(s^{t-1})\, d(s^{t-1})$. Notice that bankruptcy occurs after the intermediaries know the riskiness of their projects j and the current aggregate shock s_t has realized. Bankrupt intermediaries are liquidated. Their equity holders receive no equity returns and the government steps in to guarantee the rate of return on deposits.
- The government uses lump-sum taxes or transfers $T(s^t)$ to cover expenses and to balance its budget.
- Household wealth, $w(s^t)$, is realized. Households use current wealth to purchase consumption and make investments that will pay off next period: $M(s^t)$, $Z(s^t)$ and $D_h(s^t)$ and supply labour inelastically to financial intermediaries and nonfinancial firms.
- Each nonfinancial firms receives equity $M(s^t)/(1-\pi_m)$.

- Financial intermediaries receive deposits $d(s^t)$ and equity $z(s^t)$.
- At the end of period t, financial intermediaries allocate the resources received from household into bonds and new risky projects. Subsequently, they find out the type of risky project $j \in \{h, l\}$ they invested in and trade repurchasing agreements on government bonds in the repo market. The resulting investments into the risky projects pay returns at the beginning of period $t+1$, after shock s_{t+1} realizes.

A.2 Computation of Equilibrium

We compute a recursive formulation of the model, where the state variables at each time period are the aggregate state, s_t, and the household wealth, w_t. Our strategy is to solve for consumption as a function of the state variables using a collocation method with linear spline functions. To improve the accuracy and the speed of the computation, we use of the endogenous grid method idea of Carroll (2006).

We separate the household problem into two parts: a portfolio choice problem and an intertemporal problem. The household's portfolio choice allocates resources to the nonfinancial and financial sectors to equate expected returns of investing in these sectors. Then, given the overall resources allocated to the financial sector, the split between equity and deposits is determined to equalize expected returns from the two types of investments (for details, see Carroll (2011), Section 7 on multiple control variables).

There are two main challenges when solving the financial sector problem. First, some financial intermediaries may be constrained in their repo market trades and, second, financial intermediaries may go bankrupt when the aggregate state is realized. We consider all the possible combinations in sequence and verify which is an equilibrium. For example, an ex-ante assumption that we make is that when the aggregate state switches from good to bad, high risk intermediaries are constrained in their repo market trade and go bankrupt, while the low risk intermediaries are unconstrained and do not go bankrupt. After solving the financial intermediaries' problems, we check whether the ex-post outcome is consistent with the assumed ex-ante behavior.

A.3 Sketch of Proof for Proposition 1

To simplify notation in our derivations, we use subscripts as a short hand notation for the entire history, s^{t-1}. For example, $\tilde{b}_{j,t-1} \equiv \tilde{b}_j\left(s^{t-1}\right)$ and $b_{t-1} \equiv b\left(s^{t-1}\right)$.

Deriving the relationship between bond prices and the return to deposits in our model involves studying three possible outcomes on the repo market. Transactions of bonds either satisfy: (i) $\tilde{b}_{j,t-1} < b_{t-1}$ for both $j \in \{h, l\}$ or (ii) $\tilde{b}_{h,t-1} = b_{t-1}$ and $\tilde{b}_{l,t-1} < b_{t-1}$ or (iii) $\tilde{b}_{l,t-1} = b_{t-1}$ and $\tilde{b}_{h,t-1} < b_{t-1}$. Here, we sketch the proof of Proposition 1 for case (ii). The proof is obtained in an analogous fashion for cases (i) and (iii) and is omitted here for brevity.[22]

In case (ii), the high-risk intermediary increases the amount of resources allocated to risky investments by selling all bond holdings in the repo market.

Step 1: Some Key Relationships

In finding and characterizing the equilibrium, it is useful to define the share of resources a financial intermediary retains for risky investment in the bond market, call it x_{t-1}. Then,

$$k_{t-1} = x_{t-1}\left(z_{t-1} + d_{t-1}\right) \tag{5}$$

$$b_{t-1} = \frac{1 - x_{t-1}}{p_{t-1}}\left(z_{t-1} + d_{t-1}\right) \tag{6}$$

where the second equation was obtained from equation (1).

For the case presented here, high-risk intermediaries use all their bonds as collateral in

[22] The full derivation is available upon request from the authors.

the repo market, while low-risk intermediaries give resources against this collateral. We have:

$$\tilde{b}_{h,t-1} = b_{t-1} = \frac{1-x_{t-1}}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right) \tag{7}$$

$$\tilde{b}_{l,t-1} = -\frac{\pi_h}{\pi_l} b_{t-1} = -\frac{\pi_h}{\pi_l}\frac{1-x_{t-1}}{p_{t-1}}\left(z_{t-1}+d_{t-1}\right) \tag{8}$$

Lastly, using equations $(5)-(8)$, the resources allocated to risky investments by high-risk and low-risk intermediaries after the repo market trades are given by (9) and (10).

$$k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{h,t-1} = \left[x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right) \tag{9}$$

$$k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{l,t-1} = \left[x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right]\left(z_{t-1}+d_{t-1}\right) \tag{10}$$

Step 2: Equilibrium Conditions for the Financial Sector

In what follows, we make use of the equilibrium result $l_{t-1}=1$.

We rewrite the repo market problem given in $(P2)$ as below:

$$\max_{\tilde{b}_{j,t-1}} \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \left(\begin{array}{l} q_{j,t}\left[\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta}+(1-\delta)\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)\right] \\ +\left(b_{t-1}-\tilde{b}_{j,t-1}\right)-R^d_{t-1}d_{t-1}-W_{j,t} \end{array} \right)$$

where $\tilde{b}_{j,t-1} \in \left[-\frac{k_{t-1}}{\tilde{p}_{t-1}}, b_{t-1}\right]$ and $1_{j,t}$ is an indicator function given by $1_{j,t} \equiv \begin{cases} 1 \text{ if } V_{j,t}>0 \\ 0 \text{ otherwise} \end{cases}$

The first order conditions with respect to bond trades, $\tilde{b}_{h,t-1}$ and $\tilde{b}_{l,t-1}$, are given by:

$$\sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t\left\{q_{j,t}\tilde{p}_{t-1}\left[\theta\left(k_{t-1}+\tilde{p}_{t-1}\tilde{b}_{j,t-1}\right)^{\theta-1}+1-\delta\right]-1\right\}-\mu_{j,t-1}=0 \tag{11}$$

where $\mu_{j,t-1}$ for $j\in\{h,l\}$ are the Lagrange multipliers on the constraints $\tilde{b}_{j,t-1}\le b_{t-1}$ and they satisfy the complimentary slackness conditions: $\mu_{j,t-1}\ge 0$, $\mu_{j,t-1}\left(b_{t-1}-\tilde{b}_{j,t-1}\right)=0$.[23]

[23] In equilibrium, the constraint $-\frac{k_{t-1}}{\tilde{p}_{t-1}}\le \tilde{b}_{j,t-1}$ does not bind as returns to capital invested in risky projects

Notice that for the case we are analyzing here, $\mu_{l,t-1} = 0$ and $\mu_{h,t-1} \geq 0$. Using this, along with the expressions in (9) and (10), we can rewrite equation (11) for $j \in \{h, l\}$ as (12) and (13) below:

$$\theta \left[\left(x_{t-1} - \frac{\pi_h}{\pi_l} \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta = \frac{\sum_{s^t|s^{t-1}} 1_{l,t} \lambda_t}{\tilde{p}_{t-1} \sum_{s^t|s^{t-1}} 1_{l,t} \lambda_t q_{l,t}} \quad (12)$$

$$\theta \left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta-1} + 1 - \delta \geq \frac{\sum_{s^t|s^{t-1}} 1_{h,t} \lambda_t}{\tilde{p}_{t-1} \sum_{s^t|s^{t-1}} 1_{h,t} \lambda_t q_{h,t}} \quad (13)$$

Notice that equation (12) can be equivalently written as:

$$\left[x_{t-1} - \frac{\pi_h}{\pi_l} \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right] (z_{t-1} + d_{t-1}) = \left[\frac{1}{\theta} \left(\frac{\sum_{s^t|s^{t-1}} 1_{l,t} \lambda_t}{\sum_{s^t|s^{t-1}} 1_{l,t} \lambda_t q_{l,t} \tilde{p}_{t-1}} - 1 + \delta \right) \right]^{\frac{1}{\theta-1}} \quad (14)$$

Using equations (5) − (10) we rewrite the bond market problem $(P1)$ as below:

$$\max_{\substack{x_{t-1} \in [0,1] \\ d_{t-1} \geq 0}} \sum_{j \in \{h,l\}} \pi_j \sum_{s^t|s^{t-1}} \lambda_t V_{j,t}$$

subject to :

$$V_{l,t} = \max \left\{ \begin{array}{l} q_{l,t} \left[\left(x_{t-1} - \frac{\pi_h}{\pi_l} \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta} \\ + q_{l,t} (1 - \delta) \left(x_{t-1} - \frac{\pi_h}{\pi_l} \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \\ + \frac{1}{\pi_l} \frac{(1 - x_{t-1})}{p_{t-1}} (z_{t-1} + d_{t-1}) - R^d_{t-1} d_{t-1} - W_{l,t},\ 0 \end{array} \right\}$$

$$V_{h,t} = \max \left\{ \begin{array}{l} q_{h,t} \left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) \right]^{\theta} \\ + q_{h,t} (1 - \delta) \left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}} (1 - x_{t-1}) \right) (z_{t-1} + d_{t-1}) - R^d_{t-1} d_{t-1} - W_{h,t},\ 0 \end{array} \right\}$$

The first order conditions with respect to x_{t-1} and d_{t-1} are given by (15) and (16),

would become infinite.

respectively.[24]

$$\frac{1}{p_{t-1}} \sum_{s^t|s^{t-1}} \lambda_t 1_{l,t} = \tag{15}$$
$$\left\{\theta \left[\left(x_{t-1} - \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1 - \delta\right\}\left(1 + \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}$$
$$+\left\{\theta \left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1 - \delta\right\}\left(1 - \frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}$$

$$R^d_{t-1} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t = \tag{16}$$
$$\left\{\theta \left[\left(x_{t-1} - \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1 - \delta\right\}\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t}$$
$$+\left\{\theta \left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1 - \delta\right\}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}$$

Step 3: Bond Prices

Using (14), we rewrite the equilibrium condition for the choice of x_{t-1}, equation (15), as below:

$$\left(\frac{1}{p_{t-1}} - \frac{\pi_l}{\tilde{p}_{t-1}}\left(1 + \frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\right)\sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t$$
$$= \left\{\theta \left[\left(x_{t-1} + \frac{\tilde{p}_{t-1}}{p_{t-1}}(1-x_{t-1})\right)(z_{t-1}+d_{t-1})\right]^{\theta-1} + 1 - \delta\right\}\left(1 - \frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}$$

Using $\pi_l + \pi_h = 1$, we can simplify the left hand side of the above equation and write it equivalently as:

$$\left(1 - \frac{\tilde{p}_{t-1}}{p_{t-1}}\right)\cdot \Xi = 0 \tag{17}$$

[24] In order to obtain equation (16), we derive the first order condition with respect to deposits and simplify it by using the expression in (15).

$$\Xi \equiv \left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t}+\frac{\pi_l \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t}{\tilde{p}_{t-1}}.$$

Notice that $\Xi > 0$, unless all financial intermediaries go broke. Then, equation (17) implies that, the bond price and the repo price are equated, $\tilde{p}_{t-1} = p_{t-1}$.

Step 4: Bond Price and Return to Deposits

Next, we subtract equation (16) from equation (15) and find:

$$\begin{aligned} & \frac{1}{p_{t-1}} \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t - R^d_{t-1} \sum_{j\in\{h,l\}} \pi_j \sum_{s^t|s^{t-1}} 1_{j,t}\lambda_t \qquad (18) \\ = & \left\{\theta\left[\left(x_{t-1}-\frac{\pi_h}{\pi_l}\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\pi_h\frac{\tilde{p}_{t-1}}{p_{t-1}} \sum_{s^t|s^{t-1}} 1_{l,t}\lambda_t q_{l,t} \\ & -\left\{\theta\left[\left(x_{t-1}+\frac{\tilde{p}_{t-1}}{p_{t-1}}\left(1-x_{t-1}\right)\right)\left(z_{t-1}+d_{t-1}\right)\right]^{\theta-1}+1-\delta\right\}\frac{\tilde{p}_{t-1}}{p_{t-1}}\pi_h \sum_{s^t|s^{t-1}} 1_{h,t}\lambda_t q_{h,t} \end{aligned}$$

Using (12) and (13), equation (18) becomes $R^d_{t-1} \geq \frac{1}{p_{t-1}}$. This completes the proof of Proposition 1 for the case in which the high-risk intermediary sells all bonds in the repo market. The other cases are derived analogously, but are omitted here for brevity.

B Tables

Table 1: Calibrated Parameters

Parameter/Value	Moment[1]
$\beta = \left(\frac{1}{1.04}\right)^{1/4}$	Real interest rate of 4 percent
$\theta = 0.29$	Capital income share
$\Phi = \begin{bmatrix} 0.945 & 0.055 \\ 0.20 & 0.80 \end{bmatrix}$	Average length of expansions/contractions of business sector
$\pi_l = 0.85$, $\pi_h = 1 - \pi_l = 0.15$	Sensitivity analysis

[1]See Section 3 for details on the sources of data.

Table 2: Estimated Parameters

Parameter	Value
The following parameters are determined jointly to match the moments in Table 3	
Share of nonfinancial firms	$\pi_m = 0.7097$
Depreciation rate	$\delta = 0.0261$
Fixed factor income share	$\alpha = 0.000363$
Productivity parameters	
nonfinancial firms	$q_m(\overline{s}) = 0.9654$
	$q_m(\underline{s}) = 0.9376$
low-risk financial firms	$q_l(\overline{s}) = 0.9407$
	$q_l(\underline{s}) = 0.9397$
high-risk financial firms	$q_h(\overline{s}) = 1$ (normalization)
	$q_h(\underline{s}) = 0.3994$

Table 3: Comparison of Data and Model Moments

Moment	Data in %	Model in %
Coefficient of variation of output[1]	3.75	3.92
Coefficient of variation of household net worth[2]	8.17	8.07
Average maximum decline in output during contractions[3]	6.48	7.26
Average deposits over total household financial assets[2]	17.2	19.5
Recovery rate in case of bankruptcy[4]	42.0	41.6
Mean output share of corporate nonfinancial sector	66.9	73.1
Average capital depreciation rate in economy	2.5	2.5
Equity to asset ratio of the financial sector[2]	19.8	19.2

[1]Output is measured as the value added for the business sector from 1987Q1 to 2010Q2. This reference period is used for the other moments as well, unless otherwise stated. [2]Data on household net worth, deposits, equity and financial assets are from the U.S. Flow of Funds accounts. [3]The decline in output during contractions takes the growth trend into account. [4]The recovery rate in bankruptcy is from Acharya, Bharath, and Srinivasan (2003).

Table 4: Model Welfare and Risk Taking Relative to the Social Planner[1]

Experiment	LTCE[2] in %	Risk taking[3] in %
Optimal interest rate policy	−0.0166	4.97
Optimal policy −0.5 percentage points	−0.0265	−4.75
Optimal policy +0.5 percentage points	−0.0260	16.35

[1]The statistics are averages over 500 simulations of 750 periods each of the model economy and the social planner's problem. [2]Lifetime Consumption Equivalents (LTCE) is the percentage decrease in the optimal consumption from the social planner problem needed to generate the same welfare as the competitive equilibrium with a given interest rate policy. [3]Risk taking is the percentage deviation in the amount of resources invested in the high-risk projects in the competitive equilibrium relative to the social planner's choice. The numbers reported here are averages over expansions and contractions in our calibrated model. A positive number indicates too much risk taking, on average, relative to the social planner, while a negative number indicates less risk taking.

Table 5: SENSITIVITY ANALYSIS FOR FRACTION OF HIGH RISK INTERMEDIARIES
WELFARE AND RISK TAKING RESULTS RELATIVE TO THE SOCIAL PLANNER[1]

	LTCE[2] in %		
Experiment / π_h value	0.13	**0.15**	0.17
Optimal policy −0.5 percentage points	−0.0223	**−0.0265**	−0.0313
Optimal interest rate policy	−0.0122	**−0.0166**	−0.0219
Optimal policy +0.5 percentage points	−0.0250	**−0.0260**	−0.0291
	Risk taking[3] in %		
Experiment / π_h value	0.13	**0.15**	0.17
Optimal policy −0.5 percentage points	−6.30	**−4.75**	−3.38
Optimal interest rate policy	5.02	**4.97**	5.11
Optimal policy +0.5 percentage points	18.66	**16.35**	14.83

[1]The statistics are averages over 500 simulations of 750 periods each of the model economy and the social planner's problem. [2,3]See definitions given in notes to Table 4.

C Figures

Figure 1: Simulation Results: Model with Optimal Interest Rate Policy and Social Planner Allocations

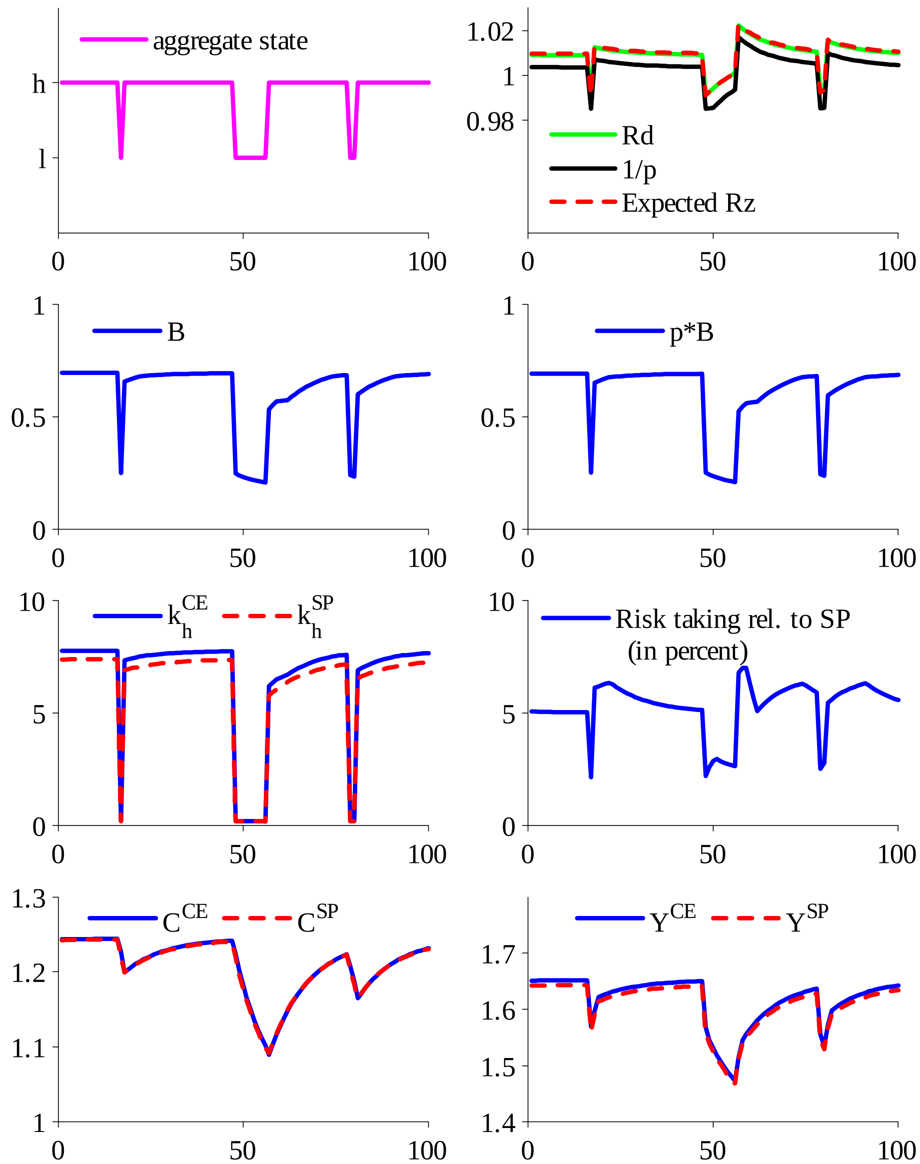

Figure 2: Model Welfare and Risk Taking Relative to the Social Planner

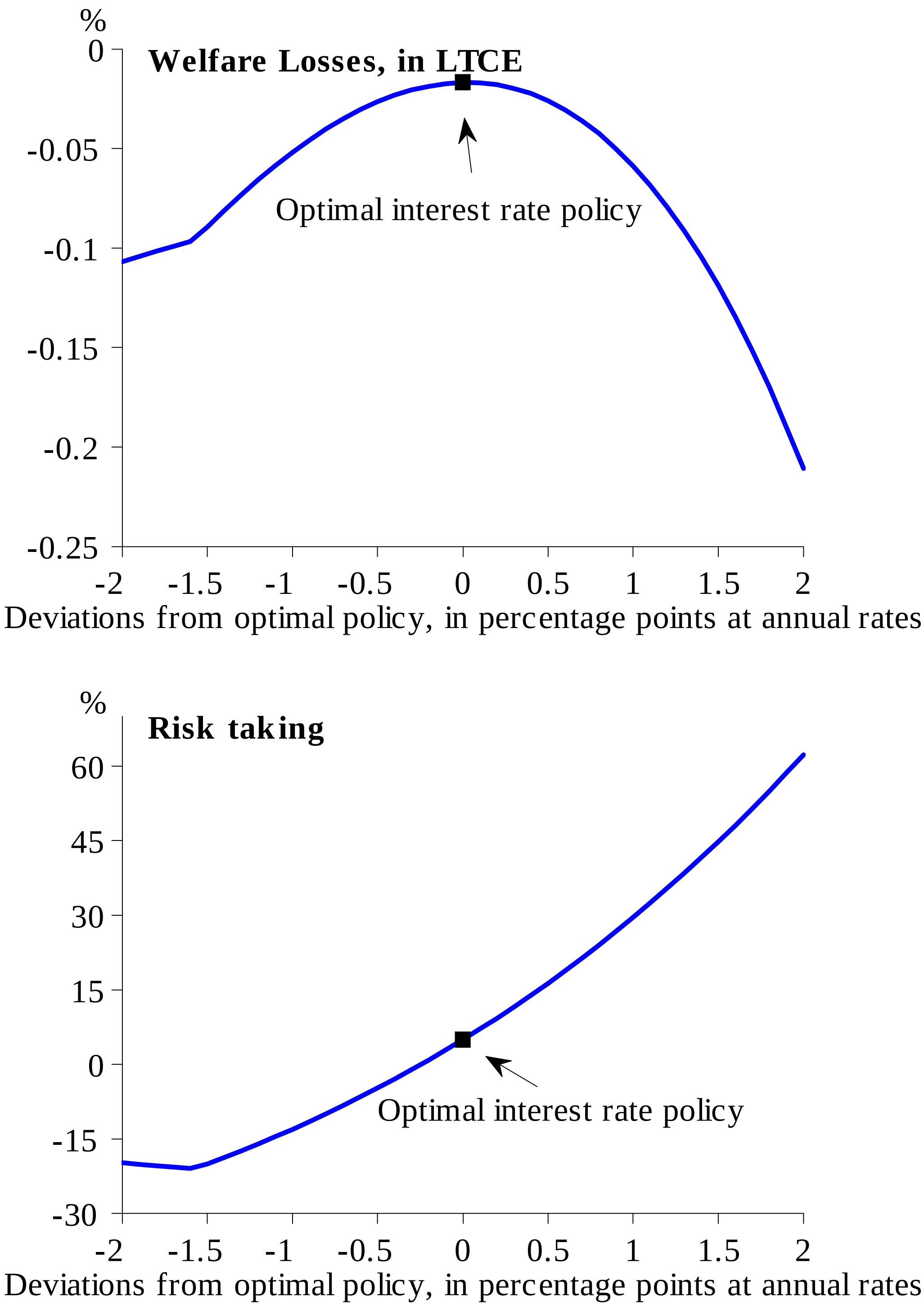

Figure 3: Simulation Results: Model with Lower than Optimal Interest Rates and Social Planner Allocations