Monetary and Fiscal Policies In a Heterogeneous-Agent Economy

Hongfei Sun*

Queen's University

August 15, 2012

Abstract

I study the effects of long-run inflation and income taxation in an economy where households face uninsurable idiosyncratic risks. I construct a tractable competitive search framework that generates dispersion of prices, income and wealth. I analytically characterize the stationary equilibrium and the policy effects on individual choices. Quantitative analysis finds that monetary and fiscal policies have distinctive effects on macro aggregates, such as output, savings, wealth dispersion, income and consumption inequalities. There can be a hump-shape relationship between welfare and the respective policies. Overall, welfare can be maximized by a deviation from the Friedman rule, paired with distortionary income taxation.

JEL Classification Codes: E0, E4, E5, E6.

Keywords: Competitive Search, Distributional Effect, Monetary and Fiscal Policies.

*Mail address: Department of Economics, Queen's University, 94 University Avenue, Kingston, Ontario, Canada, K7L 3N6. E-mail address: hfsun@econ.queensu.ca. I thank Gabriele Camera, Allen Head and Shouyong Shi for helpful comments and suggestions. I have also benefited from conversations with seminar and conference participants at Queen's University, 2011 Midwest Macroeconomic Meetings, 2011 Canadian Economic Association Meetings and 2011 Society for the Advancement in Economic Theory Meetings. I gratefully acknowledge the financial support from the Social Sciences and Humanities Research Council of Canada. All errors are my own.

1 Introduction

I construct a tractable framework of competitive search that endogenously generates dispersion of prices, wealth and income. With this framework, I investigate the effects of long-run inflation and income taxation on the macroeconomy, as well as the interaction of the two policies. Wealth and income dispersions prevail in modern economies. In the presence of such distributions, monetary and fiscal policies are likely to have uneven impacts on households, generating non-trivial effects on real activities and welfare. To maximize welfare, policy makers must have a good understanding of the distributional policy effects. In the case where monetary and fiscal authorities are independent of one another, it is also important to understand policy interactions. This is particularly relevant given the context that there have been formulations of central banks and monetary unions, as well as attempts made to form fiscal unions in recent years.

Despite its importance, it is a challenging task to study both policies in a heterogeneous-agent environment. This is because the individual decision problem is generally affected by the endogenous wealth distribution. Solving the model involves handling problems with large dimensions. Policy analysis further enhances the difficulty because it entails computing and comparing equilibrium outcomes in various policy scenarios. Therefore, it is a rare attempt in the literature to examine the distributional effects of monetary and fiscal policies simultaneously.

In my model, households and firms can trade in frictionless and frictional goods markets. The frictional market contains a variety of submarkets and is characterized by competitive search. Households make tradeoffs between the terms of trade and matching probabilities when choosing which submarket to participate in. Search is competitive in that both households and firms take as exogenous the terms of trade and the matching probabilities across all submarkets. In equilibrium, a submarket that requires a higher payment per transaction offers a higher quantity of goods per transaction and also a higher probability for a buyer to be matched for a transaction. Households face uninsurable idiosyncratic shocks on labor preferences. Money is the only asset available for self-insurance. The idiosyncratic risks lead to diverse decisions on consumption, precautionary savings, labor supply and trading strategies. In equilibrium there will arise dispersions of price, income and wealth.

I analytically characterize the stationary equilibrium and the policy effects on individual decisions. Policies include long-run money growth maintained by lump-sum transfers and a proportional income tax. The government balances budget and rebates tax revenues in a lump-sum manner. I numerically examine the policy implications on the macroeconomy. The key findings are the following: First, both policies can directly affect the intensive and extensive margins (i.e., the quantity per trade and the trading probability per buyer), and also indirectly through altering household choices of spending in the frictional market. Quantitatively, the indirect effects tend to dominate the direct ones. Inflation can stimulate spending and thus has a positive overall effect on both margins, while income taxation does the opposite. Overall, inflation increases output and consumption but decreases precautionary savings. Income taxation has the opposite effects.

Second, the effect of inflation on wealth dispersion depends on the tax regime. The positive effect tends to dominate at lower tax rates while the negative effect tends to dominate at higher tax rates. Income taxation decreases wealth dispersion regardless of the monetary policy. Moreover, inflation decreases income inequality but increases consumption inequality. Taxation has a positive effect on both income and consumption inequalities.

Finally, inflation and taxation can respectively have a hump-shape relationship with welfare. The welfare-improving role of inflation is sensitive to the status of the fiscal policy. It is important to coordinate the two policies. Welfare can be maximized by a deviation from the Friedman rule together with distortionary income taxation. When the monetary and fiscal authorities are independent authorities, a change of policy by one has non-trivial implications on the optimal policy choice of the other.

This framework embeds the model structure of Menzio, Shi and Sun (2011; henceforth MSS) in a Lagos-Wright environment (2005; henceforth LW). MSS construct a tractable monetary environment with non-degenerate money distributions. The model is block recursivity in the sense that individual decision problems can be solved independently from the endogenous distributions.[1] This impressive feature is due to a carefully-designed competitive search environment. Unlike commonly-studied bilateral bargaining in a search environment, individual traders cannot affect any of the market specifications, *i.e.* terms of trade and matching probabilities, due to the competitive nature of the search process. With given trading specifications, a household need not consider the amount of money that its potential trading partner might have, when making its optimal decisions. As a result, the household decision problem can be solved without involving the endogenous money distribution, which greatly reduces the state space and renders the model tractable.

Nevertheless, MSS abstracts away from policy considerations such as money growth. Once policies are introduced, the model will no longer be block recursive. Therefore, I introduce the Lagos-Wright feature into the model, to analyze policy effects without losing tractability. My model is block recursive even in the context of both monetary and fiscal

[1]The concept of block recursivity was first applied to economics by the seminal work of Shi (2009) on equilibrium wage-tenure contracts. Gonzalez and Shi (2010) and Menzio and Shi (2010a,b; 2011) further examine the functioning of labor markets using the notion of block recursive equilibrium.

policies, given the key features of competitive search, quasi-linear preferences and access to frictionless markets. The convenience of block recursivity is particularly important for my purpose of exploring optimal policy combination, instead of simply monetary or fiscal policy alone. Note that the LW feature is known for making the equilibrium money distribution degenerate. To avoid this, I use uninsurable idiosyncratic shocks on labor preferences to drive a non-degenerate equilibrium distribution. Without the shocks on labor preferences, the model works in a similar fashion to the competitive-search setup of Rocheteau and Wright (2005; henceforth RW), which focuses on results with degenerate distributions. The key difference between the no-preference-shock version of my model and RW is that the decision maker is a buyer/worker couple, instead of buyers and sellers being individuals with different preferences.

This paper is closely related to the literature of heterogeneous-agent economies that study the distributional effect of monetary policy. In these models, money is valued either because of the cash-in-advance constraint (*e.g.* Imrohorglu, 1992; Erosa and Ventura, 2002; Camera and Chien, 2011), or for precautionary purpose (*e.g.* Akyol, 2004; Wen, 2010; Dressler, 2011), or due to search frictions (*e.g.* Molico, 2006; Boel and Camera, 2009; Chiu and Molico, 2010). In most of these models, agents trade in Walrasian markets. Such models are not able to generate equilibrium price dispersion. The search model can be a natural environment to have dispersion of prices, *e.g.* Molico (2006) and Chiu and Molico (2010), where agents trade in decentralized markets and bargain bilaterally. In contrast to search models with bargaining, my model features competitive search, which significantly improves tractability. Finally, none of the above literature examines the distributional effects of monetary and fiscal policies simultaneously, which is in contrast the main goal of my paper.

The rest of this paper is organized as follows. Section 2 describes the physical model environment. Section 3 characterizes the stationary equilibrium. Section 4 presents analytical policy effects. Section 5 discusses numerical results. Finally, Section 6 concludes the paper.

2 A Unified Macroeconomic Framework

2.1 The environment

Time is discrete and continues forever. Each time period consists of two sub-periods. The economy is populated by a measure one of *ex ante* identical households. Each household consists of a worker and a buyer. All households consume general goods in the first sub-period and special goods in the second sub-period. There are different types of special goods. Every period a household faces a random preference shock, which determines with equal probability the type of special goods to consume in the current period. Household members share income, consumption and labor cost. The preference of a household in a time period is

$$\mathcal{U}(y, q, l) = U(y) + u(q) - \theta l, \tag{1}$$

where y is consumption of general goods, q is consumption of special goods and l is labor input in a time period. The variable θ measures the random disutility per unit of labor. It is *i.i.d.* across households and over time, and is drawn from the probability distribution $F(\theta)$ with support $[\underline{\theta}, \bar{\theta}]$, where $0 < \underline{\theta} < \bar{\theta} < \infty$. The value of θ is realized at the beginning of every period, before any decisions are made. The functions u and U are twice continuously differentiable and have the usual properties: $u' > 0$, $U' > 0$; $u'' < 0$, $U'' < 0$; $u(0) = U(0) = u'(\infty) = U'(\infty) = 0$; and $u'(0)$ and $U'(0)$ being large but finite. Households discount future with factor $\beta \in (0, 1)$. All goods are perishable across sub-periods. There is no insurance on idiosyncratic risks. Nor is borrowing or lending feasible among households. There is a fiat object called money, which is storable without cost.

General goods are traded in competitive and frictionless markets. Special goods are traded in frictional markets in that trades are decentralized and that buyers and sellers are randomly matched in pair-wise meetings. Trading frictions are driven by households' random demand for special goods. There is a measure one of competitive firms. Firms hire workers from households, who own equal shares of all firms. The labor market is competitive. Labor is hired at the beginning of a period and is used in production for both general and special goods. Each firm can organize production of general goods and one type of special goods. In each period the frictionless goods market opens in the first sub-period, followed by the frictional market in the second sub-period.[2]

Trading in a frictional market is characterized by competitive search. This market contains a variety of submarkets. Each submarket is characterized by (x, q, b, s), where (x, q) are the terms of trade and (b, s) are the respective matching probabilities for a buyer and a shop. Search is competitive in the sense that households and firms take as given the characteristics of all submarkets, when making their trading decisions. Buyers and shops are randomly matched in a pair-wise manner because households and firms cannot coordinate. A buyer can enter at most one submarket in each period. Firms have free entry to all submarkets and choose the measure of shops to operate in each submarket. The cost of operating a shop for one period is $k > 0$ units of labor. Moreover, producing q units

[2] In this environment, one can also assume that the frictionless and frictional goods markets open simultaneously in a period. The results are similar to the sequential order of markets. Here I adopt the sequential structure for expositional convenience.

of special goods requires $\psi(q)$ units of labor, where ψ is twice continuously differentiable with the usual properties: $\psi' > 0$, $\psi'' \geq 0$ and $\psi(0) = 0$. In equilibrium free entry of firms is such that the characteristics of submarkets are consistent with the specified ones. The matching technology has constant returns to scale and is such that $s = \mu(b)$. In equilibrium the matching probabilities of each submarket become functions of the terms of trade (x, q), as is shown in (4). Therefore, a submarket can be sufficiently indexed by (x, q). I impose the following assumption:

Assumption 1 *For all $b \in [0,1]$, the matching function $\mu(b)$ satisfies: (i) $\mu(b) \in [0,1]$, with $\mu(0) = 1$ and $\mu(1) = 0$, (ii) $\mu'(b) < 0$, and (iii) $[1/\mu(b)]$ is strictly convex, i.e., $2(\mu')^2 - \mu\mu'' > 0$.*

I focus on the steady state equilibrium and suppress the time index throughout the paper. The per capita money stock is fixed at M for now. I will allow money growth and income taxation when I analyze policy effects in section 4. Labor is the numeraire. In particular, let m denote the real value of a household's money balance measured in terms of labor units. Let w denote the normalized wage rate, which is the nominal wage rate divided by the money stock M. Then the dollar amount associated with a balance m is $(wM)m$.

2.2 A firm's decision

In the frictionless market, a representative firm takes the general-good price as given and chooses output Y to maximize profit. For simplicity, it takes one unit of labor to produce one unit of general goods. Let p be the price of general goods, measured in terms of labor. In the frictional market, the firm takes the terms of trade for each submarket, (x, q), as given and chooses the measure of shops, $dN(x, q)$, to set up in each submarket. Recall that a shop is matched by a buyer with probability $s(x, q)$. For a particular shop in the submarket, the operational cost is k units of labor and the expected cost of production is $\psi(q)s(x, q)$ units of labor. A shop's expected revenue is $xs(x, q)$, where the revenue x is measured in labor units. The firm's total profit in a period is

$$\pi = \max_{Y} \{pY - Y\} + \max_{dN(x,q)} \left\{ \int \{xs(x,q) - [k + \psi(q)s(x,q)]\} dN(x,q) \right\}. \tag{2}$$

The first item on the right-hand side denotes the firm's profit in the frictionless market and the second item its profit in the frictional market. Free entry of firms implies that the firm earns zero profit and thus $p = 1$ in equilibrium. Zero-profit in the frictional market requires

$$s(x,q)\left[x-\psi(q)\right]\leq k \quad \text{and} \quad dN(x,q)\geq 0, \tag{3}$$

where the two inequalities hold with complementary slackness. In particular, if the expected profit of operating a shop $s(x,q)[x-\psi(q)]-k<0$, then the firm will choose $d\,N(x,q)=0$. If $s(x,q)[x-\psi(q)]-k=0$, the firm is indifferent across any $dN(x,q)\in(0,\infty)$. It is not an equilibrium to have $s(x,q)[x-\psi(q)]-k>0$ because it would attract the firm to choose $d\,N(x,q)=\infty$, which contradicts zero-profit. As is common in the competitive search literature,[3] I focus on equilibria where condition (3) also holds for submarkets not visited by any buyer. Given that the trading probabilities $b,s\in[0,1]$, condition (3) implies

$$s(x,q)=\mu(b(x,q))=\begin{cases}\frac{k}{x-\psi(q)}, & \text{if } k<x-\psi(q)\\ 1, & \text{if } k\geq x-\psi(q).\end{cases} \tag{4}$$

The free-entry condition pins down the matching probabilities in a submarket as functions of the terms of trade. Indeed, a submarket can be sufficiently indexed by the terms of trade, (x,q).

2.3 A household's decision

2.3.1 Decision in the frictionless market

Let $W(m,\theta)$ be a household's value at the beginning of a period with real money balance m and the random realization θ. Given price p and the characteristics of all frictional submarkets, a household maximizes its value by choosing consumption of general goods $y\geq 0$, labor input $l\geq 0$, the real balance to spend in the frictional market $z\geq 0$, and precautionary savings $h\geq 0$.[4] If the household's buyer is matched with a shop in the frictional market, then the buyer spends z and the household carries h into the following period. Otherwise, the household carries a balance $z+h$ into the following period. Moreover, $z+h\leq\bar{m}$, where $\bar{m}$ is the maximum real money balance that a household can carry

[3]For example, Moen (1997), Acemoglu and Shimer (1999), and Menzio, Shi and Sun (2011). Given such beliefs off the equilibrium, markets are complete in the sense that a submarket is inactive only if the expected revenue of the only shop in the submarket is lower than its expected cost given that some buyers are present in the submarket. Such a restriction can be justified by a "trembling-hand" argument that an infinitely small measure of buyers appear in every submarket exogenously.

[4]Without the θ-shock, the household has no incentive to choose $h>0$, not even when θ is permanently heterogeneous across households. With this preference shock, the household saves (more than any unspent balance z) in a low-θ state to help ease future disutility of labor. Therefore, the role of the preference shock here is not only to generate endogenous distributions but also to induce precautionary savings.

across periods. I assume $0 < \bar{m} < U'^{-1}\left(\bar{\theta}\right)$.[5] The dividend Π is paid to the household at the end of a period. In equilibrium $\Pi = 0$ because firms earn zero profit.

The value $W\left(m,\theta\right)$ satisfies the following Bellman equation:

$$\begin{aligned} W\left(m,\theta\right) &= \max_{(y,l,z,h)} \quad \left\{U\left(y\right) - \theta l + V\left(z,h\right)\right\} \\ & \quad\; s.t. \quad py + z + h \leq m + l. \end{aligned} \tag{5}$$

The constraint in the above is a budget constraint.[6] The function $V\left(z,h\right)$ is the household's value at the beginning of the second sub-period, i.e., immediately before the frictional market opens. Because the analysis on the decisions of frictional trading is much involved, I will postpone fully characterizing V until the next sub-section. In Lemma 3, I show that V is differentiable and concave in z and h. For now, I take such information as given. Given $U' > 0$, the budget constraint must hold with equality and thus

$$l = y + z + h - m, \tag{6}$$

where I have incorporated the equilibrium price $p = 1$. For now I assume that the choice of l is interior, which I will verify later. Using (6) to eliminate l in the objective function

[5] The assumption of an exogenous upper bound $\bar{m}$ is critical for providing regulation on an agent's labor choice and for the LW feature to be effective in this model. If we directly impose an upper bound on the labor choice, say $\bar{l}$, agents with low enough realization of θ will choose $l = \bar{l}$. This means we cannot rewrite the agent's decision problem as (7), which further implies that the decisions in the frictional market sometimes depend on m. Then the model is no longer block recursive because the equilibrium wage rate, and hence the submarket specifications (i.e. terms of trade and matching probabilities) will be affected by the equilibrium money distribution. In other words, conditions (4) and (33) will no longer hold and must be adjusted to incorporate the impact of the money distribution. If we do not impose any upper bound on l or m, the agent may choose to exert an infinite amount of labor and hold an infinite amount of money when $\theta = \underline{\theta}$. This will render the problem at hand not interesting. Therefore, only a carefully chosen upper bound $\bar{m}$ can help the model maintain block recursivity in an interesting way.

[6] Note that the household's spending in the frictional market is not constrained by its initial money holdings of the period. This is because in this environment both money and firm IOUs can be used in all transactions. Firm IOUs take the form of a firm's promise of wage payments at the end of a period, in terms of money. Firm IOUs are settled in a central clearinghouse at the end of a period. Such IOUs are enforceable because firms are large (in the sense that each of them owns a positive measure of shops) and thus they have deterministic revenues and costs, although individual shops face matching risks. Firms last for one period and new ones are formed at the beginning of the next. Thus firm IOUs can be circulated for only one period. Nevertheless, personal IOUs of households are not accepted as a medium of exchange because households face idiosyncratic preference and matching risks and there is no enforcement on their IOUs. No particular type of goods is *cash goods* in this environment because both fiat money and firm IOUs can be used in all transactions. This is in contrast with standard money search models, where goods traded in the frictional markets are considered cash goods. In these models, fiat money must be used as a medium of exchange to overcome the lack of double coincidence of wants and record-keeping of individual traders.

yields

$$W(m,\theta) = \theta m + \max_{y}\{U(y) - \theta y\} + \max_{z,h}\{V(z,h) - \theta(z+h)\}. \tag{7}$$

The optimal choices must satisfy the following first-order conditions:

$$U'(y) \leq \theta, \quad \text{and} \quad y \geq 0 \tag{8}$$

$$V_z(z,h)\begin{cases} \leq \theta, & \text{and} \quad z \geq 0 \\ \geq \theta, & \text{and} \quad z \leq \bar{m} - h, \end{cases} \tag{9}$$

$$V_h(z,h)\begin{cases} \leq \theta, & \text{and} \quad h \geq 0 \\ \geq \theta, & \text{and} \quad h \leq \bar{m} - z \end{cases} \tag{10}$$

where all sets of inequalities hold with complementary slackness. Given $0 < \bar{m} < U'^{-1}(\bar{\theta})$, it follows that

$$\theta \leq \bar{\theta} < U'(\bar{m}) < U'(0)$$

for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Then condition (8) implies that the choice of y is always interior and satisfies

$$U'(y) = \theta. \tag{11}$$

Clearly, the household's current money balance m does not affect the optimal choices of y, z or h, although it does affect l. Let the policy functions be $y(\theta)$, $z(\theta)$, $h(\theta)$ and $l(m,\theta)$. Note that $z(\theta) + h(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and that $m \leq \bar{m}$. Therefore, (6) and (11) imply that $l(m,\theta) \geq U'^{-1}(\bar{\theta}) - \bar{m} > 0$ for all (m,θ). That is, all households' choices of l are interior given that $U'^{-1}(\bar{\theta}) > \bar{m}$. Given (7), the value function W is

$$W(m,\theta) = W(0,\theta) + \theta m, \tag{12}$$

where

$$W(0,\theta) = U(y(\theta)) - \theta y(\theta) + V(z(\theta), h(\theta)) - \theta[z(\theta) + h(\theta)]. \tag{13}$$

The preceding exposition proves the following lemma:

Lemma 1 *The value function W is continuous and differentiable in (m,θ). It is also affine in m.*

2.3.2 Decision in the frictional market

The household chooses whether to participate in the frictional market. If yes, then it chooses which submarket to enter and search for a trade. Given balances z and h, the household is faced with the following problem at the beginning of the second sub-period:

$$\max_{x,q} \left\{ b(x,q)\left[u(q) + \beta E\left[W(z-x+h,\theta)\right]\right] + \left[1-b(x,q)\right]\beta E\left[W(z+h,\theta)\right]\right\}, \tag{14}$$

where $q \geq 0$, $x \leq z$ and $b(x,q)$ is determined by (4). It is convenient to use condition (4) to eliminate q in the above objective function. Given linearity of W, the problem in (14) simplifies to

$$\max_{x\leq z,\ b\in[0,1]} b\left\{u\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right) - \beta E(\theta)\,x\right\} + \beta E\left[W(z+h,\theta)\right]. \tag{15}$$

The optimal choices satisfy the following first-order conditions

$$\frac{u'\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right)}{\psi'\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right)} - \beta E(\theta) \geq 0, \quad \text{and} \quad x \leq z, \tag{16}$$

$$u\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right) - \beta E(\theta)\,x + \left[\frac{u'\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right)}{\psi'\left(\psi^{-1}\left(x-\frac{k}{\mu(b)}\right)\right)}\right]\frac{kb\mu'(b)}{[\mu(b)]^2} \leq 0, \quad \text{and} \quad b \geq 0, \tag{17}$$

where the two sets of inequalities hold with complementary slackness. It has been taken into account in condition (17) that $b = 1$ cannot be an equilibrium outcome. This is because $b = 1$ implies $s = 0$. This further implies that firms choose $d\,N(z,q) = \infty$ and earn strictly positive profit, which violates free entry in equilibrium. Let the policy functions be $x(z)$, $b(z)$ and $q(z)$, where $q(z)$ is implied by condition (4):

$$q(z) = \psi^{-1}\left(x(z) - \frac{k}{\mu(b(z))}\right). \tag{18}$$

If $b(z) = 0$, then the choices of x and q are irrelevant. In this case, the household chooses not to participate in the frictional submarket. Without loss of generality, I impose $x(z) = z$ if $b(z) = 0$.

Now consider z such that $b(z) > 0$. It is obvious from (15) that the optimal choices are independent of z if the money constraint does not bind, i.e., $x(z) < z$. Define $\Phi(q) \equiv$

$u'(q)/\psi'(q)$. If $x(z) < z$, then (16) holds with equality. Then conditions (16) and (18) imply

$$q^* = \Phi^{-1}[\beta E(\theta)]. \tag{19}$$

Given q^*, using (18) to eliminate x in (17) yields

$$u(q^*) - \beta E(\theta)\left[\psi(q^*) + \frac{k}{\mu(b^*)}\right] + \left[\frac{u'(q^*)}{\psi'(q^*)}\right]\frac{kb^*\mu'(b^*)}{[\mu(b^*)]^2} = 0. \tag{20}$$

It is straightforward to show that the left-hand side of (20) is strictly increasing in b^*. Moreover, $b^* > 0$ exists and is unique if $E(\theta)$ satisfies

$$u\left(\Phi^{-1}[\beta E(\theta)]\right) - \beta E(\theta)\left[\psi\left(\Phi^{-1}[\beta E(\theta)]\right) + k\right] > 0. \tag{21}$$

Given unique values of q^* and b^*, x^* is uniquely determined by

$$x^* = \psi(q^*) + \frac{k}{\mu(b^*)}. \tag{22}$$

Therefore, if condition (21) holds, then $x(z) = z$ for all $z < x^*$ and $x(z) = x^*$ for all $z \geq x^*$. If condition (21) fails to hold, then $x(z) = z$ for all $z \geq 0$. Define $\hat{z}$ as the maximum value such that $x(z) = z$. Thus $\hat{z} = x^*$ if (21) holds and $\hat{z} = \infty$ otherwise.

In this environment, it is not necessary for the household to choose z higher than the amount x that it plans to spend in the frictional market. Without loss of generality, I impose $x(z) = z$ in the rest of the analysis. In particular, consider $z \in [0, \hat{z}]$. Given such z, the problem in (15) becomes

$$B(z) + \beta E[W(z+h, \theta)], \tag{23}$$

where

$$B(z) = \max_{b\in[0,1]} b\left\{u\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta) z\right\}. \tag{24}$$

The value $B(z)$ is the household's expected trade surplus. If $b > 0$, it must be the case that $q > 0$ and that the surplus from trade is strictly positive:

$$u\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right) - \beta E(\theta) z > 0. \tag{25}$$

Moreover, the optimal choice of b satisfies condition (17) given $x = z$.

Lottery choice. It is necessary to mention that the value function $B(z)$ may not be concave in z because the objective function in (24) may not be jointly concave in its state and choice variables, (z, b). This objective function involves the product between the choice variable b itself and a function of b. Even if both of these two terms are concave, the product may not be jointly concave. Above all, it is unclear whether either of the two terms is a concave function of z, given that b is a choice variable and is yet to be determined. To make the household's value function concave, I introduce lotteries with regards to the balance z, as in Menzio, Shi and Sun (2011). In particular, lotteries are available every period immediately before trading in the frictional market takes place.

A lottery is characterized by (L_1, L_2, π_1, π_2). If a household plays the lottery, it will win the prize L_2 with probability π_2. The household loses the lottery with probability π_1, in which case it receives a payment of L_1. There is a complete set of lotteries available. Given z, a household's optimal choice of lottery solves:

$$\tilde{V}(z) = \max_{(L_1, L_2, \pi_1, \pi_2)} \{\pi_1 B(L_1) + \pi_2 B(L_2)\} \tag{26}$$

subject to

$$\pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0;$$
$$\pi_1 + \pi_2 = 1; \quad \pi_i \in [0, 1] \text{ for } i = 1, 2.$$

Denote the policy functions as $L_i(z)$ and $\pi_i(z)$, respectively, where $i = 1, 2$. If the household is better off not playing any lottery, it is trivial to see that $L_1(z) = L_2(z) = z$.

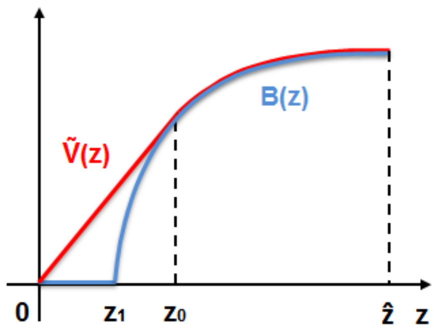


Figure 1. Lottery Choice

Figure 1 illustrates how the lottery can help make the value function $\tilde{V}(z)$ concave, even though the function $B(z)$ has some strictly convex part. It is intuitive to see that a household will choose to play a lottery if it has a very low balance. As is shown in Figure 1, for any balance $z \in (0, z_0)$, it is optimal for the household to participate in the lottery offering the prize z_0. The lottery makes $\tilde{V}(z)$ linear whenever $B(z)$ is strictly convex. The properties of z_0 are presented in part (iii) of Lemma 2.

2.3.3 Properties of value and policy functions

Lemma 2 *The value function $B(z)$ is continuous and increasing in $z \in [0, \hat{z}]$. The value function $\tilde{V}(z)$ is continuous, differentiable, increasing and concave in $z \in [0, \hat{z}]$. For z such that $b(z) = 0$, the value function $B(z) = 0$. In this case, the choice of q is irrelevant. There exists $z > 0$ such that $b(z) > 0$ if and only if there exists $q > 0$ that satisfies*

$$u(q) - \beta E(\theta)[\psi(q) + k] > 0. \tag{27}$$

For z such that $b(z) > 0$, the value function $B(z)$ is differentiable, $B(z) > 0$ and $B'(z) > 0$. Moreover, the following results hold: (i) The policy functions $b(z)$ and $q(z)$ are unique and strictly increasing in z. In particular, $b(z)$ solves

$$u(q(z)) - \beta E(\theta) z + \left[\frac{u'(q(z))}{\psi'(q(z))}\right] \frac{kb(z)\mu'(b(z))}{[\mu(b(z))]^2} = 0, \tag{28}$$

where

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right). \tag{29}$$

Moreover, $b(z)$ strictly decreases in $E(\theta)$ while $q(z)$ strictly increases in $E(\theta)$; (ii) There exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z \in (z_1, \hat{z}]$; (iii) There exists $z_0 > z_1$ such that a household with $z < z_0$ will play the lottery with the prize z_0. Moreover, $B(z_0) = \tilde{V}(z_0) > 0$, $B'(z_0) = \tilde{V}'(z_0) > 0$ and $b(z_0) > 0$.

Lemma 2 (see Appendix A for a proof) summarizes the properties of the household's value and policy functions in the frictional market. According to part (i), the optimal choices of (q, b) are strictly increasing in z when the household chooses $b > 0$ to participate in frictional trading. In this case, the higher a balance the household spends, the higher a quantity it obtains and the higher the matching probability at which it trades. As a result, households endogenously sort themselves into different submarkets based on their balances to spend. For any given z, a higher value of $E(\theta)$ implies a lower matching probability for the buyer and a higher amount of goods to be purchased by the buyer. The intuition is the following: Given higher $E(\theta)$, it becomes more costly for firms to hire labor. Firms respond by setting up fewer shops in a submarket but increasing quantity produced per trade. This helps save the fixed cost of operating shops and steer more labor into production. All else equal, fewer shops in a submarket lead to a higher matching probability for a shop, which tends to increase a firm's revenue. Thus the firm can afford to offer a higher quantity per trade, even though it requires a higher labor input. In this case, households face a lower matching probability for a buyer. Nevertheless, the households are compensated by an increase in the quantity per purchase.

Recall that V is the value of a household at the beginning of the second sub-period before trading decisions are made. Given (12), (23), (24) and (26), V is given by

$$V(z,h) = \tilde{V}(z) + \beta E[W(0,\theta)] + \beta E(\theta)(z+h). \tag{30}$$

Clearly V is linear in h. Recall from (7) that the household chooses z and h to solve the following problem:

$$\max_{z,h} \{V(z,h) - \theta(z+h)\}. \tag{31}$$

It follows that $h(\theta) > 0$ if and only if θ is such that $\tilde{V}'(z(\theta)) = 0$, that is, if $z(\theta) = x^*$ as defined in (22). In this case, $h(\theta) = \bar{m} - x^*$. For all θ such that $z(\theta) < x^*$, it must be that $h(\theta) = 0$. Then (6) implies

$$l(m,\theta) = \begin{cases} y(\theta) + \bar{m} - m, & \text{if } z(\theta) = x^* \\ y(\theta) + z(\theta) - m, & \text{if } z(\theta) < x^*. \end{cases} \tag{32}$$

Given Lemma 1 and Lemma 2, it is trivial to derive the following lemma:

Lemma 3 *(i) V is continuous and differentiable in (z,h). The function $V(\cdot,h)$ is increasing and concave in $z \in [0,\hat{z}]$, with $V(z,h) \geq \beta E[W(0,\theta)] > 0$ for all z. Moreover, $V(z,\cdot)$ is affine in h. If θ is such that $z(\theta) = x^*$, then $h(\theta) = \bar{m} - x^*$. Otherwise, $h(\theta) = 0$; (ii) Policy functions $y(\theta)$, $z(\theta)$ and $h(\theta)$ are decreasing functions. The policy function $l(m,\theta)$ is decreasing in both m and θ.*

3 Stationary Equilibrium

Definition 1 *A stationary equilibrium consists of household values $(W, B, \tilde{V}, V)$ and choices $(y, l, z, h, (q, b), (L_1, L_2, \pi_1, \pi_2))$; firm choices $(Y, dN(x,q))$; price p and wage rate w. These elements satisfy the following requirements: (i) Given the realizations of shocks, asset balances, prices and terms of trade, a household's choices solve (7), (24), (26) and (30), which induce the value functions $W(m,\theta)$, $B(z)$, $\tilde{V}(z)$ and $V(z,h)$; (ii) Given prices and terms of trade, firms maximize profit and solve (2); (iii) Free entry condition: The function $s(x,q)$ satisfies (4); (iv) All labor markets, general-good markets and money markets clear; (v)*

Stationarity: All quantities, prices and distributions are time invariant; (vi) Symmetry: Households in the same idiosyncratic state make the same optimal decisions.

The above definition is self-explanatory. The labor-market-clearing condition implies that the equilibrium normalized wage rate w^* is determined by

$$(w^*)^{-1} = \int_{\underline{\theta}}^{\bar{\theta}} h(\theta)\, dF(\theta) + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \pi_i(z(\theta))\,[1 - b(L_i(z(\theta)))]\, L_i(z(\theta))\, dF(\theta)\,. \tag{33}$$

I derive the above market-clearing condition in Appendix D. Given the equilibrium definition, I have the following theorem (see Appendix B for a proof):

Theorem 2 *A stationary equilibrium exists. It is unique if and only if the lottery choices* $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ *are unique for all* $z(\theta)$*. Moreover, the following results hold: (i) The general-good consumption* $y(\theta) > 0$ *for all* θ*; (ii) If there does not exist* $q > 0$ *that satisfies condition (27), then* $z(\theta) = 0$ *for all* θ*. Otherwise,* $z(\theta) > 0$ *for some* θ*.*

According to Theorem 2, frictionless markets are always active, while frictional markets are not. A necessary condition for frictional markets to be used is that condition (27) holds for some $q > 0$. This condition depends on the preferences and the production technology for special goods, the discount factor and the value of $E(\theta)$. Intuitively, if the utility derived from consuming special goods is too low, or if the production cost of special goods is too high, consumption of special goods can become too costly, especially considering the uncertainty involved in obtaining such goods. Similarly, if $E(\theta)$ is too high, then the cost of labor is high, which drives up the cost of producing special goods and hence suppresses the demand. These results are consistent with the findings in Camera (2000). In a model without distributional components, Camera shows that the frictional market is used in equilibrium when households can have sufficiently high expected consumption relative to that in the frictionless market.

Note that this equilibrium is remarkably tractable. None of the decision problems (7), (24), (26) and (30) is affected by the endogenous money distribution. Therefore, one can solve these decision problems first, and then use the household optimal decisions to derive the equilibrium aggregates, together with income and wealth distributions. This model property is called block recursivity.

4 Policy Effects

I now analyze the effects of monetary and fiscal policies. Consider that the money stock per capita evolves according to $M' = \gamma M$, where $\gamma \geq \beta$ is the money growth rate and M' is the money stock of the next period. Money growth is achieved by a lump-sum transfer from the government to households, and vice versa for money contraction. The government also imposes a proportional tax rate $\tau \in [0, 1)$ on wage income. The government balances its budget every period. All tax revenues are redistributed from the government to households in a lump-sum manner. Transfers are made at the beginning of each period. All tax payments and transfers are made with money. The money market opens in the second subperiod of a period.

First, it is straightforward to show that $\partial y(\theta)/\partial \tau \leq 0$ and $\partial y(\theta)/\partial \gamma = 0$. The former is a standard income effect and the latter is straightforward because the choice if y is not affected by any monetary factor as in (8). Second, policies directly affect equilibrium trading strategies, $q(z)$ and $b(z)$. The effect on $q(z)$ is called the intensive-margin effect and the one on $b(z)$ the extensive-margin effect. Even with policies, all the results in Lemma 2 still hold, except that the policy functions $b(z)$ and $q(z)$ are jointly determined by

$$u(q(z)) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z + \left[\frac{u'(q(z))}{\psi'(q(z))}\right] \frac{kb(z)\mu'(b(z))}{[\mu(b(z))]^2} = 0 \tag{34}$$

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right), \tag{35}$$

instead of (28) and (29). Then follows a proposition on policy effects (see Appendix C for a proof):

Proposition 1 *For all z such that $b(z) > 0$, the following results hold:*

(i) Given z, fiscal policy τ has a positive direct effect on the intensive margin and a negative direct effect on the extensive margin, i.e., $\frac{\partial q(z;\tau)}{\partial \tau} > 0$ and $\frac{\partial b(z;\tau)}{\partial \tau} < 0$;

(ii) Given z, monetary policy γ has a negative direct effect on the intensive margin and a positive direct effect on the extensive margin, i.e., $\frac{\partial q(z;\gamma)}{\partial \gamma} < 0$ and $\frac{\partial b(z;\gamma)}{\partial \gamma} > 0$;

(iii) Fiscal policy has a negative effect on spending z, i.e., $\frac{\partial z(\theta;\tau)}{\partial \tau} < 0$. By decreasing $z(\theta)$, fiscal policy τ has a negative indirect effect on both margins b and q. The effect of monetary policy on z is ambiguous and hence the indirect effect of monetary policy on both margins is also ambiguous;

(iv) On the extensive margin, the overall fiscal policy effect is negative. The fiscal policy effect on the intensive margin is ambiguous given the opposing direct and indirect effects.

Proposition 1 characterizes both direct and indirect policy effects on intensive and extensive margins, $b(z)$ and $q(z)$. Each policy has a direct impact on the choices of b and q, as well as an indirect one through affecting the choice of spending z. Part (i) summarize the direct effects of proportional income taxes. A higher income tax rate τ makes households frugal on spending. For any given balance, a household chooses to visit a submarket that offers a higher quantity of goods per trade, which is a positive effect on the intensive margin. In such a submarket, a firm's cost of production per trade is higher. Thus it reduces overall cost by setting up a smaller measure of shops in this submarket. This imposes a negative effect along the extensive margin.

Part (ii) of Proposition 1 lists the direct monetary policy effects on the margins. In particular, the real value of a money balance over time decreases with money growth. A household responds by sending its buyer to a submarket with a higher matching probability b, in order to increase the chance of spending money in the current period. In such a submarket, the matching probability for a shop is lower, which all else equal implies a lower profit for firms. Zero profit condition requires that firms must be compensated by producing a lower quantity per trade. These results of monetary policy are standard and have been well-documented in the money search literature.

Part (iii) shows that income taxes reduce spending z, which is an income effect. Thus taxation also has a negative indirect effect on the margins because b and q are increasing in z. Money growth has an ambiguous effect on z. Intuitively, inflation tax weakens a household's incentives for precautionary savings, which causes a positive effect on z as the household switches some savings (future consumption) to spending on current consumption. On the other hand, money growth also has a negative effect on z as the household has the incentives to reduce spending in the frictional market. This is because an unmatched participation in the frictional market will result in the household carrying the unspent money balance into the future and bearing the inflation tax. If the household values consumption of special goods high enough, the first effect dominates and thus inflation can have a positive indirect effect on both margins by increasing $z(\theta)$. The result in part (iv) is self-explanatory and follows directly from the previous parts.

Finally, it is worthwhile mentioning that all the results in parts (i), (iii) and (iv) of Proposition 1 are novel analytical results in the current literature on search-theoretic models of money. The literature has rarely analyzed the effect of fiscal policy on frictional trading strategies, let alone in a heterogeneous-agent environment.

5 Numerical Results

I further analyze the policy effects using numerical examples. I employ the following functional forms:

$$u(c) = \frac{(c+a)^{1-\sigma} - a^{1-\sigma}}{1-\sigma}; \quad U(c) = U_0 \frac{(c+a)^{1-\sigma_u} - a^{1-\sigma_u}}{1-\sigma_u};$$
$$\psi(q) = q^{\chi}; \quad \mu(b) = (1-b^{\rho})^{1/\rho}; \quad F(\theta) \text{ is uniform on } \theta \in \left[\underline{\theta}, \bar{\theta}\right]. \tag{36}$$

As the benchmark computation, I adopt the following parameter values:

β	U_0	a	σ	σ_u	χ	k	ρ	$\bar{m}$	$\underline{\theta}$	$\bar{\theta}$	γ	τ
0.96	20	10^{-3}	1.01	2	1.5	0.01	1	2	0.5	1.5	$[\beta, 2]$	$[0, 0.25]$

(37)

The model period is set to be one year. The discount factor β is chosen to match an annual interest rate of 4%. For the utility functions, I take $\sigma = 1.01$ as a normalization. The value $\sigma_u = 2$ is typical in the macro literature. The constant a is used to satisfy $U(0) = 0$. For the production function, χ is chosen such that the production function takes the form of output $Q = L^{2/3}$ as in a standard RBC model. The cost k is set to a small number. The matching technology is the so-called telegraph matching function. With $\rho = 1$, the number of matches in a submarket with N_b buyers and N_s shops is $N_b N_s/(N_b + N_s)$. I restrict my attention to policy parameters $\gamma \in [\beta, 2]$ and $\tau \in [0, 0.25]$. The parameters $(\bar{m}, \bar{\theta})$ are such that all households $\theta \leq \bar{\theta}$ have interior labor choices under the considered policy values.[7] This is important for tractability because it allows one to transform the decision problem (5) into (7). This way, the household's choices of $(y, z, h, q, b, L_1, L_2, \pi_1, \pi_2)$ are independent of the state variable m and also the money distribution. After the benchmark case, I will also discuss results from variations on the values of $(U_0, \sigma_u, k, \rho, \underline{\theta}, \bar{\theta})$.

Computation strategy. This model is block recursive even with monetary and fiscal policies. The decision problems are listed in (41)-(44), none of which is affected by equilibrium distributions. For simulations, one can first solve these problems, and then derive the equilibrium wage rate, the government transfer and the macro aggregates using the formulas presented in Appendix D.

Policy functions. Panel A of Figure 2 depicts a household's optimal lottery choice given balance z. In particular, $z_0 = 0.027$. That is, any household with $z \in (0, 0.027)$ will

[7] With both policies in place, an interior choice of labor for all households requires $l(\bar{m}, \bar{\theta}) = \left[y(\bar{\theta}) + z(\bar{\theta}) + h(\bar{\theta}) - \bar{m} - T\right] / (1-\tau) > 0$, where the transfer T is derived in Appendix D. The values of $(\bar{m}, \bar{\theta})$ chosen here satisfy this condition.

play the lottery of receiving either a payment of 0.027 or nothing. For households with $z > 0.027$, the functions $L_1(z)$ and $L_2(z)$ almost always coincide, indicating that the lottery is almost never played at higher z values. Moreover, for the policy parameters considered here, a household with any $\theta \in [\underline{\theta}, \bar{\theta}]$ never chooses $z(\theta) \leq 0.027$. Therefore, the lottery choice is quantitatively negligible in this numerical exercise. For expositional convenience, I will ignore lotteries when presenting the rest of the numerical results. Nevertheless, all computation results presented in this paper are carried out with the lottery choice.

Panel B of Figure 2 depicts the policy functions under various policy regimes. In the top right panel of Figure $2-B$, the blue curve coincides with the green curve and is completely blocked by the latter. In the bottom three panels, all three colored curves are not always discernible in the graphs because they often coincide with one another. In each panel, a shift from the blue curve to the red one represents the effect of an increased income tax rate (from $\tau = 0$ to $\tau = 0.25$) given a money growth rate ($\gamma = 0.96$). Similarly, a shift from the blue curve to the green one represents the effect of an increased money growth rate (from $\gamma = 0.96$ to $\gamma = 1.18$) given the fiscal policy ($\tau = 0$). A few observations follow immediately:

S1. $y(\theta)$, $z(\theta)$ and $h(\theta)$ are decreasing functions, while $b(z)$ and $q(z)$ are increasing functions. This is consistent with the results in Lemmas 2 and 3;

S2. (a) Consider a given γ. For any θ, a higher τ decreases $z(\theta)$ and $y(\theta)$, but increases savings $h(\theta)$. The effect of τ on z is consistent with part (iii) of Proposition 1.

(b) Consider a given τ. Money growth increases the transaction balance $z(\theta)$ but decreases savings $h(\theta)$. It has no effect on $y(\theta)$. According to part (iii) of Proposition 1, the effect of γ on z can be either positive or negative. Here the numerical results suggest that the positive effect dominates.

(c) There is equilibrium price dispersion as the equilibrium prices $z(\theta)/q(z(\theta))$ vary with the realizations of θ.

(d) Neither γ nor τ has a significant impact on the functions $b(z)$, $q(z)$ or the price function $z/q(z)$. This suggests that the direct policy effects summarized in parts (i)-(ii) of Proposition 1 can be quantitatively small.

The results in S2 are intuitive. All else equal, a higher income tax rate makes it more costly to supply labor. Accordingly, the household saves more and becomes more frugal on spending. However, the higher tax rate stimulates precautionary savings because it helps alleviate the elevated disutility of labor supply in a higher-θ state. Inflation has no effect on consumption of goods traded in the frictionless market, which is a standard result. All else equal, inflation tax causes the household to save less yet spend more on goods traded in the frictional market.

Aggregate margins, output and labor.

S3. Inflation has a positive effect on both of the aggregate intensive and extensive margins, while the tax rate has a negative effect on them. Inflation increases aggregate output and labor in the frictional market, while the income taxes decrease such aggregates.

Figure 3 depicts the policy effects on aggregate intensive and extensive margins. The first column is for the intensive margin, i.e. average quantity per trade in the frictional market, and the second column is for the extensive margin, i.e. volume of transactions in the frictional market. The first row shows the monetary policy effect and the second row is for the fiscal policy. Figure 4 shows the policy effects on aggregate output and labor in the frictional market. Recall from S2 and Figure 2 that the direct policy effects on the optimal choices of (b, q) are quantitatively small and are dominated by the indirect policy effects through z. Since (b, q) are increasing functions of z, a higher γ causes both to rise, which further leads to a positive effect on both margins as is shown by Figure 3. The boost to the margins results in a rise in both output and labor. Finally, the policy effects on output and labor in the frictionless market are standard. The monetary policy has no effect on either aggregate and the fiscal policy imposes a negative effect on them. I omit their numerical characterizations due to limited space.

The positive relationship between inflation and output in the top left panel of Figure 4 is in strong contrast to the results from the previous literature. Typically, a monetary search model either delivers a negative inflation-output relationship or a hump-shape one. This is because fiat money is required as a medium of exchange in such models. As a result, inflation tax eventually will bring down output as inflation gets severe. Similarly, cash-in-advance models typically find a negative relationship between inflation and output.[8] In contrast, here in this model money is used for precautionary purposes rather than transaction purposes. Households can use money and/or firm credit to buy goods. Inflation causes households to increase their desired trading probabilities and to decrease their precautionary savings, the latter of which is obvious from the top middle panel in

[8] Molico (2006) reports a hump-shape relationship between inflation and output in a search environment with endogenous money distributions. Camera and Chien (2011) find a negative relationship between inflation and output in a cash-in-advance environment with endogenous money distributions. Moreover, the negative effect of inflation on output by no means limits to models with heterogeneous agents. It is also common among monetary models with degenerate money distributions.

Figure $2 - B$. Therefore, at very high inflation rates, precautionary savings go to zero. Moreover, the trading probabilities for a buyer in the frictional market approach one and thus households carry little unspent money balances over time. Together, the entire economy functions as if it were a cash-less economy. As a result, output increases with inflation and stays flat at very high money growth rates.

This is consistent with the empirical findings for the U.S. and some other countries, suggesting a positive long-run relationship between inflation and output at low inflation rates and little effects at high inflation rates (see King and Watson, 1992; Bullard and Keating, 1995; McCandless and Weber, 1995; Ahmed and Rogers, 2000; Rapach, 2003). As is indicated by this model, the little real effect found at higher inflation rates can be explained by a switch of means of payment. That is, people may seek to adopt other reliable currencies or gold for payments and savings, when a country's own currency undergoes fast growth.

Wealth distribution.

S4. Inflation has a negative effect on average wealth while income taxation has a positive effect on it. The effect of inflation on wealth dispersion depends on the income tax rate. At low tax rates, the positive effect tends to dominate. At intermediate rates, there is a hump-shape relationship between inflation and wealth dispersion. At higher tax rates, the negative effect tends to dominate.[9] Given money growth, income taxation reduces wealth dispersion.

Figure 5 depicts the policy effect of inflation on wealth distribution. Here a household's wealth is interpreted as its beginning-of-period real money balances after receiving the government transfer T given by equation (49). Therefore, the average wealth consists of aggregate precautionary savings, aggregate unspent balances and the transfer. Recall from S2 that higher inflation causes households to save less (lower h) and spend more (higher z) at a higher frequency (higher b). Thus inflation decreases aggregate savings. Its effect on

[9] Note in particular that inflation tends to have a positive effect on wealth dispersion at zero tax rate. This is in contrast with the results from the literature that examines the distributional effect of monetary policy (note that this literature typically abstracts away from distortionary income taxes). For example, in a search-theoretic model with bargaining, Molico (2006) shows that dispersion in money holdings first decreases and then increases as the inflation rate rises. Moreover, Chiu and Molico (2010) establish a negative relationship between inflation and the dispersion of the money distribution. In a model where heterogeneous agents use money to self-insure against liquidity shocks, Dressler (2011) demonstrates that inflation increases dispersion in money balances. In a cash-in-advance environment with heterogeneous agents, Camera and Chien (2011) show that inflation reduces wealth dispersion when money is the only asset, but has little effect on wealth inequality when bonds are introduced. Moreover, none of the above papers consider the relevance of a fiscal policy regime.

aggregate unspent balances can be ambiguous. On one hand, households plan on spending more, which means the level of unspent balances held by a household is also higher. On the other hand, households also choose to trade with a higher probability, which reduces the chance of holding an unspent balance across periods. The government transfer includes the monetary component to achieve money growth and the fiscal component from taxation on labor income. Both components increase with inflation. The former is because of money injection and the latter is because aggregate labor increases as inflation rises. Overall, the negative effect of inflation dominates, which indicates that the negative impact of inflation on precautionary savings is the dominating force. Now consider the positive relationship between income taxation and average wealth. Recall from S2 that taxation makes households save more (higher h) and spend less (lower z) at a lower frequency (lower b). Moreover, higher tax rates reduce aggregate labor and thus the fiscal component of government transfers. Altogether, the positive effect of income taxation dominates, which again is likely to the result of the dominating effect on precautionary savings.

Now consider the policy effects on the coefficient of variation of wealth. Since all households receive the same amount of transfers, wealth dispersion critically depends on the dispersion of precautionary savings and unspent balances. A rise in inflation tends to increase dispersion in household unspent balances (due to trading frictions) but decrease dispersion in household savings, which is suggested to a certain extent in Figure 2 by the changes in the functions $z(\theta)$ and $h(\theta)$ under various policy regimes. Nevertheless, as the tax rate rises, households increase savings, which allows the effect of inflation on savings to make a stronger presence. As is shown by the top right panel of Figure 5, the positive effect of inflation on dispersion in household unspent balances tends to dominate when the tax rate is low (*e.g.* $\tau = 0$). Then at higher tax rates, the negative effect on savings tends to dominate. In contrast, income taxation unambiguously decreases wealth dispersion.

Income and consumption inequalities.

S5. Inflation has a negative effect on income inequality but a positive effect on consumption inequality.[10] Income taxation has a positive effect on both income inequality and consumption inequality.

Figure 6 reports policy effects on the respective coefficients of variation of household disposable income and consumption. Inflation reduces income inequality because of the

[10] Camera and Chien (2011) also show that inflation lowers income inequality. Nevertheless, there is no consensus in the empirical literature relating inflation to income distribution. Galli and Hoeven (2001) provide an extensive review over this literature and refer to the mixed results as the "inflation-inequality puzzle". Also in Camera and Chien (2011), lower inflation can increase consumption inequality when agents are not allowed to borrow.

redistributive effect of lump-sum transfers to sustain money growth. This negative effect strengthens with higher taxes because income taxation suppresses the incentives to supply labor. This accentuates the redistributive effect of inflation. Inflation increases consumption inequality because it stimulates participation in the frictional market. Income taxation increases income inequality. One interpretation is that the negative impact of taxation on aggregate labor income overpowers its effect on the standard deviation of income. Income taxation has a positive effect on consumption inequality, which is likely to be related to its influence on income inequality.

Welfare.

S6. There can be a hump-shape relationship between inflation and welfare at a given tax rate. The welfare-improving role of inflation strengthens as the tax rate increases. Similarly, there is a hump-shape relationship between income taxation and welfare at a given inflation rate. Overall, welfare can be maximized by a deviation from the Friedman rule, paired with distortionary income taxation.

Figure 7 illustrates the welfare effects. Welfare is defined as the weighted average of the life-time discounted value W given by (7). Inflation has a positive welfare effect through the following channels: increasing output, reducing income inequality, and reducing wealth inequality at higher tax rates. On the other hand, inflation also has a negative welfare effects by reducing savings, increasing consumption inequality and increasing wealth dispersion at lower tax rates. Overall, at a given tax rate, inflation can improve welfare at lower money growth rates but reduce welfare at higher rates. The higher the tax rate, the more prominent the positive effect of inflation on welfare. Income taxation also has both positive and negative impacts on welfare. The positive welfare effect of taxation comes through increasing savings and decreasing wealth inequality. The negative effect of taxation includes decreasing output and increasing income and consumption inequalities. Altogether, at a given inflation rate, income taxation can also improve welfare at lower tax rates but end up reducing it at higher tax rates.

The results in S1-S6 are robust to variation of parameter values satisfying the restriction that the labor choices of all households are strictly positive. For the sake of limited space, I only report in Table 1 the optimal monetary and fiscal policies under various parameter values. The benchmark case is given in (37). For each of the other cases, only the parameter value(s) different from the benchmark case is(are) listed. In the benchmark, the optimal policy regime is $\gamma^* = 0.98$ and $\tau^* = 0.1$. First, note that welfare can be maximized by a deviation from the Friedman rule (i.e. $\gamma = \beta = 0.96$), together with distortionary income taxation. Secondly, the optimal policy seems very sensitive to changes in the variability of the θ-shock. Recall $\underline{\theta} = 0.5$ and $\bar{\theta} = 1.5$ in the benchmark. All else equal and keeping $E(\theta) = 1$ constant, the optimal policy goes from $\gamma^* = 0.96$ and $\tau^* = 0.03$ given $\theta \in [0.8, 1.2]$, to $\gamma^* = 1.01$ and $\tau^* = 0.17$ given $\theta \in [0.3, 1.7]$, and to $\gamma^* = 1.03$ and $\tau^* = 0.22$ given $\theta \in [0.2, 1.8]$. Since inflation has no effect on real activities in the frictionless market whatsoever, all of the non-trivial effects of long-run inflation summarized in S1-S6 are due to trading frictions. This suggests that the trading frictions can play an important role in reconciling empirical observations on the macroeconomy.

It is clear that monetary and fiscal policies often have opposite yet asymmetric effects on macro aggregates. Money growth directly affects the intertemporal consumption choices while income taxation directly affects labor choices across idiosyncratic states. To maximize welfare, a policy maker must choose the policy pair such that they optimally augment each other's positive welfare effects. If the monetary and fiscal authorities are independent of each other, a policy change by one often implies that the other should adjust its policy accordingly. For example, in the benchmark case, the optimal money growth rate is $\gamma^* = 0.96$ given $\tau = 0$, $\gamma^* = 0.98$ given $\tau = 0.1$, and $\gamma^* = 1$ given $\tau = 0.25$. Finally, there is a large literature on the welfare cost of inflation. This literature typically abstracts away from considerations of the fiscal policy regime. The results obtained here suggest that such an abstraction is very likely to produce biased evaluations of monetary policy.

6 Conclusion

I have constructed a tractable framework of competitive search that endogenously generates dispersion of prices, income and wealth. This model is used to study the implications of monetary and fiscal policies in an environment with heterogeneous agents. With competitive search, a household's decision problems can be solved independently from the endogenous wealth distribution, which brings the model significant tractability. Analytical and quantitative results suggest that monetary and fiscal policies have distinctive effects on real activities and welfare. Welfare-maximization requires an optimal policy mix. If the monetary and fiscal authorities are separate identities, a change of policy by one has a non-trivial implication on the optimal policy choice of the other.

Appendix

A Proof of Lemma 2

Given (24), it is straightforward to see that the value function $B(z)$ is continuous. Moreover, $B(z) \geq 0$ for all $z \geq 0$, where the equality holds if and only if $b = 0$. If $b = 0$, the choice of q is irrelevant. Since B is continuous on a closed interval $[0, \hat{z}]$, the lotteries in (30) make $\tilde{V}$ concave (see Appendix F in Menzio and Shi, 2010b, for a proof). I prove differentiability of $\tilde{V}$ in the proof of part (iii).

For part (i), define the left-hand side of (17) as $LHS(b)$ and impose $x = z$:

$$LHS(b) \equiv u(q) - \beta E(\theta) z + \left[\frac{u'(q)}{\psi'(q)}\right] \frac{kb\mu'(b)}{[\mu(b)]^2}, \tag{38}$$

where q is given by (18) with $x = z$. It is straightforward to derive that

$$\begin{aligned} LHS(b=0) &= u\left(\psi^{-1}(z-k)\right) - \beta E(\theta) z, \\ &= u(q) - \beta E(\theta)[\psi(q) + k], \end{aligned}$$

where (18) yields $q = \psi^{-1}(z-k)$ given $b = 0$. Thus the above implies that $LHS(b=0) > 0$ if and only if there exists $q > 0$ such that condition (27) holds. Moreover, one can further derive $LHS(b=1) = -\infty$, and

$$\begin{aligned} LHS'(b) &= u'(q) q'(b) + \frac{u''(q)\psi'(q) - u'(q)\psi''(q)}{[\psi'(q)]^2} q'(b) \left(\frac{kb\mu'(b)}{[\mu(b)]^2}\right) \\ &\quad + k\left[\frac{u'(q)}{\psi'(q)}\right] \frac{\mu(b)[\mu'(b) + b\mu''(b)] - 2b[\mu'(b)]^2}{[\mu(b)]^3} < 0. \end{aligned}$$

Given all the above results, condition (27) implies that there exists $z > 0$ such that $b > 0$. Furthermore, the above results imply that the policy function $b(z)$ is unique, which further implies that $q(z)$ is also unique given (18). Given $x = z$, (16) implies

$$\frac{u'(q)}{\psi'(q)} - \beta E(\theta) > 0. \tag{39}$$

Therefore, for z such that $b > 0$,

$$\frac{\partial LHS(b;z)}{\partial z} = \frac{u'(q)}{\psi'(q)} - \beta E(\theta) + \frac{kb\mu'(b)[u''(q)\psi'(q) - u'(q)\psi''(q)]}{[\mu(b)]^2[\psi'(q)]^3} > 0. \tag{40}$$

This implies that an increase of z shifts the entire function $LHS(b)$ upwards. Because $LHS'(b) < 0$, it follows that $b'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, (17) holds with equality. Total differentiating (17) by z yields

$$\begin{aligned} 0 &= u'(q)\,q'(z) - \beta E(\theta) + \frac{kb\mu'(b)\left[u''(q)\,\psi'(q) - u'(q)\,\psi''(q)\right]}{\left[\mu(b)\right]^2\left[\psi'(q)\right]^3}\,q'(z) \\ &\quad + k\,\frac{u'(q)}{\psi'(q)}\left[\frac{\mu(b)\left[\mu'(b) + b\mu''(b)\right] - 2b\left[\mu'(b)\right]^2}{\left[\mu(b)\right]^3}\right]b'(z)\,. \end{aligned}$$

Given $b'(z) > 0$ and Assumption 1, rearranging the above yields $q'(z) > 0$ for all z such that $b > 0$. Given $b > 0$, one can derive that

$$B'(z) = b'(z)\left[u(q(z)) - \beta E(\theta)\,z\right] + b(z)\left[\frac{u'(q(z))}{\psi'(q(z))} - \beta E(\theta)\right] > 0.$$

This is because $b'(z) > 0$ and the trade surplus, $u(q(z)) - \beta z E(\theta)$, is strictly positive given $b > 0$, and also condition (39). Obviously, $b(z)$ is strictly decreasing in $E(\theta)$, given the results about $LHS(b)$ in part (ii). Then (29) implies that $q(z)$ is strictly increasing in $E(\theta)$.

For part (ii), note that the previous proof has established that $b(z)$ is continuous and increasing in all $z \in [0, \hat{z}]$. In particular, $b(z)$ is strictly increasing in z if $b > 0$. It is obvious from (18) that $b(z) = 0$ for all $z \in [0, k]$. Continuity of $b(z)$ implies that there exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$.

I now prove part (iii) and the differentiability of $\tilde{V}$ together. If $b(z) = 0$ for all z, then obviously $\tilde{V}(z)$ is differentiable. Now consider the case where there exists z such that $b(z) > 0$, i.e., condition (27) holds. It is obvious that $B(z)$ is differentiable for all z such that $b(z) > 0$. Consider z such that $b(z) > 0$. Recall that a concave function has both left-hand and right-hand derivatives (see Royden, 1988, pp113-114). Let $\tilde{V}'(z^-)$ and $\tilde{V}'(z^+)$ be the left-hand and right-hand derivatives, respectively. Suppose $\tilde{V}'(z^-) > \tilde{V}'(z^+)$ for some z such that $b(z) > 0$. Then $\tilde{V}$ is strictly concave at such z, which implies $\tilde{V}(z) = B(z)$. It follows that $B'(z^-) \geq \tilde{V}'(z^-) > \tilde{V}'(z^+) \geq B'(z^+)$, where the first and the last inequalities follow from the construction of lotteries. However, $B'(z^-) > B'(z^+)$ contradicts the differentiability of B. Therefore, the value function $\tilde{V}(z)$ is differentiable for all z such that $b(z) > 0$. Part (ii) has established that there exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z > z_1$. This has two implications: First, $B'\left(z_1^-\right) = 0$ because $b(z) = B(z) = 0$ all $z \in [0, z_1]$. Second, $B'\left(z_1^+\right) > 0$ because $b(z) > 0$ in the right neighborhood of z_1. Therefore, B is strictly convex but not differentiable at z_1 because $0 = B'(z^-) < B'(z^+)$. Strict convexity of B at z_1 implies that there is a lottery over the region $z \in [0, z_1]$. Let the winning prize of this lottery be z_0. Then all households with $z \in (0, z_0)$ will play this lottery and receive zero payment if they lose. Moreover, it must be the case that $z_0 > z_1$, $b(z_0) > b(z_1) > 0$ and $B(z_0) = \tilde{V}(z_0) > 0$. Given $b(z_0) > 0$, both value functions are differentiable at z_0 and $B'(z_0) = \tilde{V}'(z_0) > 0$. Therefore, $\tilde{V}$ is differentiable for all $z \in [0, z_0]$ because of the lottery. Moreover, $\tilde{V}$ is also differentiable for all $z > z_0$ because $b > 0$ for all $z > z_0$. **QED**

B Proof of Theorem 2

Recall the normalized wage rate w^* as given in (33). Note that all the policy functions on the right-hand side of (33) are independent of w^*. Thus $w^* > 0$ obviously exists. It follows that a stationary equilibrium exists and is characterized by w^*. It is unique if and only if the lottery choices $\{L_1(z(\theta)), L_2(z(\theta)), \pi_1(z(\theta)), \pi_2(z(\theta))\}$ are unique for all $z(\theta)$. Part (i) follows from (11). For part (ii), recall from Lemma 2 that there exists $z > 0$ such that the policy function $b(z) > 0$ if any only if condition (27) holds for some $q > 0$. Therefore, if (27) does not hold, then $b(z) = 0$ for all z. Moreover, $B(z) = B'(z) = \tilde{V}(z) = \tilde{V}'(z) = 0$ for all z. In this case, the household never trades in the frictional market, which renders no need to hold a positive balance for transaction purposes. Thus $z(\theta) = 0$ for all θ. Consider the case where condition (27) holds for some $q > 0$. In this case, there exists $z > 0$ such that the policy function $b(z) > 0$, according to Lemma 2. Note that condition (9) implies that $z(\theta) > 0$ if $V_z(0, h) \geq \theta$. If $V_z(0, h) < \theta$, then $z(\theta) = 0$ is optimal. If $z(\theta) > 0$, $b(L_2(z(\theta))) > 0$ follows from construction of the lottery. **QED**

C Proof of Proposition 1

Given policies γ and τ, the household's decision problem is given by:

$$\begin{aligned} W(m, \theta) \quad = \quad & \max_{(y,l,z,h)} \quad \{U(y) - \theta l + V(z, h)\} \\ & s.t. \;\; y + z + h \leq m + (1 - \tau) l + T. \end{aligned}$$

Given interior choice of l, the above reduces to

$$W(m, \theta) = \frac{\theta(m + T)}{1 - \tau} + \max_{y} \left\{ U(y) - \frac{\theta y}{1 - \tau} \right\} + \max_{z,h} \left\{ V(z, h) - \frac{\theta(z + h)}{1 - \tau} \right\}. \quad (41)$$

Moreover,

$$V(z, h) = \tilde{V}(z) + \beta E[W(0, \theta)] + \frac{\beta E(\theta)}{\gamma(1 - \tau)}(z + h) \quad (42)$$

$$\begin{aligned} \tilde{V}(z) &= \max_{(L_1,L_2,\pi_1,\pi_2)} \{\pi_1 B(L_1) + \pi_2 B(L_2)\} \qquad (43) \\ & \quad s.t. \quad \pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0; \\ & \quad \quad \pi_1 + \pi_2 = 1; \quad \pi_i \in [0,1] \text{ for } i = 1,2 \end{aligned}$$

$$B(z) = \max_{b\in[0,1]} b\left\{u\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z\right\}. \qquad (44)$$

It follows immediately that policy functions $b(z)$ and $q(z)$ are given by (34) and (35). Define the left-hand side of (34) as

$$LHSP(b) \equiv u(q) - \frac{\beta E(\theta)}{\gamma(1-\tau)} z + \left(\frac{u'(q)}{\psi'(q)}\right) \frac{kb\mu'(b)}{[\mu(b)]^2}, \qquad (45)$$

where q is given by (35). Following the same procedure as the proof of Lemma 2, one can show that Lemma 2 also applies to this case with policies γ and τ, except that condition (27) is replaced by

$$u(q) - \frac{\beta E(\theta)}{\gamma(1-\tau)} [\psi(q) + k] > 0, \qquad (46)$$

and (28) and (29) replaced by (34) and (35). As is the case with $LHS'(b) < 0$, we have $LHSP'(b) < 0$. Then parts (i) and (ii) of this proposition follow trivially given $b > 0$.

For part (iii), to analyze policy effects on z, note from (41), (42), (43) and (44) that the decision involving z is given by

$$\max_z \left\{\tilde{V}(z) + \frac{\beta E(\theta)/\gamma - \theta}{1-\tau} z\right\}.$$

It is clear from part (ii) of Lemma 2 that $b(z) > 0$ implies $z > 0$. Consider interior choice of $z < \bar{m}$, which satisfies the following first-order condition:

$$\tilde{V}'(z) + \frac{\beta E(\theta)/\gamma - \theta}{1-\tau} = 0.$$

Given the lottery, if $\tilde{V}(z)$ is strictly concave at z, then $\tilde{V}(z) = B(z)$ and $\tilde{V}'(z) = B'(z)$. Moreover, the optimal choice of z satisfies

$$B'(z) + \frac{\beta E(\theta)/\gamma - \theta}{1-\tau} = 0.$$

That is,

$$b\left[\frac{u'\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right)}{\psi'\left(\psi^{-1}\left(z - \frac{k}{\mu(b)}\right)\right)} - \frac{\beta E(\theta)}{\gamma(1-\tau)}\right] + \frac{\beta E(\theta)/\gamma - \theta}{1-\tau} = 0. \qquad (47)$$

If $\tilde{V}(z)$ is linear at z, then a lottery is employed at z. In this case, $\tilde{V}'(z)$ is trivially determined by the slopes of $\tilde{V}$ at the lottery prize points, $L_1(z)$ and $L_2(z)$. At both of these points, $\tilde{V}(z)$ is strictly concave. Without loss of generality, I focus on z such that (47) holds and $B''(z) < 0$.

For the fiscal policy effect, total differentiating (47) yields

$$\begin{aligned} 0 &= B''(z)\,dz + \left[\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)}{\gamma(1-\tau)} + \frac{kb\mu'(b)\left[u''(q)\psi'(q) - u'(q)\psi''(q)\right]}{[\mu(b)]^2[\psi'(q)]^3}\right]\frac{\partial b(z;\tau)}{\partial\tau}d\tau \\ &\quad + \frac{(1-b)\beta E(\theta)/\gamma - \theta}{(1-\tau)^2}d\tau. \end{aligned} \tag{48}$$

Similar to (39) and (40), one can show that

$$\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)}{\gamma(1-\tau)} > 0$$

and that the term within the square bracket of (48) is strictly positive. Then (47) implies that $\theta > \beta E(\theta)/\gamma > (1-b)\beta E(\theta)/\gamma$. Moreover, we have $\frac{\partial b}{\partial\tau} < 0$ according to part (i) of this proposition. Given $B''(z) < 0$, the above equation implies that $\frac{dz}{d\tau} < 0$ given θ. That is, $\frac{\partial z(\theta;\tau)}{\partial\tau} > 0$.

For the monetary policy effect, total differentiate (47):

$$\begin{aligned} 0 &= B''(z)\,dz + \left[\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)}{\gamma(1-\tau)} + \frac{kb\mu'(b)\left[u''(q)\psi'(q) - u'(q)\psi''(q)\right]}{[\mu(b)]^2[\psi'(q)]^3}\right]\frac{\partial b(z;\gamma)}{\partial\gamma}d\gamma \\ &\quad - \frac{(1-b)\beta E(\theta)}{(1-\tau)\gamma^2}d\gamma. \end{aligned}$$

where $q = \psi^{-1}(z - k/\mu(b))$. Again, the term within the square bracket is strictly positive. Moreover, we have $\frac{\partial b}{\partial\gamma} > 0$ according to part (ii) of this proposition. Given $B''(z) < 0$, the above equation implies that $\frac{dz}{d\gamma} > 0$ if

$$\left[\frac{u'(q)}{\psi'(q)} - \frac{\beta E(\theta)}{\gamma(1-\tau)} + \frac{kb\mu'(b)\left[u''(q)\psi'(q) - u'(q)\psi''(q)\right]}{[\mu(b)]^2[\psi'(q)]^3}\right]\frac{\partial b(z;\gamma)}{\partial\gamma} - \frac{(1-b)\beta E(\theta)}{(1-\tau)\gamma^2} > 0.$$

The above part (iii) shows that z is strictly decreasing in τ. Moreover, part (i) of Lemma 2 that the policy functions $b(z)$ and $q(z)$ are strictly increasing in z. Thus, the fiscal policy has a negative indirect effect on both margins through its effect on z. The overall effect of τ is given by

$$\frac{db}{d\tau} = \frac{\partial b\left(z\left(\theta;\tau\right);\tau\right)}{\partial \tau} + b'\left(z\right)\frac{\partial z\left(\theta;\tau\right)}{\partial \tau} < 0$$

because $\frac{\partial b(z;\tau)}{\partial \tau} < 0$, $b'\left(z\right) > 0$ and $\frac{\partial z(\theta;\tau)}{\partial \tau} < 0$ for all z such that $b\left(z\right) > 0$. Then rest of the results in this proposition follow trivially. **QED**

D Government Transfers and Market Clearing

In this Appendix, I further characterize the market-clearing conditions and the formula for the government transfer. The analysis in this Appendix is carried out with the monetary and fiscal policies. For the benchmark case without any policy, one can simply apply $\gamma = 1$ and $\tau = 0$ to all the derivations in what follows.

With policies, the definition of a stationary equilibrium must satisfy one more condition that the government balances its budget every period. For money growth, the household receives a dollar amount of $\left(\gamma - 1\right)M$, which is equivalent to $\left(\gamma - 1\right)M/\left(wM'\right) = \left(\gamma - 1\right)/\left(w\gamma\right)$ units of labor. For income taxation, the amount of the government transfer in terms of labor units is τLS. Altogether, the total real transfer received by a household is given by

$$T^* = \frac{\gamma - 1}{w^*\gamma} + \frac{\tau}{\gamma}LS. \tag{49}$$

The market-clearing condition for the general-good market is

$$Y = \int_{\underline{\theta}}^{\bar{\theta}} y\left(\theta\right)dF\left(\theta\right). \tag{50}$$

The market-clearing condition for the labor market is aggregate demand for labor, LD, is equal to aggregate supply of labor, LS. Consider LD first. A household's realization of θ determines the money balance $z\left(\theta\right)$. Given this money balance, the resulted money balance after lotteries is $L_i\left(z\left(\theta\right)\right)$, $i = 1, 2$, which takes place with probability $\pi_i\left(z\left(\theta\right)\right)$. Thus the measure of such households is $N_b\left(\theta, i\right) = \pi_i\left(z\left(\theta\right)\right)dF\left(\theta\right)$. The measure of shops corresponding to the households holding $L_i\left(z\left(\theta\right)\right)$ is given by

$$N_s\left(\theta, i\right) = \pi_i\left(z\left(\theta\right)\right)dF\left(\theta\right)b\left(L_i\left(z\left(\theta\right)\right)\right)/\left[\mu\left(b\left(L_i\left(z\left(\theta\right)\right)\right)\right)\right],$$

which is derived from $b/\mu\left(b\right) = N_s/N_b$ given the constant-return-to-scale matching technology. Then for each shop, the expected labor demand is $k + \psi\left(q\right)\mu\left(b\right)$, which is used to compute the aggregate demand for labor in the frictional market. Thus, LD is given by

$$LD = Y + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \frac{\pi_i \left(z\left(\theta\right)\right) b\left(L_i\left(z\left(\theta\right)\right)\right)}{\mu\left(b\left(L_i\left(z\left(\theta\right)\right)\right)\right)} \left[k + \psi\left(q\left(L_i\left(z\left(\theta\right)\right)\right)\right) \mu\left(b\left(L_i\left(z\left(\theta\right)\right)\right)\right)\right] dF\left(\theta\right). \tag{51}$$

The firm's zero-profit condition (3) implies that for $i = 1, 2$,

$$k + \psi\left(q\left(L_i\left(z\left(\theta\right)\right)\right)\right) \mu\left(b\left(L_i\left(z\left(\theta\right)\right)\right)\right) = L_i\left(z\left(\theta\right)\right) \mu\left(b\left(L_i\left(z\left(\theta\right)\right)\right)\right).$$

Then (51) can be transformed to

$$LD = \int_{\underline{\theta}}^{\bar{\theta}} y\left(\theta\right) dF\left(\theta\right) + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \pi_i\left(z\left(\theta\right)\right) b\left(L_i\left(z\left(\theta\right)\right)\right) L_i\left(z\left(\theta\right)\right) dF\left(\theta\right). \tag{52}$$

The aggregate labor supply is given by

$$LS = \int_{\underline{\theta}}^{\bar{\theta}} \int l\left(m, \theta\right) dG_a\left(m\right) dF\left(\theta\right),$$

where $G_a\left(m\right)$ is the money distribution at the beginning of a period. Given $l\left(m, \theta\right)$ from (32),

$$LS = \int_{\underline{\theta}}^{\bar{\theta}} \int \frac{1}{1-\tau} \left[py\left(\theta\right) + z\left(\theta\right) + h\left(\theta\right) - m - T^*\right] dF\left(\theta\right) dG_a\left(m\right).$$

Use (49) to substitute for T^* in the above. Also recall the constraint for the household's lottery choice, $\pi_1\left(z\left(\theta\right)\right) L_1\left(z\left(\theta\right)\right) + \pi_2\left(z\left(\theta\right)\right) L_2\left(z\left(\theta\right)\right) = z\left(\theta\right)$. It follows that

$$\left(1 - \tau + \frac{\tau}{\gamma}\right) LS = \int_{\underline{\theta}}^{\bar{\theta}} \left[y\left(\theta\right) + h\left(\theta\right) + z\left(\theta\right)\right] dF\left(\theta\right) - \int m dG_a\left(m\right) - \frac{\gamma - 1}{w^* \gamma}. \tag{53}$$

The household's beginning-of-the-period balance m consists of precautionary savings and if any, the transactional balance unspent due to matching frictions. Thus,

$$\int m dG_a\left(m\right) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{h\left(\theta\right)}{\gamma} dF\left(\theta\right) + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \pi_i\left(z\left(\theta\right)\right) \left[1 - b\left(L_i\left(z\left(\theta\right)\right)\right)\right] \frac{L_i\left(z\left(\theta\right)\right)}{\gamma} dF\left(\theta\right). \tag{54}$$

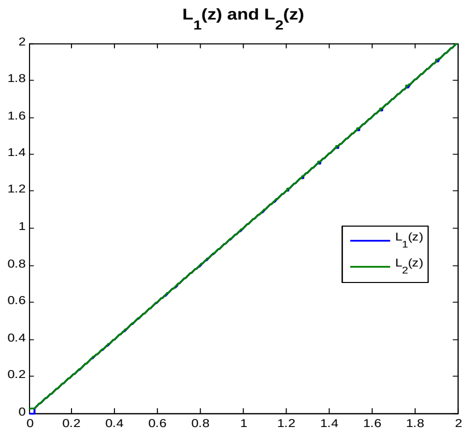
The labor-market clearing requires $LD = LS$. Thus (52)-(54) together solve for the normalized wage rate in the steady state:

$$
\begin{aligned}
(w^*)^{-1} \;=\; & \int_{\underline{\theta}}^{\bar{\theta}} [h(\theta) + \tau y(\theta)]\, dF(\theta) + \tau \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \pi_i(z(\theta))\, b(L_i(z(\theta)))\, L_i(z(\theta))\, dF(\theta) \\
& + \sum_{i=1}^{2} \int_{\underline{\theta}}^{\bar{\theta}} \pi_i(z(\theta))\, [1 - b(L_i(z(\theta)))]\, L_i(z(\theta))\, dF(\theta). \qquad (55)
\end{aligned}
$$

Note that the formula in (33) is clearly given by setting $\tau = 0$ in the above equation. Given that the labor market clears, the money market clears by Walras' law.

Figure 2. Policy functions

A. The lottery choice



B. Optimal choices under various policy regimes

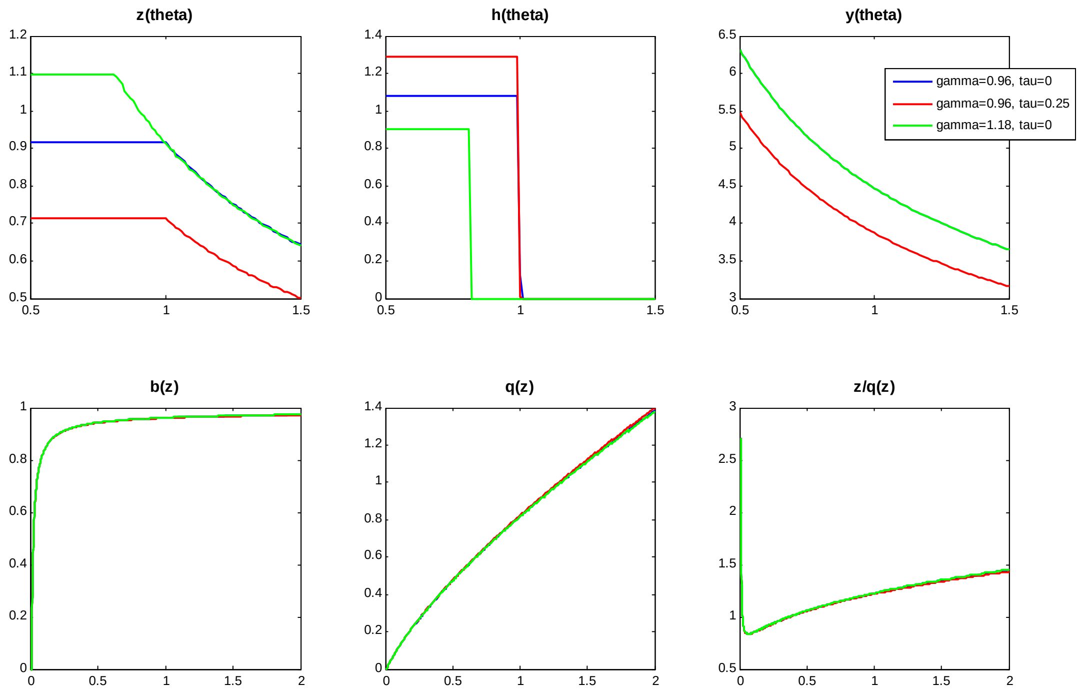


Figure 3. Aggregate intensive and extensive margins

A. Average quantity per trade B. Volume of transactions

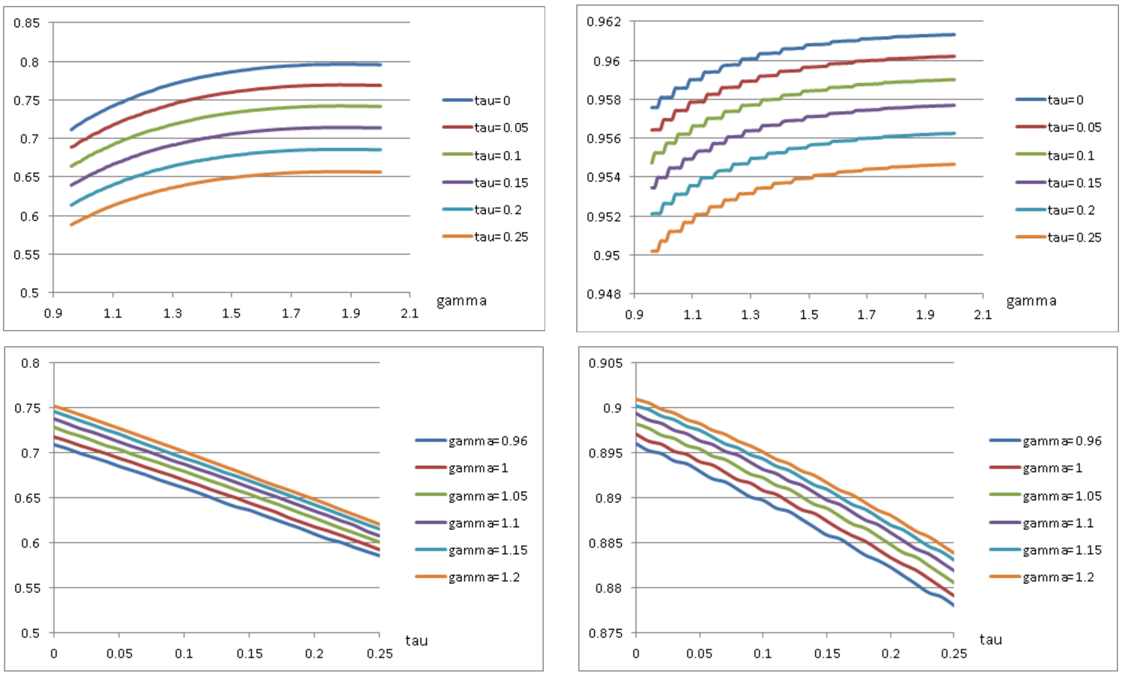


Figure 4. Output and labor and in the frictional market

A. output B. labor

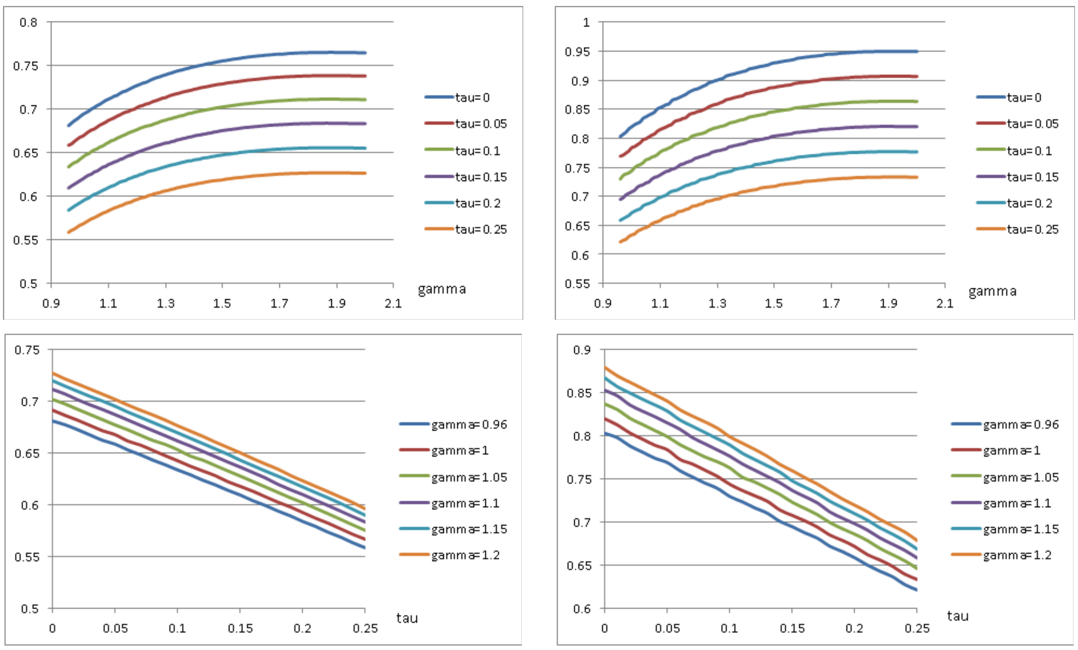


Figure 5. Wealth distribution

A. Average wealth

B. Coefficient of variation of wealth

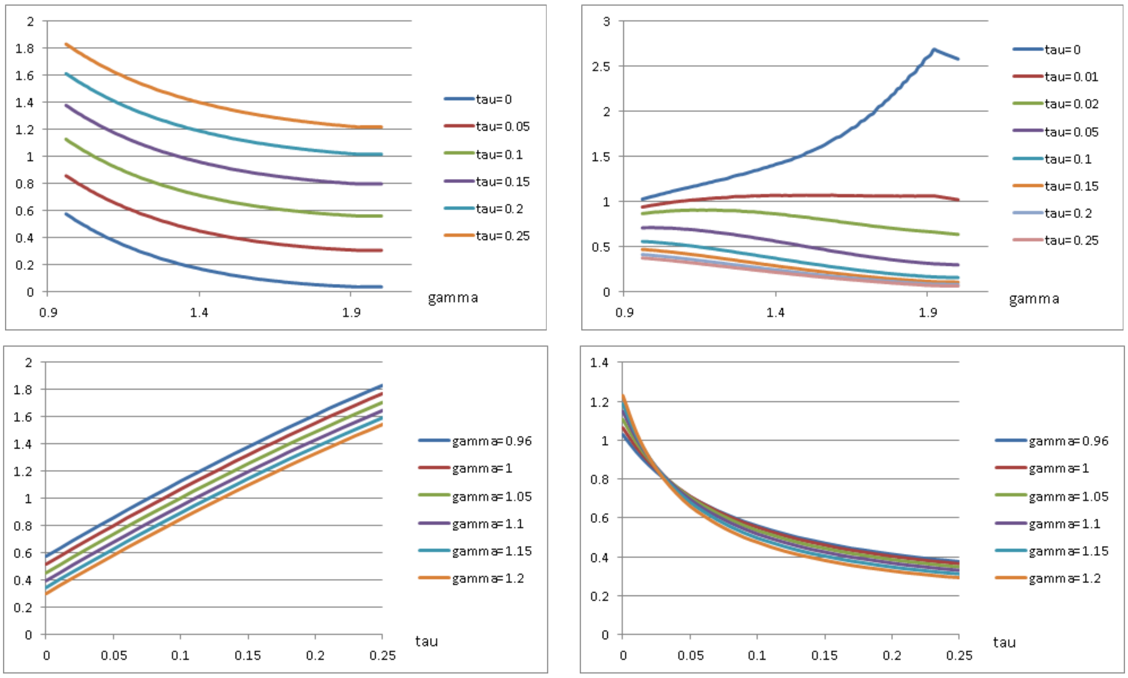


Figure 6. Income and consumption inequalities

A. Income inequality

B. Consumption inequality

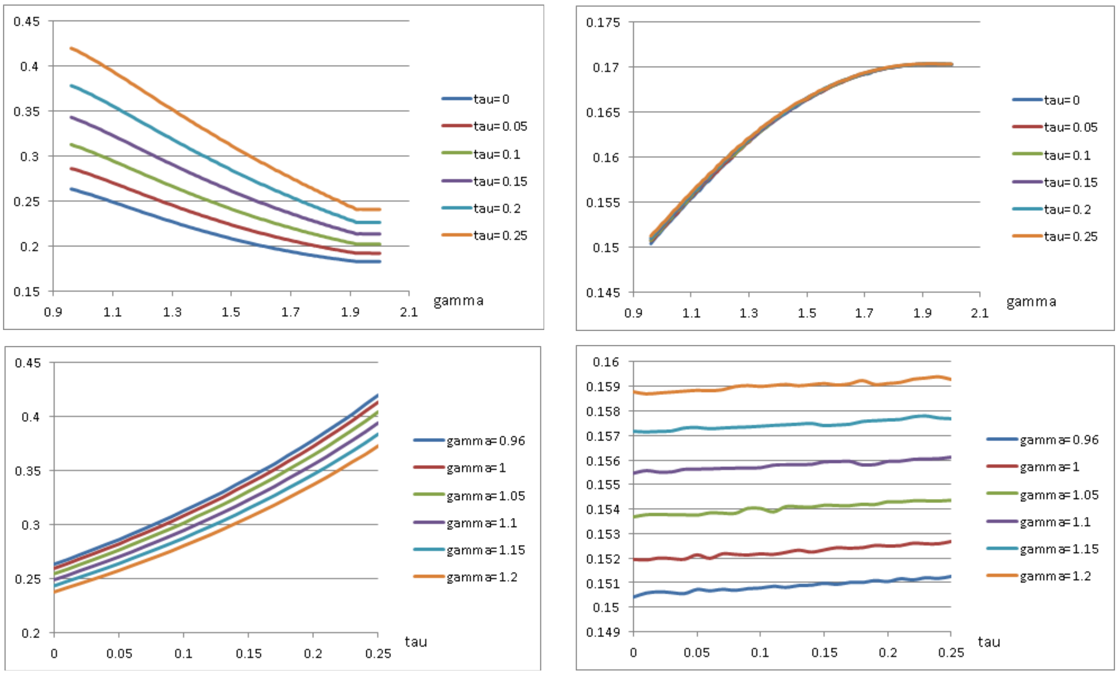


Figure 7. Welfare

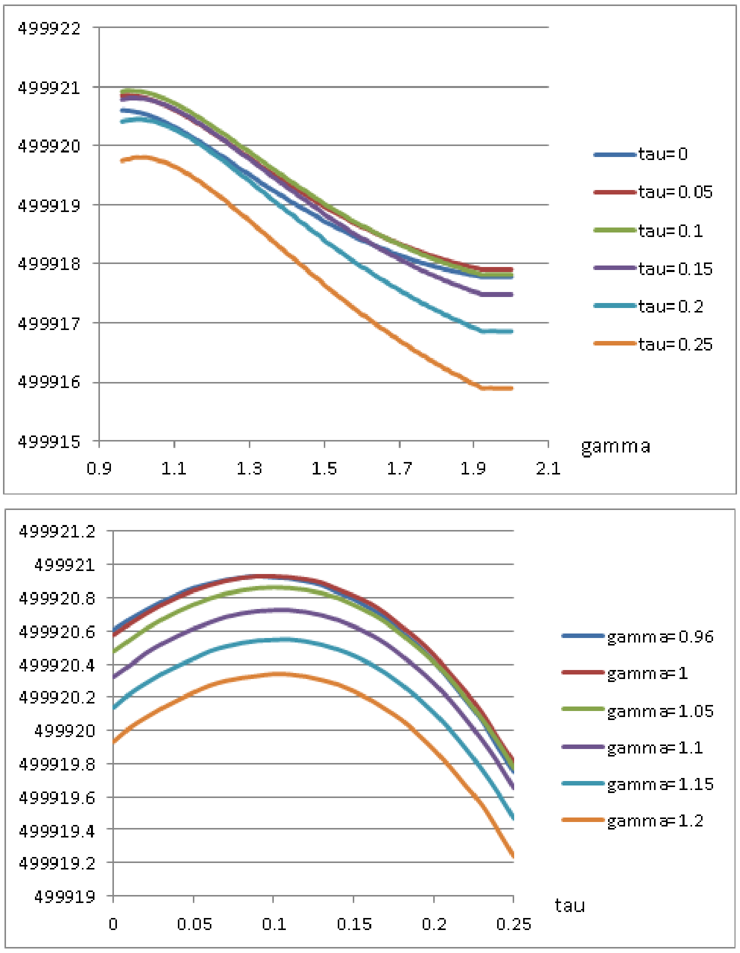


Table 1. Optimal policy

Parameters	γ^*	τ^*		Parameters	γ^*	τ^*
benchmark	0.98	0.1		$\sigma_u = 0.5$	0.99	0.18
$k = 10^{-3}$	0.97	0.1		$U_0 = 12$	0.99	0.11
$k = 0.5$	1	0.07		$U_0 = 100$	0.98	0.07
$\rho = 0.5$	0.98	0.08		$\theta \in [0.8, 1.2]$	0.96	0.03
$\rho = 1.5$	0.98	0.1		$\theta \in [0.3, 1.7]$	1.01	0.17
$\sigma_u = 1.01$	0.98	0.1		$\theta \in [0.2, 1.8]$	1.03	0.22

References

[1] Acemoglu, D. and R. Shimer, 1999. "Efficient Unemployment Insurance," *Journal of Political Economy* **107**, 893-928.

[2] Akyol, A., 2004. "Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk," *Journal of Monetary Economics* **51**, 1245-1269.

[3] Boel, P. and G. Camera, 2009. "Financial sophistication and the distribution of the welfare cost of inflation," *Journal of Monetary Economics* **56**, 968-978.

[4] Camera, G., 2000. "Money, Search and Costly Matchmaking," *Macroeconomic Dynamics* **4**, 289-323.

[5] Camera, G. and Y. Chien, 2011. "Understanding the Distributional Impact of Inflation," Purdue University.

[6] Chiu, J. and M. Molico, 2010. "Liquidity, redistribution, and the welfare cost of inflation," *Journal of Monetary Economics* **57**, 428-438.

[7] Dressler, S., 2011. "Money Holdings, Inflation, and Welfare in a Competitive Market," *International Economic Review* **52**, 407-423.

[8] Erosa, A. and G. Ventura, 2002. "On Inflation as a Regressive Consumption Tax," *Journal of Monetary Economics* **49**, 761-795.

[9] Galli, R. and R. Hoeven, 2001. "Is inflation bad for income inequality: The importance of the initial rate of inflation," Working Paper, the University of Lugano, Switzerland.

[10] Gonzalez, F. M. and S. Shi, 2010. "An Equilibrium Theory of Learning, Search and Wages," *Econometrica* **78**, 509-537.

[11] Imrohoroglu, A., 1992. "The welfare cost of inflation with imperfect insurance," *Journal of Economics Dynamics and Control* **16**, 79-91.

[12] Kiyotaki, N. and R. Wright, 1993. "A Search-Theoretic Approach to Monetary Economics," *American Economic Review* **83**, 63-77.

[13] Lagos, R. and R. Wright, 2005. "A Unified Framework for Monetary Theory and Policy Analysis," *Journal of Political Economy* **113**, 463-484.

[14] Menzio, G. and S. Shi, 2010a. "Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations," *American Economic Review: Papers and Proceedings* **100**.

[15] Menzio, G. and S. Shi, 2010b. "Block Recursive Equilibria for Stochastic Models of Search on the Job," *Journal of Economic Theory* **145**, 1453-1494.

[16] Menzio, G. and S. Shi, 2011. "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy* **119**, 468-510.

[17] Menzio, G., S. Shi and H. Sun, 2011. "A Monetary Theory with Non-Degenerate Distributions," University of Toronto Working Paper 425.

[18] Moen, E., 1997. "Competitive Search Equilibrium," *Journal of Political Economy* **105**, 694-723.

[19] Molico, M., 2006. "The Distribution Of Money And Prices In Search Equilibrium," *International Economic Review* **47**, 701-722.

[20] Rocheteau, G. and R. Wright, 2005. "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica* **73**, 175-202.

[21] Royden, H. L., 1988. "Real Analysis." New York: Macmillan.

[22] Shi, S., 2009. "Directed Search for Equilibrium Wage-Tenure Contracts," *Econometrica* **77**, 561-584.

[23] Wen, Yi, 2010. "Liquidity Demand and Welfare in a Heterogeneous-Agent Economy," manuscript, Federal Reserve Bank of St. Louis.

[24] Williamson, S. and R. Wright, 2010a. "New Monetarist Economics: Models," forthcoming in Benjamin Friedman and Michael Woodford, eds., *Handbook of Monetary Economics* **3A**, Second Edition.

[25] Williamson, S. and R. Wright, 2010b. "New Monetarist Economics: Methods," *Federal Reserve Bank of St. Louis Review* **92(4)**, 265-302.