# Imported Inputs and the Gains from Trade*


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## Abstract

The bulk of international trade takes place in intermediate inputs as opposed to goods for final consumption. Studies of firm-level data show that there is substantial heterogeneity in the share of inputs that are imported by different firms, and that a firm's productivity increases with the quantity and variety of inputs that it imports. This paper develops a model to quantify the contributions of firm-level productivity gains from importing to aggregate productivity and welfare gains from trade. In the model, heterogeneous firms choose the fraction of their inputs to import. Importing a higher fraction of inputs raises firm-level productivity, but requires higher up-front fixed costs. Therefore, firms with different inherent profitability will vary in how much they import and the productivity they gain from doing so. This heterogeneity provides aggregate productivity and welfare gains from trade that would not exist in a world in which firms used identical input bundles. These gains are consistent with data on historical trade liberalization episodes that show large firm-level productivity gains attributed to higher imports of intermediate inputs.



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# 1 Introduction

Intermediate inputs comprise about half of international trade in goods for most industrialized countries, and international trade theory has for a long time dealt explicitly with goods used in production as distinct from trade in goods for final consumption.[1] In addition, models of trade in intermediate inputs have been useful in studying the relationship between trade and growth.[2] Until recently, the bulk of this work has been based on models in which all producers use an identical bundle of imported and domestic goods. However, a recent literature examining firm- and plant-level data has found that imported inputs are concentrated among relatively few producers, and there is substantial heterogeneity in import shares among them.[3] Understanding the producer-level decisions behind these outcomes is important for understanding the behavior of trade in intermediate goods at the aggregate level and how the gains from importing these goods are distributed across different producers.

This paper develops a model of trade in intermediate goods in which heterogeneous plants decide on how much of their inputs to import. In the model, plants producing a final good use a continuum of intermediate inputs, any of which can be produced domestically or abroad. Intermediate goods are produced in different countries with different technologies, as in the Ricardian model of Eaton and Kortum (2002). For each input, a plant chooses whether to pay a cost to optimally source the input from the cheapest location, or to simply purchase it domestically. A fraction of those inputs sourced will be imported. Buying a higher fraction of inputs from the cheapest source lowers the prices paid for intermediate inputs, so a plant that sources (and hence imports) a higher fraction of inputs appears more productive - it produces more output with the same expenditure on inputs - than a plant that sources fewer inputs. Plants differ in their underlying efficiency, and more efficient plants choose to source a higher share of their inputs, while the least efficient choose to purchase everything domestically. Importing plants, therefore, are larger and more productive, both because they take advantage of the productivity gains of optimally sourcing inputs, and because they tend to be more efficient producers. The model generates cross-sectional dispersion in import shares and plant size, which depends on the parameters governing underlying heterogeneity and the benefits and costs of importing. I calibrate the model to match moments of the distributions of import shares and sizes among manufacturing plants in Chile, and then analyze the response of plant-level and aggregate productivity and welfare to changes in the

[1] Trade in intermediate goods plays a role in, among others, Sanyal and Jones (1982), Ethier (1982), Krugman and Venables (1995), and Eaton and Kortum (2002).

[2] See, for example, Grossman and Helpman (1990) and Rivera-Batiz and Romer (1991).

[3] Kasahara and Lapham (2007) have documented these facts for Chile, while similar facts can be found in Kurz (2006) and Bernard, Jensen, and Schott (2009) for the US, Biscourp and Kramarz (2007) for France, Amiti and Konings (2007) for Indonesia, and Halpern, Koren, and Szeidl (2009) for Hungary.

variable costs of importing.

The model can account for the large within-plant productivity gains following trade liberalization found, for example, by Amiti and Konings (2007) in Indonesian manufacturing firms after tariffs were reduced in the mid 90s. In my model, as the relative price of imports falls, plants raise the share of inputs they import, taking advantage of higher cost savings, which increases the effective productivity with which they produce output. This productivity gain is quantitatively similar in magnutide to that estimated by Amiti and Konings (2007).

A literal interpretation of the production technology in my model is that imports are perfect substitutes for domestic inputs, but are available at a lower cost, so that importing a larger share lowers the average cost of production. More broadly, imported inputs could also yield productivity gains because imports are of higher quality than comparable domestic inputs, or because imported goods are imperfect substitutes for domestic goods. The quality explanation, for example discussed in Grossman and Helpman (1991), is studied in plant-level data for Mexico by Kugler and Verhoogen (2009). Imperfect substitutability would generate gains from input variety as in Ethier (1982) and Romer (1990). Halpern, Koren, and Szeidl (2009) use data on the number of goods Hungarian firms import to measure the relative magnitudes of the quality and substitutability channels. Goldberg, Khandelwal, Pavcnik, and Topalova (2010) also measure the benefits of input variety using data on Indian firms, though they measure the effects on the number of products firms produce, not on their productivity. Using data from Argentina, Gopinath and Neiman (2011) show that the adjustment of the number of inputs firms import accounts for high-frequency movements in productivity. In a model that combines the decisions to import and export, Kasahara and Lapham (2007) assume plants gain from importing through the variety effect, but the number of imports each importing plant uses is fixed.

The fixed costs of importing in my model are meant as a stand-in for costs of using imported goods that do not depend on the amount purchased. These can include the costs of finding suppliers, or the costs of testing and finding out whether an imported product is an appropriate substitute for a domestic one. In my model, fixed costs are necessary to get the result that only some plants import, and the shape of the fixed cost as a function of the share of inputs imported generates the differences in import shares observed in the data. These results follow in the same way that models of producers' decisions to export, such as those in Melitz (2003) and Chaney (2008) have used fixed costs to segment firms into exporters and nonexporters. The form of the fixed costs of importing I use generates a profit maximization problem at the plant level that shares features of the model in Arkolakis (2008), in which a producer pays an increasing cost to export to a larger fraction of consumers.

Section 2 below sets out the model. Section 3 relates parameters to the cross-sectional distribution of import shares and plant size, and performs a calibrated numerical simulation. In this section, I also perform counterfactual exercises to evaluate the model's response to a trade liberalization.

# 2 Model

The model economy consists of $N$ countries in which production takes place in two stages: internationally tradeable intermediate goods are produced with labor, and a final, nontradable good is produced using labor and intermediate goods. The final good is produced by heterogeneous plants that differ in their efficiency and in the fraction of goods they choose to import. All producers are perfectly competitive.

## 2.1 Production and prices of intermediate goods

Intermediate goods production is as in the Ricardian model of Eaton and Kortum (2002). Producers in each country have technologies to produce a continuum of goods labelled $\omega \in [0,1]$. Good $\omega$ can be produced in country $i$ with labor, with efficiency $z_i(\omega)$. Denoting the wage rate in country $i$ as $w_i$, the cost of producing a unit of good $\omega$ in country $i$ is $\frac{w_i}{z_i(\omega)}$. As in Eaton and Kortum (2002), $z_i(\omega)$ is the realization of a random variable drawn independently and identically across $\omega$ and $i$ from a Frechet distribution,

$$\Pr\left(z_i(\omega) \leq z\right) = e^{-T_i z^{-\theta}}$$

Here, $T_i$ controls the level and $\theta$ the dispersion of efficiency draws within country $i$.

Intermediate goods are tradeable, but producers face proportional trade costs when selling internationally: in order to sell one unit of any good $\omega$ in country $j$, a producer in country $i$ must produce $\tau_{ij}$ units, where $\tau_{ij} > 1$ if $i \neq j$ and $\tau_{ii} = 1$. Since producers are perfectly competitive, the price of a good sold from $i$ to $j$ is

$$p_{ij}(\omega) = \frac{\tau_{ij} w_i}{z_i(\omega)}$$

The distribution of prices of goods that country $j$ can potentially buy from country $i$ is:

$$\Pr\left(p_{ij}(\omega) \leq p\right) = 1 - e^{-T_i (\tau_{ij} w_i)^{-\theta} p^{\theta}}$$

## 2.2 Input sourcing and final good production

A continuum of mass one of heterogeneous plants produce the final good in each country using labor $\ell$ and a composite of intermediate inputs $x$, according to:

$$y = z^{1-\alpha-\eta}\ell^{\alpha}x^{\eta}$$

where $\alpha + \eta < 1$. Although plants are perfectly competitive and produce a homogeneous final good, plants with different efficiencies will coexist because of decreasing returns to scale. Plants' efficiencies $z$ are distributed in country $j$ according to a Pareto distribution with density

$$h_j(z) = \zeta \underline{z}_j^{\zeta} z^{-\zeta-1} \tag{1}$$

The composite intermediate input is given by the Cobb-Douglas aggregator:

$$x = \exp\left(\int_0^1 \log \tilde{x}(\omega)\, d\omega\right)$$

where $\tilde{x}(\omega)$ refers to units of good $\omega$ and $x$ is units of the composite input.

### 2.2.1 Two extremes

If a plant were to buy every intermediate good from the cheapest location worldwide, then, as Eaton and Kortum (2002) show, the fraction of country $j$ plants' intermediate input expenditures that is spent on goods from country $i$ would be:

$$\lambda_{ij} = \frac{T_i(\tau_{ij}w_i)^{-\theta}}{\sum_k T_k(\tau_{kj}w_k)^{-\theta}} \tag{2}$$

and the price of a unit of the composite input in country $j$ would be:

$$p_j^s = \left(\sum_i T_i(\tau_{ij}w_i)^{-\theta}\right)^{-1/\theta} \tag{3}$$

In contrast, if a plant purchased all intermediate goods domestically, then the price $p_j^d$ of the composite input bundle would be equal to just the country $j$ term in the expression above for $p_j^s$,

$$p_j^d = \left(T_j w_j^{-\theta}\right)^{-1/\theta} \tag{4}$$

which is higher than $p_j^s$ for any $\theta > 0$.

### 2.2.2 Costly sourcing of imports

To introduce differences in importing behavior across plants, suppose that it is costly to source each input from the cheapest location (hereafter referred to simply as "sourcing") rather than just buy it from domestic suppliers. This cost is a stand-in for the costs of searching for and maintaining a relationship with a foreign supplier. Specifically, if a plant sources a fraction $n$ of its inputs, it has to pay $g(n) = b(f^n - 1)$ units of output. I assume $f > 1$, so that the total cost a plant pays is increasing and convex in the fraction of goods sourced. In addition $g(0) = 0$, so the fixed cost associated with sourcing nothing and purchasing everything from domestic suppliers is normalized to zero. The benefit of sourcing a larger fraction of inputs is that it lowers the price index of the input bundle: if a plant in country $j$ sources $n$ inputs, then the price index among those $n$ goods is given by $p_j^s$ as defined in (3), while the price index for the remainder of the inputs is given by $p_j^d$, in (4).[4] Therefore, the price for a unit of the overall input bundle if a plant sources $n$ of the inputs is

$$p_j(n) = \left(p_j^s\right)^n \left(p_j^d\right)^{1-n} \tag{5}$$

Since $p_j^s < p_j^d$, $p_j(n)$ is decreasing in $n$: plants that source a higher fraction of their inputs face lower per-unit input costs. Using (3) and (4),

$$p_j(n) = (\lambda_{jj})^{n/\theta} T_j^{-1/\theta} w_j \tag{6}$$

where $\lambda_{jj}$, still given by the formula in (2), is the fraction of expenditures on the $n$ *sourced* inputs that is purchased domestically, and $1 - \lambda_{jj}$ is the fraction of these goods imported. Overall then, a plant that sources $n$ of its inputs spends a fraction $n(1 - \lambda_{jj})$ on imported inputs, and $n\lambda_{jj} + 1 - n$ on domestic goods.

The input sourcing and production choices of plants can be separated into two steps: first choose input quantities to maximize variable profit, given a sourcing policy (i.e., given an $n$), then choose $n$ to maximize overall profits given the optimal quantity decisions.

The variable profit of a plant in country $j$ with efficiency $z$ that has chosen to source $n$ of its inputs is given by:

$$\tilde{\pi}_j(z, n) = \max_{\ell, x} P_j z^{1-\alpha-\eta} \ell^\alpha x^\eta - w_j \ell - p_j(n) x$$

where $P_j$ is the price of the final good in country $j$. The profit maximizing choices of inputs

[4] Although this model is static, there is an implicit timing assumption: plants choose the fraction $n$ of inputs to source before the realization of intermediate good producers' efficiencies $z_i(\omega)$. This assumption generates the formula for the price index $p_j(n)$ in (5), and allows the closed-form solution to plants' optimal choice of $n$ below.

and output are:

$$
\begin{aligned}
\ell_j(z,n) &= z\left(P_j\left(\frac{\alpha}{w_j}\right)^{1-\eta}\left(\frac{\eta}{p_j(n)}\right)^{\eta}\right)^{\frac{1}{1-\alpha-\eta}} \\
x_j(z,n) &= z\left(P_j\left(\frac{\alpha}{w_j}\right)^{\alpha}\left(\frac{\eta}{p_j(n)}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha-\eta}} \\
y_j(z,n) &= z\left(P_j^{\alpha+\eta}\left(\frac{\alpha}{w_j}\right)^{\alpha}\left(\frac{\eta}{p_j(n)}\right)^{\eta}\right)^{\frac{1}{1-\alpha-\eta}}
\end{aligned}
$$

These choices can be written,

$$
\begin{aligned}
\ell_j(z,n) &= \frac{\alpha}{w_j} z v_j \gamma_j^n && (7) \\
x_j(z,n) &= \frac{\eta}{p_j(n)} z v_j \gamma_j^n && (8) \\
y_j(z,n) &= \frac{1}{P_j} z v_j \gamma_j^n && (9)
\end{aligned}
$$

where

$$
\begin{aligned}
v_j &= \left(P_j\left(\frac{\alpha}{w_j}\right)^{\alpha}\left(T_j^{1/\theta}\frac{\eta}{w_j}\right)^{\eta}\right)^{1/(1-\alpha-\eta)} \\
\gamma_j &= \lambda_{jj}^{\frac{-\eta}{\theta(1-\alpha-\eta)}}
\end{aligned}
$$

Maximized variable profit given $n$ is then:

$$\tilde{\pi}_j(z,n) = (1-\alpha-\eta)\, z v_j \gamma_j^n$$

Since $\lambda_{jj} \leq 1$, $\gamma_j \geq 1$, so given $n$, a plant with higher $z$ is larger in terms of labor, intermediate expenditure, and outputs, and has higher profits.

Now, the choice of $n$ and total profit are determined by

$$\pi_j(z) = \max_{n\in[0,1]} \tilde{\pi}_j(z,n) - P_j b\left(f^n - 1\right)$$

Variable profit rises exponentially with $n$, at rate $\gamma_j$, while the fixed cost of sourcing inputs also rises exponentially at rate $f$. As shown in the appendix, (1) a sufficient condition for the solution to this problem to exist and be unique is that $f > \gamma_j$; and (2) if that is the case, then the optimal choice of $n$ for a plant with efficiency $z$ in country $j$ is:

$$n_j(z) = \begin{cases} 0 \text{ if } z \leq z_j^0 \\ \psi_j \log z + \phi_j \text{ if } z \in \left(z_j^0, z_j^1\right) \\ 1 \text{ if } z \geq z_j^1 \end{cases} \tag{10}$$

where

$$\begin{aligned} \psi_j &= \frac{1}{\log f_j - \log \gamma_j} \\ \phi_j &= \psi_j \log \left( \frac{(1-\alpha-\eta) v_j \log \gamma_j}{P_j b_j \log f_j} \right) \end{aligned}$$

and

$$\begin{aligned} z_j^0 &= \exp\left(\frac{-\phi_j}{\psi_j}\right) \\ z_j^1 &= \exp\left(\frac{1-\phi_j}{\psi_j}\right) \end{aligned}$$

The solution takes the form of two cutoffs, $z_j^0$ and $z_j^1$, with $z_j^0 < z_j^1$: plants with efficiency below $z_j^0$ source none of their inputs globally and purchase everything from domestic suppliers, while plants with efficiency above $z_j^1$ source all of their inputs, and purchase a fraction $\lambda_{jj}$ domestically. Between these two thresholds, the fraction of inputs sourced is linear in the log of efficiency, with slope $\psi_j = \frac{1}{\log f - \log \gamma_j}$. Notice that the sufficient condition for existence and uniqueness of this solution $(f > \gamma_j)$ also implies that $n_j(z)$ is increasing in $z$: more efficient plants source a higher share of their inputs. In this range, the import share, $n_j(z)(1-\lambda_{jj})$, is increasing with $z$, so more efficient plants import a larger share of their intermediate inputs. Also, since size – measured by either labor, output, or total intermediate expenditures – is increasing in efficiency $z$ (from (7)-(9), size and import share are positively related. In practice, when calibrating this model, the moments I match relating size and importing behavior guarantee that $f > \gamma_j$.

### 2.2.3 Heterogeneity in import shares

Figure 1 illustrates an example for the functional form of $n_j(z)$, juxtaposed against the distribution of efficiency levels $h_j(z)$ (the parameters are those calibrated below). The interaction of the exogenous heterogeneity in efficiency and the choice of $n$ generates a distribution for import shares, $s_j(z) = (1-\lambda_{jj}) n_j(z)$. Among those plants that import a

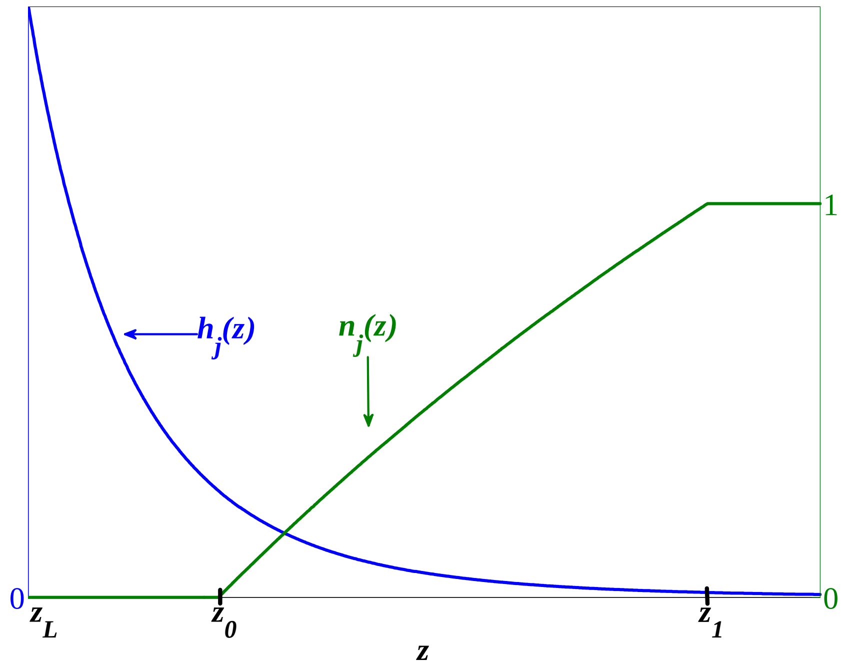


Figure 1: Heterogeneity in efficiency and import shares

positive amount, ignoring for a moment the restriction that $n_j(z) \leq 1$,

$$\begin{aligned}
\Pr\left(s_j(z) \geq s | s_j(z) \geq 0\right) &= \frac{\Pr\left(\left(1-\lambda_{jj}\right)\left(\psi_j \log z + \phi_j\right) \geq s\right)}{\Pr\left(\left(1-\lambda_{jj}\right)\left(\psi_j \log z + \phi_j\right) \geq 0\right)} \\
&= \exp\left(-\zeta \frac{s}{\left(1-\lambda_{jj}\right)\psi_j}\right)
\end{aligned}$$

Therefore, the cumulative distribution function of import shares, for $s > 0$, is:

$$\begin{aligned}
G_j(s) &= \Pr\left(s_j \leq s | s_j \geq 0\right) \\
&= \begin{cases} 1-\exp\left(-\zeta \frac{s}{(1-\lambda_{jj})\psi_j}\right) \text{ if } 0 < s < 1-\lambda_{jj} \\ 1 \text{ if } s \geq 1-\lambda_{jj} \end{cases} \qquad (11)
\end{aligned}$$

The distribution of import shares is an exponential distribution with parameter $\frac{\zeta}{(1-\lambda_{jj})\psi_j}$ up to the point $1-\lambda_{jj}$, and has a mass point at $1-\lambda_{jj}$. The mass of this point is equal to the fraction of plants that source all of their intermediate inputs, and hence import a share $1-\lambda_{jj}$ of them (those with $z$ above $z_j^1$).

## 2.3 Market clearing and equilibrium

A representative consumer in each country $j$ values consumption of the final good, inelastically supplies labor at the level $\bar{L}_j$, and receives the profits of all final good plants. The consumer therefore is willing to consume whatever level of output plants produce.

The market clearing condition for labor requires that the labor used by intermediate goods producers plus the labor used by final good plants in each country $j$ adds up to $\bar{L}_j$. Since intermediate goods producers are perfectly competitive, their total payments to labor equal their sales, which are:

$$\sum_k \left( \int \lambda_{jk} n_k(z)\, p_k(n_k(z))\, x_k(z, n_k(z))\, h_k(z)\, dz \right) + \int (1 - n_j(z))\, p_j(n_j(z))\, x_j(z, n_j(z))\, h_j(z)\, dz \tag{12}$$

The first term in (12) is total sales to plants in all countries sourcing intermediate goods from country $j$. Plants in each country $k$ with efficiency $z$ spend a fraction $\lambda_{jk} n_k(z)$ of their intermediate expenditures $p_k(n_k(z))\, x_k(z, n_k(z))$ on $j$'s goods, and there are mass $h_k(z)$ of plants with each efficiency level $z$. The second term is additional sales to final good plants within $j$ of the goods that they decide not to source, and hence must purchase from country $j$'s intermediate good producers. Each plant in $j$ with efficiency $z$ spends a fraction $1 - n_j(z)$ of its intermediate expenditures on its own country's intermediate goods in this way.

Payments to labor by final good plants is given by:

$$w_j \int \ell_j(z, n_j(z))\, h_j(z)\, dz \tag{13}$$

So the labor market clearing condition states that (12) and (13) equal total payments to labor:

$$\begin{aligned} w_j \bar{L}_j = {} & \sum_k \left( \int \lambda_{jk} n_k(z)\, p_k(n_k(z))\, x_k(z, n_k(z))\, h_k(z)\, dz \right) \\ & + \int (1 - n_j(z))\, p_j(n_j(z))\, x_j(z, n_j(z))\, h_j(z)\, dz \\ & + \int w_j \ell_j(z, n_j(z))\, h_j(z)\, dz \end{aligned}$$

Finally, balanced trade requires that country $j$'s exports,

$$\sum_{k \neq j} \left( \int \lambda_{jk} n_k(z)\, p_k(n_k(z))\, x_k(z, n_k(z))\, h_k(z)\, dz \right)$$

equal its imports,

$$\int \left(1-\lambda_{jj}\right) n_j\left(z\right) p_j\left(n_j\left(z\right)\right) x_j\left(z, n_j\left(z\right)\right) h_j\left(z\right) dz$$

which, rearranging, gives

$$\begin{aligned} & \sum_k \left(\int \lambda_{jk} n_k\left(z\right) p_k\left(n_k\left(z\right)\right) x_k\left(z, n_k\left(z\right)\right) h_k\left(z\right) dz\right) \\ = & \int n_j\left(z\right) p_j\left(n_j\left(z\right)\right) x_j\left(z, n_j\left(z\right)\right) h_j\left(z\right) dz \end{aligned}$$

An equilibrium consists of a set of wages $w_j$ and final good prices $P_j$ such that, given the plant-level decisions characterized in the previous subsection, the market clearing condition for labor and the trade balance condition hold for each country.

Using the plant-level input decisions in (7)-(8) and (10), the labor market clearing condition and trade balance condition in each country $j$ can be written in terms of two moments of the distribution of $z$,

$$w_j \bar{L}_j = \alpha \mu_{Yj} + \sum_k \lambda_{jk} \eta \mu_{Mk} + \eta\left(\mu_{Yj} - \mu_{Mj}\right)$$

and

$$\sum_k \lambda_{jk} \mu_{Mk} = \mu_{Mj}$$

where

$$\begin{aligned} \mu_{Yj} &= v_j \int z \gamma_j^{n_j(z)} h_j\left(z\right) dz \\ &= v_j \zeta \underline{z}_j^{\zeta} \left(\frac{\left(\underline{z}_j\right)^{1-\zeta}}{\zeta-1} + \left(\gamma_j \left(z_j^1\right)^{1-\zeta} - \left(z_j^0\right)^{1-\zeta}\right)\left(\frac{1}{\zeta-1} + \frac{1}{1+\psi_j \log \gamma_j - \zeta}\right)\right) \end{aligned}$$

and

$$\begin{aligned} \mu_{Mj} &= v_j \int z n_j\left(z\right) \gamma_j^{n_j(z)} h_j\left(z\right) dz \\ &= v_j \zeta \underline{z}_j^{\zeta} \left(\gamma_j \left(z_j^1\right)^{1-\zeta}\left(\frac{1}{1+\psi_j \log \gamma_j - \zeta} + \frac{1}{\zeta-1}\right) - \frac{\psi_j\left(\left(z_j^1\right)^{1-\zeta}\gamma_j - \left(z_j^0\right)^{1-\zeta}\right)}{\left(1+\psi_j \log \gamma_j - \zeta\right)^2}\right) \end{aligned}$$

The two moments are related to aggregate revenue of final-good producing plants (which is equal to $\mu_{Yj}$) and aggregate imports of intermediate goods (which is equal to $\left(1-\lambda_{jj}\right)\eta\mu_{Mj}$).

Total final consumption $C_j$ is then

$$C_j = (1-\eta)\frac{\mu_{Yj}}{P_j} - H_j$$

where $H_j = b_j \left(\int f_j^{n_j(z)} g_j(z)\, dz - 1\right)$ is the aggregate quantity of output used by plants to pay the fixed costs of sourcing inputs.

## 2.4 The link between importing and productivity

In this model, importing raises plant-level productivity when input expenditures are measured across plants using common price deflators, as is standard in plant-level data sets. Sourcing some inputs (and importing a fraction of those inputs sourced) lowers the prices on average that a plant pays for its input bundle. Productivity then appears higher at plants that import some of their inputs because they produce more output with the same expenditures on inputs, compared to plants that purchase all of their inputs domestically.

The output of a plant in country $j$ with efficiency $z$ can be written:

$$y_j(z, n_j(z)) = z^{1-\alpha-\eta} \ell_j(z, n_j(z))^{\alpha} (X_j(z, n_j(z)))^{\eta} p_j(n_j(z))^{-\eta}$$

where $X_j(z, n_j(z))$ are expenditures on intermediate goods by the plant,

$$X_j(z, n_j(z)) = p_j(n_j(z))\, x_j(z, n_j(z))$$

Therefore, total factor productivity measured at the plant level is

$$\frac{y_j(z, n_j(z))}{\ell_j(z, n_j(z))^{\alpha} (X_j(z, n_j(z)))^{\eta}} = z^{1-\alpha-\eta} p_j(n_j(z))^{-\eta}$$

so that plants who choose higher $n_j$, and hence pay a lower input price $p_j$, appear more productive.

As a function of the import expenditure share, $s_j(z) = n_j(z)(1-\lambda_{jj})$, the gain (in logs) in productivity for a plant relative to sourcing none of its inputs (and purchasing them all from domestic suppliers) is, using the form for $p_j(n)$ in (6):

$$\log\left(\frac{p_j(n_j(z))}{p_j(0)}\right)^{-\eta} = \frac{s_j(z)}{1-\lambda_{jj}}\frac{\eta}{\theta}\log\left(\frac{1}{\lambda_{jj}}\right)$$

Since $\frac{\log(1/\lambda_{jj})}{1-\lambda_{jj}}\frac{\eta}{\theta} > 0$, the productivity gain of importing is increasing in a plant's import share. The magnitude of this productivity effect depends directly on two parameters – $\eta$, the share of intermediate inputs in total costs, and $\theta$, the degree of heterogeneity in the prices of intermediate inputs – as well as the fraction of sourced inputs that are optimally purchased domestically in equilibrium, $\lambda_{jj}$. The lower is $\theta$, the greater the dispersion in prices of intermediate inputs, so the greater is the incentive to exploit comparative advantage by sourcing inputs. The factor $\frac{\log(1/\lambda_{jj})}{1-\lambda_{jj}}$ is decreasing in $\lambda_{jj}$, so that the less a country spends on imports (among the fraction that plants source optimally), the lower is the productivity gain from sourcing inputs.

# 3 Quantitative Analysis

In this section, I analyze the model's quantitative implications for productivity and welfare in a numerical experiment with two countries. I calibrate several parameters to cross-sectional facts from Chilean plant-level data, so I take the two countries to be Chile (country 1) and the rest of the world (country 2).

## 3.1 Calibration

Table 1 summarizes the parameter values. I choose the share parameters in production, $\eta = 0.5$ and $\alpha = 0.35$, so that 50% of gross output goes to intermediate input expenditures, and 70% of value-added (gross output net of intermediate expenditures) is paid to labor. I set $\bar{L}_1$ and $\bar{T}_1$ to 1, and set $\bar{L}_2 = \bar{T}_2 = 100$, and assume that $\tau_{12} = 1$, that is, there is no variable trade cost for Chile to export to the rest of the world.

The remaining parameters determine the levels and dispersion of importing and size among importing and nonimporting plants in the model. These are the inbound trade cost, $\tau_{21}$, the dispersion in intermediate good efficiencies $\theta$, the shape parameter $\zeta$ for the Pareto distribution of final good efficiencies, and the parameters of the fixed cost function $b$ and $f$. I choose these five parameters so that the matches averages of moments in Chilean manufacturing plant-level data over the period 1987-1996, as described in the following subsections. The lower bounds of the Pareto distribution of final good efficiencies $\underline{z}_1$ and $\underline{z}_2$ are set to 1.

Table 1: Calibration

| parameter | value | role |
|---|---|---|
| $\eta$ | 0.50 | intermediate share of gross output |
| $\alpha$ | 0.35 | labor share of gross output |
| $\underline{z}_j$ | 1.00 | lower bound of distribution of plant efficiences |
| $\tau_{21}$ | 1.17 | variable per-unit import cost |
| $\theta$ | 4.31 | shape parameter in distribution of intermediate efficiencies |
| $\zeta$ | 6.00 | shape parameter of distribution of final good efficiencies |
| $b$ | 0.05 | level parameter in importing fixed cost function |
| $f$ | 14.38 | curvature parameter in importing fixed cost function |

#### 3.1.1 Average and standard deviation of import shares

Given the distribution of import shares $G_1(s)$ derived in (11), the average import share in country 1 (Chile) is

$$\begin{aligned}\bar{s}_1 &= \int_0^1 s dG_1(s) \\ &= (1-\lambda_{11})\frac{\psi_1}{\zeta}\left(1-e^{-\zeta/\psi_1}\right)\end{aligned}$$

The variance of import shares, similarly, is

$$\begin{aligned}\sigma_1^2 &= \int_0^1 s^2 dG_1(s) - (\mu_1^s)^2 \\ &= (1-\lambda_{11})^2 \frac{2\psi_1}{\zeta}\left(\frac{\psi_1}{\zeta} - e^{-\zeta/\psi_1}\left(\frac{\psi_1}{\zeta}+1\right)\right) - (\bar{s}_1)^2\end{aligned}$$

These two statistics uniquely pin down the two factors $\frac{\psi_1}{\zeta} = \frac{1}{\zeta(\log f_1 - \log \gamma_1)}$ and $\lambda_{11}$.

#### 3.1.2 The dispersion in imports among importers

For a given import share $s$, a high efficiency plant would be larger than a low efficiency plant, measured by labor used or inputs purchased. But high efficiency plants also choose high import shares. Therefore, the dispersion in heterogeneity in size from the exogenous variation in $z$ is magnified through the dispersion in import shares generated by the curvature of the fixed cost function. The relationship between dispersion in size and the curvature parameter $f$ is most easily seen in ratios of percentiles of the distribution of imports among importing plants.

Let $z_j^{(q)}$ be the $q$th percentile of the conditional distribution of efficiencies among plants with nonzero import shares in country $j$, that is, the level above which there are $(100-q)\,\%$

of the importing plants:

$$\begin{aligned}
\Pr\left(z \geq z_j^{(q)} | z \geq z_j^0\right) &= 1 - \frac{q}{100} \\
z_j^{(q)} &= \left(z_j^0\right)\left(1 - \frac{q}{100}\right)^{-1/\zeta}
\end{aligned}$$

Since total import purchases $M_j(z) = (1 - \lambda_{jj}) n_j(z) p_j(n_j(z)) x_j(z, n_j(z))$ are a monotonically nondecreasing function of $z$, the $q$th percentile of the distribution of imports among importing plants is given by $M_j^{(q)} = M_j\left(z_j^{(q)}\right)$. As long as $q$ is not so large that $z_j^{(q)} \geq z_j^1$ (so that the import share for the plant at the $q$th percentile is interior), this quantity is given by:

$$\begin{aligned}
M_j^{(q)} &= (1 - \lambda_{jj}) n_j\left(z_j^{(q)}\right) p_j\left(n_j\left(z_j^{(q)}\right)\right) x_j\left(z_j^{(q)}, n_j\left(z_j^{(q)}\right)\right) \\
&= (1 - \lambda_{jj})\left(\psi_j \log z_j^{(q)} + \phi_j\right) \eta z_j^{(q)} \upsilon_j \gamma_j^{\psi_j \log z_j^{(q)} + \phi_j} \\
&\left(z_j^{(q)}\right)^{1+\psi_j \log \gamma_j} \upsilon_j \eta (1 - \lambda_{jj})\left(\psi_j \log z_j^{(q)} + \phi_j\right) \gamma_j^{\phi_j}
\end{aligned}$$

Now, consider the ratio of two percentiles, $q$ and $r$:

$$\begin{aligned}
\frac{M_j^{(q)}}{M_j^{(r)}} &= \left(\frac{z_j^{(q)}}{z_j^{(r)}}\right)^{1+\psi_j \log \gamma_j} \frac{\psi_j \log z_j^{(q)} + \phi_j}{\psi_j \log z_j^{(r)} + \phi_j} \\
&= \left(\frac{100 - r}{100 - q}\right)^{\left(1+\psi_j \log \gamma_j\right)/\zeta} \frac{\psi_j \log\left(\left(z_j^0\right)\left(1 - \frac{q}{100}\right)^{-1/\zeta}\right) - \psi_j \log z_j^0}{\psi_j \log\left(\left(z_j^0\right)\left(1 - \frac{r}{100}\right)^{-1/\zeta}\right) - \psi_j \log z_j^0} \\
&= \left(\frac{100 - r}{100 - q}\right)^{\left(1+\psi_j \log \gamma_j\right)/\zeta} \frac{\log\left(\frac{100-q}{100}\right)}{\log\left(\frac{100-r}{100}\right)}
\end{aligned}$$

So given two percentiles of the distribution of imports, their ratio pins down the factor:

$$\frac{1 + \psi_j \log \gamma_j}{\zeta} = \frac{\log f}{\zeta\left(\log f - \log \gamma_j\right)}$$

Given a mean import share for Chile, $\bar{s}_1$, which determines $\zeta(\log f - \log \gamma_1)$, the ratio of any two interior percentiles of the distribution of import expenditures, $\frac{M_1^{(q)}}{M_1^{(r)}}$, can be used to uniquely identify $f$.

Given a mean import share, a larger $f$ makes the ratio $\frac{M_1^{(q)}}{M_1^{(r)}}$ larger for any two percentiles, $q > r$. For a given dispersion of import shares, a larger $f$ makes it more costly for large plants to raise their import ratio, so dispersion in size grows without increasing the dispersion in import shares.

#### 3.1.3 The fraction of plants importing

Plants with efficiency draws above $z_j^0$ use imported inputs. The fraction of plants doing so, $F_j^{im} \in [0,1]$, is:

$$\begin{aligned} F_j^{im} &= \Pr\left(z \geq z_j^0\right) \\ &= \underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta} \\ &= \underline{z}_j^{\zeta} e^{\zeta\phi_j/\psi_j} \end{aligned}$$

With the average import share pinning down the ratio $\frac{\zeta}{\psi_1}$, a target for $F_1^{im}$ yields $\phi_1 = \psi_1 \log\left(\frac{(1-\alpha-\eta)v_1 \log\gamma_1}{P_1 b \log f}\right)$.

#### 3.1.4 The average size of importing relative to nonimporting plants

The total expenditures on inputs by a plant with efficiency $z$ are:

$$\begin{aligned} X_j(z) &= (1-\lambda_{jj})\, n_j(z)\, p_j(n_j(z))\, x_j(z, n_j(z)) + (1-n_j(z))\, p_j(n_j(z))\, x_j(z, n_j(z)) \\ &= \eta z v_j \gamma_j^{n_j(z)} \end{aligned}$$

The average size, measured by total inputs, of importing plants is

$$\bar{X}_j^m = \frac{1}{1-\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}} \int_{z_j^0}^{\infty} \eta z v_j \gamma_j^{n_j(z)} h_j(z)\, dz$$

while the average size of nonimporting plants is

$$\bar{X}_j^d = \frac{1}{\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}} \int_{\underline{z}_j}^{z_j^0} \eta z v_j h_j(z)\, dz$$

The ratio of these two can be written (see appendix for derivation):

$$\frac{\bar{X}_j^m}{\bar{X}_j^d} = \frac{1-F_j^{im}}{F_j^{im}} \frac{1}{\left(F_j^{im}\right)^{(1-\zeta)/\zeta} - 1} \left(\frac{\zeta-1}{\zeta}\frac{1}{\varsigma_{2j}-1}\left(e^{\left(\varsigma_{2j}-1\right)\varsigma_{1j}} - 1\right) + \lambda_{jj}^{\frac{-\eta}{\theta(1-\alpha-\eta)}} e^{\varsigma_{1j}(1-\zeta)/\zeta}\right) \tag{14}$$

where $F_j^{im}$ is the fraction of plants importing, $\varsigma_{1j} = \zeta/\psi_j$ and $\varsigma_{2j} = \left(1+\psi_j \log\gamma_j\right)/\zeta$ are parameter combinations that are pinned down by the average import share and the ratio of import percentiles derived above, and $\lambda_{jj}$ is country $j$'s own-import share

Therefore, given targets for the other moments, the ratio of the average size of importing plants relative to nonimporting plants in Chile, $\frac{\bar{X}_1^m}{\bar{X}_1^d}$, identifies $\zeta$ through equation (14).

### 3.1.5 Chilean Manufacturing Data and Model Fit

I choose the parameters $\tau_{21}, \theta, f, \zeta$, and $b$ to match five moments in the model – the average import share, the standard deviation of the import share, the fraction of plants importing, the 75/25 percentile ratio of imports, and the average size of importers relative to nonimporters – to the data from Chile's manufacturing census over the period 1987-1996.[5] Table 2 displays the data for these moments.

Table 2: Chilean Manufacturing Plant Data Moments, 1987-1996

| year | fraction importing (%) | average import share (%) | s.d. of import share (%) | 75/25 ratio | size ratio |
|---|---|---|---|---|---|
| 1987 | 24.4 | 33.4 | 26.8 | 16.0 | 6.9 |
| 1988 | 23.7 | 31.5 | 25.8 | 13.8 | 4.6 |
| 1989 | 21.2 | 31.6 | 26.1 | 16.6 | 4.7 |
| 1990 | 20.4 | 32.9 | 26.3 | 13.0 | 4.4 |
| 1991 | 21.2 | 32.3 | 26.8 | 13.6 | 4.2 |
| 1992 | 23.4 | 33.1 | 26.7 | 15.8 | 3.7 |
| 1993 | 24.3 | 33.8 | 26.7 | 14.9 | 3.9 |
| 1994 | 26.4 | 33.5 | 27.6 | 17.3 | 4.4 |
| 1995 | 23.9 | 34.5 | 28.3 | 16.1 | 4.0 |
| 1996 | 24.2 | 33.8 | 27.5 | 17.0 | 4.4 |
| average | 23.3 | 33.0 | 26.9 | 15.4 | 4.5 |

On average, about 23% of plants report purchasing positive amounts of imported inputs. Among these plants, the average import share is 33% of total intermediate input expenditures, and the standard deviation of import shares across plants is about 27%. The average 75/25 ratio indicates that the importer at the 75th percentile imports about 15.4 times as much as the importer at the 25th percentile of the distribution of import expenditures. And relative to nonimporting plants, importing plants are on average 4.5 times as large as measured by their total expenditures on intermediate inputs.

Figures 2 and 3 show the cumulative distributions of the import share and (de-meaned) log imports for each year in the data, along with the model's predictions. Choosing parameters to match the five moments discussed above does a fairly good job at fitting the entire

[5]The data are from the *Encuesta Nacional Industrial Anual*, from Chile's *Instituto Nactional de Estadistica*. These are the data used in Kasahara and Rodrigue (2008), and were described in detail in Liu (1993).

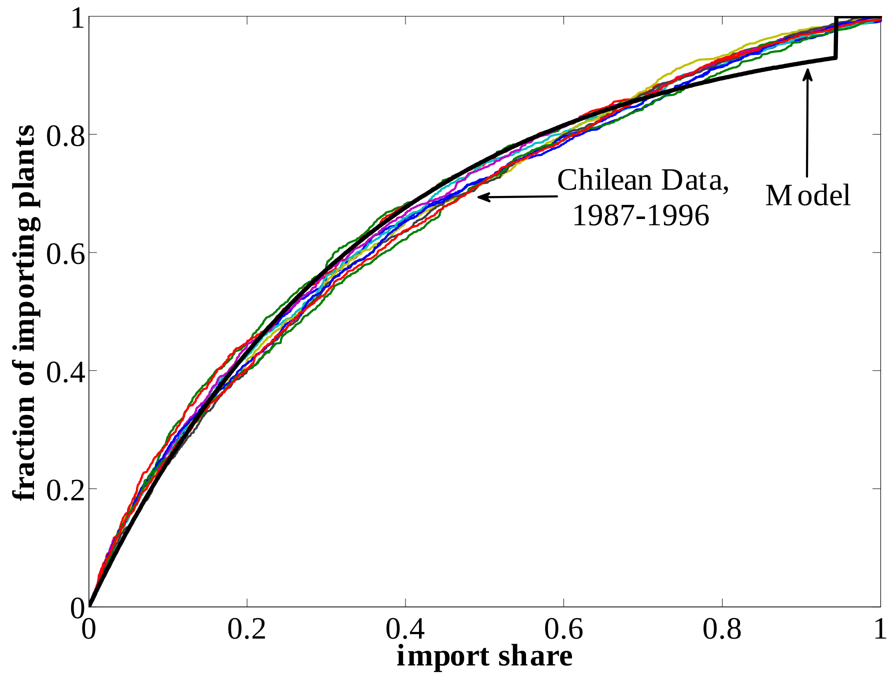


Figure 2: Cumulative distribution of import share

cross-sectional distribution in import shares and size among importers in the data, except that the model generates too many plants who import all their inputs (with an import share of $1 - \lambda_{11} = 0.94$). Among these plants, there is one less source of heterogeneity in import expenditures, hence the abrupt compression in the model's distribution at the right end of Figure 3.

### 3.1.6 The productivity advantage of importing

In my model, plants gain by importing through lowering the price index for the input bundle they purchase. Looking across plants within a period, plants that import a higher share of their inputs appear more productive, even aside from the fact that plants with inherently higher efficiency $z$ have higher import shares. Although calibrated to match moments on heterogeneity in size and import shares (and not productivity measures), my model's structure links the calibrated parameters to an implied gain in productivity from importing.

Several recent empirical studies have estimated this kind of productivity advantage of importing in plant-level data, including Amiti and Konings (2007) using Indonesian data, Halpern, Koren, and Szeidl (2009) using Hungarian data, and Kasahara and Rodrigue (2008)

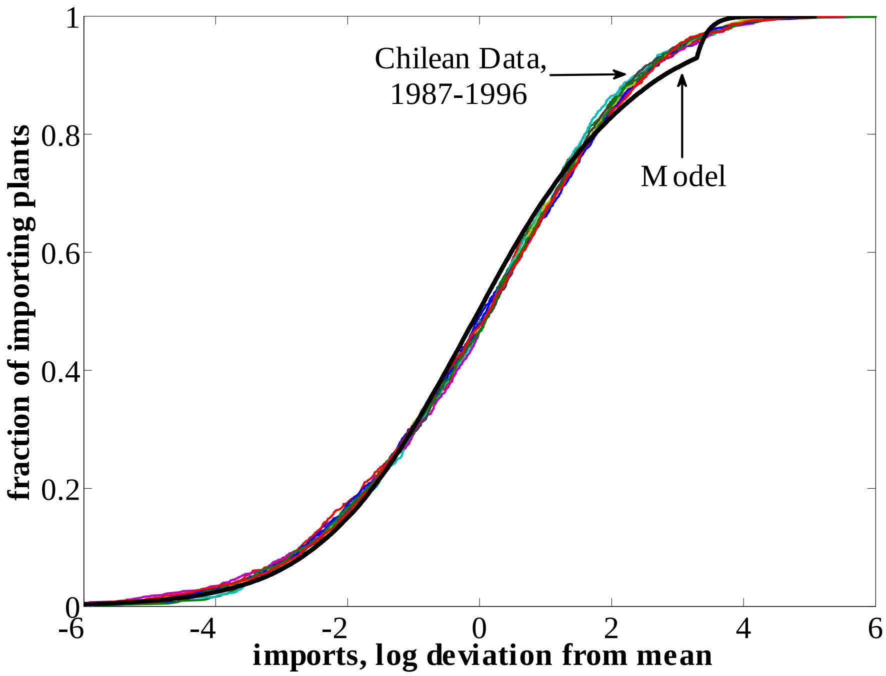


Figure 3: Cumulative distribution of log imports, relative to mean

using a subset of the Chilean data considered here.[6] These papers all estimate production functions that relate a plant's output to its factor inputs and intermediate expenditures, along with indicators of whether the plant imports any of its inputs, or its import expenditure share (or both). In my model, as described in subsection 2.4, the plant-level production technology can be represented as a function of inputs $\ell$ and $x$ and a plant's import share $s$ as follows:

$$\log y_1(z) = \log z + \alpha \log \ell_1(z) + \theta \log x_1(z) + s_1(z) \frac{\eta}{\theta(1-\lambda_{11})} \log\left(\frac{1}{\lambda_{11}}\right) \tag{15}$$

The percentage gain in productivity for a plant with productivity $z$ that uses imported inputs relative to not using imported inputs (or, equivalently, relative to a plant with the same $z$ who for some reason does not use imported inputs) is given by $s(z) \frac{\eta}{\theta(1-\lambda_{11})} \log\left(\frac{1}{\lambda_{11}}\right) > 0$. In the calibrated model, $\frac{\eta}{\theta(1-\lambda_{11})} \log\left(\frac{1}{\lambda_{11}}\right) = 0.35$, which implies that a plant gains 3.5% in productivity by increasing its import share by 10 percentage point (that is, gains 3.5%

[6] Although they do not estimate the direct producer-level productivity gain from importing, Goldberg, Khandelwal, Pavcnik, and Topalova (2010), using data on Indian firms, find that lower input tariffs, and hence higher expenditures on imported inputs, lead firms to create more new products. They argue that this is because the cost of production decreases (which similar to the increase in productivity considered here), so that producing new products becomes profitable.

in output given the quantities of all its inputs). The average productivity gain across all importing plants is given by $\bar{s}_1 \frac{\eta}{\theta(1-\lambda_{11})} \log\left(\frac{1}{\lambda_{11}}\right) = 0.117$, so that an importing plant on average is 11.7% more productive than a nonimporting plant, controlling for differences in their exogenous efficiency. Kasahara and Rodrigue (2008) report similar numbers in their analysis of the Chilean plant data. Using a continuous import share variable, their range of estimates imply that raising the import share by 10 percentage points raises productivity by 0.5% to 2.7%. Using a discrete import status variable, they find that importing raises productivity on average by between 18% and 21%. Results of similar magnitudes are reported in Halpern, Koren, and Szeidl (2009) and Amiti and Konings (2007).

### 3.2 Counterfactual: Unilateral Trade Liberalization

I consider variation in the variable importing cost, $\tau_{21}$. I analyze the effects of this change on plant-level productivity and aggregatewelfare. To compare the model's predictions to a model without heterogeneity in import shares, I also solve a model in which all plants import, with the parameters $\theta$ and $\tau_{21}$ recalibrated to generate the same aggregate import share and trade elasticity - the elasticity of imports with respect to a change in the variable cost $\tau_{21}$ - at $\tau_{21} = 1.17$. This means that the reduction in $\tau_{21}$ generates the same growth in trade in both models. As Arkolakis, Costinot, and Rodriguez-Clare (2012) show, comparing welfare gains of trade cost reductions across models requires keeping the growth in trade the same. This experiment allows one to ask whether considering heterogeneity in import behavior generates additional gains from trade.

Figure 4 plots the change in welfare, measured by aggregate consumption, against the change in the trade cost $\tau_{21}$, for both models. The two models generate different predictions for the welfare gains of trade cost reductions that vary with the size of the reduction. For example, a 20 percent reduction in $\tau_{21}$ generates a welfare gain of about 6 percent in the model with heterogeneity in import shares, while it only generates a 4.5 percent welfare gain in the model with all plants importing.

## 4 Conclusion

The model presented here captures the heterogeneity in the use of imported intermediate inputs prevalent in studies of plant- and firm-level data. The model has relatively few parameters that are easily related to observable moments of plant-level data, and accounts for the plant-level productivity gain of importing observed in the data. The model is also useful for counterfactual exercises, and can generate substantial within-plant productivity gains

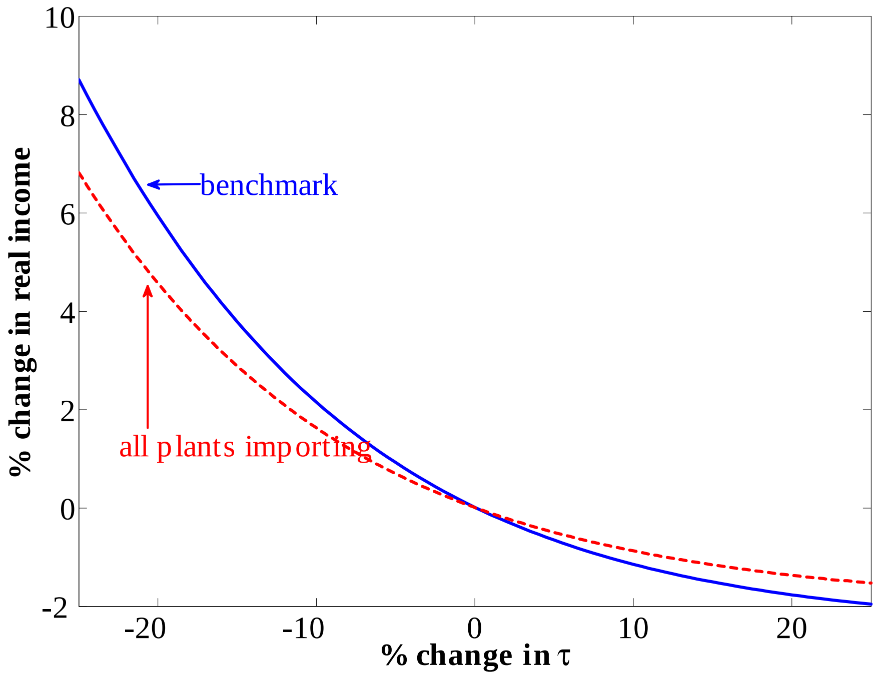


Figure 4: Welfare gains in model with import heterogeneity and in model with all plants importing

from trade liberalization. Useful extensions would include incorporating both the importing and exporting decisions in a unified model, along the lines of Kasahara and Lapham (2007), and embedding the model here in a multi-country setup in which both the production and the purchasing decisions of imported inputs are jointly studied.

# 5 Appendix

## 5.1 Choice of $n$

A plant with productivity $z$ in country $j$ solves the problem:

$$\pi_j(z) = \max_{n\in[0,1]} \tilde{\pi}_j(z,n) - P_j b(f^n - 1)$$

The Lagrangian of this problem is:

$$L = \tilde{\pi}_j(z,n) - b(f^n - 1) + \lambda_0(n - 0) + \lambda_1(1 - n)$$

where $\lambda_0, \lambda_1 \geq 0$, and the first order necessary condition is:

$$\frac{\partial \tilde{\pi}_j(z,n)}{\partial n} - P_j b f^n \log f = \lambda_1 - \lambda_0 \tag{16}$$

where the derivative of variable profit is given by:

$$\frac{\partial \tilde{\pi}_j(z,n)}{\partial n} = (1-\alpha-\eta) z v_j \gamma_j^n \log \gamma_j$$

From the complementary slackness conditions $\lambda_0 n = 0$ and $\lambda_1(1-n) = 0$, it is clear that only one of $\lambda_0$ or $\lambda_1$ can be positive.

For $\lambda_0 > 0$ and $\lambda_1 = 0$, $\frac{\partial \tilde{\pi}_j(z,n)}{\partial n} < b f^n \log f$, so:

$$(1-\alpha-\eta) z v_j \gamma_j^n \log \gamma_j < P_j b f^n \log f$$

while when $\lambda_1 > 0$ and $\lambda_0 = 0$,

$$(1-\alpha-\eta) z v_j \gamma_j^n \log \gamma_j > P_j b f^n \log f$$

Define two cutoff $z$ levels:

$$z_j^0 = \frac{P_j b \log f}{(1-\alpha-\eta) v_j \log \gamma_j}$$

$$z_j^1 = \frac{f}{\gamma_j} \frac{P_j b \log f}{(1-\alpha-\eta) v_j \log \gamma_j}$$

These come from the first order condition at equality for $n = 0$ and $n = 1$. Since $z^1 = z^0 \frac{f}{\gamma_j}$, $z^1 > z^0$ as long as $f > \gamma_j$, which is the condition assumed in the text.

Now, for $z < z^0$, the left hand side of the first order condition (16) is:

$$\begin{aligned}
& (1-\alpha-\eta)\, z v_j \gamma_j^n \log\gamma_j - P_j b f^n \log f \\
= \; & \frac{P_j b \log f}{z^0} z\gamma_j^n - P_j b f^n \log f \\
= \; & b \log f P_j \left( \frac{z}{z^0}\gamma_j^n - f^n \right) \\
< \; & 0 \text{ for all } n
\end{aligned}$$

This implies $\lambda_0 > 0$ (and hence $\lambda_1 = 0$), so the optimal $n(z) = 0$.

Similarly, for $z > z^1$, the left hand side of (16) is positive for all $n$, implying $\lambda_1 > 0$ (and hence $\lambda_0 = 0$), so the optimal $n(z) = 1$.

For $z \in (z^0, z^1)$, the solution to the first order condition at equality is an interior solution, given by:

$$(1-\alpha-\eta)\, z v_j \gamma_j^n \log\gamma_j = P_j b f^n \log f$$

Taking logs of both sides and rearranging,

$$n_j(z) = \frac{1}{(\log f - \log\gamma_j)} \left( \log z + \log\left( \frac{(1-\alpha-\eta)\, v_j \log\gamma_j}{P_j b \log f} \right) \right)$$

which leads to the solution given in (**??**).

Now, to check the second order condition at this solution, the second derivative of the profit function is:

$$\begin{aligned}
& \frac{\partial^2 \tilde{\pi}_j(z,n)}{\partial n^2} - \frac{\partial^2 b(f^n - 1)}{\partial n^2} \\
= \; & \frac{\partial \tilde{\pi}_j(z,n)}{\partial n} \log\gamma_j - b f^n (\log f)^2
\end{aligned}$$

For the range where $n$ is interior, $\frac{\partial \tilde{\pi}_j(z,n)}{\partial n} = b f^n \log f$, so the second derivative of profit evaluated at the solution is:

$$\begin{aligned}
& \frac{\partial \tilde{\pi}(z,n)}{\partial n} \log\gamma_j - b f^n (\log f)^2 \\
= \; & b f^n \log f \left( \log\gamma_j - \log f \right) \\
< \; & 0
\end{aligned}$$

which is true again by the assumption that $f > \gamma_j$.

## 5.2 Average size of importing plants relative to nonimporting plants

The average size of importing plants is given by

$$\begin{aligned}
\bar{X}_m &= \frac{1}{1-G_j\left(z_j^0\right)}\int_{z_j^0}^{\infty}\eta z \upsilon_j \gamma_j^{n_j(z)} g_j(z)\,dz \\
&= \frac{1}{\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}}\eta\upsilon_j\left(\int_{z_j^0}^{z_j^1} z\gamma_j^{\psi_j\log z+\phi_j}\zeta\underline{z}_j^{\zeta}z^{-\zeta-1}dz+\gamma_j\int_{z_j^1}^{\infty}z\zeta\underline{z}_j^{\zeta}z^{-\zeta-1}dz\right) \\
&= \frac{1}{\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}}\eta\upsilon_j\left(\frac{\gamma_j^{\phi_j}\zeta\underline{z}_j^{\zeta}}{1+\psi_j\log\gamma_j-\zeta}\left(\left(z_j^1\right)^{1+\psi_j\log\gamma_j-\zeta}-\left(z_j^0\right)^{1+\psi_j\log\gamma_j-\zeta}\right)+\frac{\gamma_j\zeta\underline{z}_j^{\zeta}}{\zeta-1}\left(z_j^1\right)^{1-\zeta}\right)
\end{aligned}$$

while the average size of nonimporting plants is

$$\begin{aligned}
\bar{X}_d &= \frac{1}{G_j\left(z_j^0\right)}\int_{\underline{z}_j}^{z_j^0}\eta z\upsilon_j g_j(z)\,dz \\
&= \frac{1}{1-\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}}\eta\upsilon_j\zeta\underline{z}_j^{\zeta}\frac{1}{1-\zeta}\left(\left(z_j^0\right)^{1-\zeta}-\underline{z}_j^{1-\zeta}\right)
\end{aligned}$$

The ratio of these two is

$$\frac{\bar{X}_m}{\bar{X}_d}=\frac{\frac{1}{\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}}\eta\upsilon_j\left(\frac{\gamma_j^{\phi_j}\zeta\underline{z}_j^{\zeta}}{1+\psi_j\log\gamma_j-\zeta}\left(\left(z_j^1\right)^{1+\psi_j\log\gamma_j-\zeta}-\left(z_j^0\right)^{1+\psi_j\log\gamma_j-\zeta}\right)+\frac{\gamma_j\zeta\underline{z}_j^{\zeta}}{\zeta-1}\left(z_j^1\right)^{1-\zeta}\right)}{\frac{1}{1-\underline{z}_j^{\zeta}\left(z_j^0\right)^{-\zeta}}\eta\upsilon_j\zeta\underline{z}_j^{\zeta}\frac{1}{1-\zeta}\left(\left(z_j^0\right)^{1-\zeta}-\underline{z}_j^{1-\zeta}\right)}$$

which can be simplified to yield:

$$\begin{aligned}
\frac{\bar{X}_m}{\bar{X}_d} &= \frac{1-F_j^{im}}{F_j^{im}}\frac{\zeta-1}{\left(F_j^{im}\right)^{(1-\zeta)/\zeta}-1}\left(\frac{1}{1+\psi_j\log\gamma_j-\zeta}\left(\left(e^{1/\psi_j}\right)^{1+\psi_j\log\gamma_j-\zeta}-1\right)+\frac{\gamma_j}{\zeta-1}\left(e^{1/\psi_j}\right)^{1-\zeta}\right) \\
&= \frac{1-F_j^{im}}{F_j^{im}}\frac{1}{\left(F_j^{im}\right)^{(1-\zeta)/\zeta}-1}\left(\frac{\zeta-1}{\zeta}\frac{1}{\varsigma_{2j}-1}\left(e^{\left(\varsigma_{2j}-1\right)\varsigma_{1j}}-1\right)+\lambda_{jj}^{\frac{-\eta}{\theta(1-\alpha-\eta)}}e^{\varsigma_{1j}(1-\zeta)/\zeta}\right)
\end{aligned}$$

where the parameter combinations I already know are:

$$\begin{aligned}
\varsigma_{1j} &= \frac{\zeta}{\psi_j} \\
\varsigma_{2j} &= \frac{1+\psi_j\log\gamma_j}{\zeta}
\end{aligned}$$

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