Firm boundaries and financing with/under opportunistic stakeholder behaviour *

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Abstract

We explore the impact of strategic behaviour between three major stakeholders, namely equity holders, debt holders, and a supplier of a critical input, on the choice of a firm's capital structure and its organisational design, determining in-house production versus outsourcing the procurement of the critical input. We show that an opportunistic coalition of the supplier and debt holders can trigger strategic bankruptcy even when the firm is solvent. Equity holders respond to this by either eliminating the supplier by producing the input in-house, or by reducing the exposure to debt by funding the firm's capital requirement through equity. Both responses create inefficiency since production of the input in-house is costlier and debt is cheaper than equity. We show that the debt-equity ratio in equilibrium varies positively with (a) profitability of the cash flow and (b) marginal cost of the supplier's input, but negatively with (c) riskiness of the cash flow and (d) equity holders' costs of producing the input in-house.

Key Words: Incomplete Contracts, Opportunistic Behaviour, Bankruptcy, Capital Structure.
JEL Classification: D0, C7, G3, L2

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1 Introduction

A vast literature on capital structure examines the interactions between real and financial markets. A common theme in this literature is that a firm may use the composition of its financial claims as an instrument to deal with various imperfections arising from agency problems, informational asymmetries, regulation, non-competitive markets, unions, etc.[1]

More recently, the stakeholder theory of capital structure has been discussed in several theoretical and empirical studies.[2] The basic idea behind this theory is that a firm is a collection of its stakeholders (workers, suppliers, creditors, customers, equity holders, etc.) and the relationships among them determines its financial and investment policies. For example, Titman (1984) argues that, liquidation of a bankrupt firm hurts the long term prospects of its suppliers and customers due to "switching costs" (for example, the costs of locating new business partners. See Williamson (1975)). Naturally stakeholders take such costs into account when deciding to sell inputs or buy from firms running the risks of bankruptcy in the near future. Consequently, the volume of business of highly leveraged firms with exposure to bankruptcy is reduced. The fear of losing long term relationships due to such post-bankruptcy switching costs in turn prompts some firms to reduce their levels of debt. Following Titman (1984), the role of financing decisions in shaping relationships and bargaining power among stakeholders during bankruptcy has been extensively discussed in the literature (see references in the previous footnote).

A common feature in this literature is that each stakeholder acts alone to formulate strategic plans. The possible formation of coalitions of stakeholders has not been recognised in the analysis of a firm's choice of its capital structure. In reality, however, alliances are often attractive. By expanding its mere size and leverage, a coalition of stakeholders may attain more power and hence extract greater benefits than individual stakeholders.

There are many real life examples that illustrate the power of coalitions in the face of bankruptcy. One commonly cited example is the Chapter 11 bankruptcy filing of Olympia and York, where a breakaway coalition of creditors, suppliers and other initial stakeholders took over the former's Canary Wharf venture in London UK, and eventually drove out the original equity holders. The tussle involving the equity holders of LTV and its senior and junior creditors during bankruptcy is another case study that supports this point (*LTV's creditors support bankruptcy exit* 1992).[3]

[1] The earlier literature on this subject began with Jensen and Meckling (1976) and was developed by a series of papers in the 80s and 90s. See Harris and Raviv (1991) and Allen and Winton (1995) for an early summary of this literature. For a more recent survey, see Frank and Goyal (2008).

[2] The following represents a small part of the growing literature in this area. Bronars and Deere (1991), Dasgupta and Sengupta (1993), Perotti and Spier (1993), Hennessy and Livdan (2009), Brander and Spencer (1989), Brander and Lewis (1986), Appelbaum (1993) and Matsa (2010) provide theoretical analysis, whereas Titman and Wessels (1988), Kale and Shahrur (2007) and Banerjee, Dasgupta, and Kim (2008), among others, provide empirical studies of the prevalence of low levels of debt. For a recent survey of this literature, see Parsons and Titman (2008).

[3] Other instances of opportunistic behaviour, bargaining and conflicts among sub-groups of stakeholders (which, at times, resulted in strategic bankruptcy) include the following: (i) the opportunistic multiple party bankruptcy bargaining, involving Wheeling-Pittsburgh Steel Corporation, its United Steelworkers of America workers, and its creditors, (ii) the case of Campeau where suppliers triggered bankruptcy (*Two deadlines intensifying pressures on Campeau* 1990), (iii) the case of GM, where a debt-equity swap deal was initially rejected by a tough bargaining position by the creditors and the unions (*Bondholders push GM to the brink of bankruptcy* 2009), (iv) the case

However the existing literature cited in footnote 2 ignores group behaviour and only considers individual stakeholders in isolation. We attempt to fill this gap by showing that the emergence of stakeholder coalitions becomes critical in the face of bankruptcy, and this may affect a firm's ex-ante choice of capital structure and organisational design. Hence we analyse the choice of capital structure and organisational design made by the firm at the start through a non-cooperative lens, as it anticipates the possibility of coalition formation in the case of bankruptcy.

We endogenise the formation of opportunistic coalitions between three stakeholders, namely equity holders, debt holders and a supplier of a critical input.[4] We examine the co-determination of such coalitions along with the choice of a firm's capital structure and its organisational design. In particular by capital structure we mean the debt-equity ratio, and by organisational design we mean the choice between procuring input from an outside supplier (outsourcing) and producing such input in-house. We analyse a previously undiscovered cost of the co-existence of debt holders and suppliers, which induces the firm to use expensive equity over cheaper debt and expensive in-house production over cheaper outsourcing.

In our set up, the supplier provides a critical input to the production process so he is capable of running the firm with the debt holders. This ability to shut out the equity holders creates a potential for conflict when the firm is on the verge of financial distress and gives rise to a novel cost of debt. This creates a tradeoff between cheap debt on the one hand and the possibility of a strategic stakeholder coalition (summed up in proposition 2) leading to the co-determination of a firm's capital structure and its organisational design as described in the following paragraph.

If a firm issues more debt instead of equity to finance its capital requirements, it benefits as equity is more expensive than debt.[5] On the other hand, the cost of debt is the increased likelihood of strategic bankruptcy triggered by a coalition of the supplier and debt holders, which results in the loss of surplus as equity holders are shut out. The threat of strategic bankruptcy is dealt with in one of two ways – (a) firms with a low capital requirement respond by reducing the debt-equity ratio, whereas (b) firms with a large capital requirement shut out the outside supplier by producing the input in-house, and continue to rely on debt.

In our set up strategic bankruptcy is costly because it is a regime where the supplier pushes the equity and debt holders to what their payoffs would be in case of non-strategic bankruptcy, where non-strategic bankruptcy is the traditional bankruptcy regime where equity holders receive nothing and debt holders receive the value of the residual assets. As a result, in the state of bankruptcy, the equity holders withdraw their inputs thereby reducing the value of the firm. The incremental

of Skeena Cellulose Inc., which after severe financial woes partially attributed to a creditor-labour coalition, was taken over by its creditors, who later formed a joint venture with the British Columbia government as the majority stakeholder, (v) Continental Airlines' use of strategic bankruptcy threats to extract union concessions, and (vi) Texaco's attempts to use strategic insolvency to avoid payment of damages to its arch rival Pennzoil.

[4] In this paper, we use the term outside "supplier" to represent a third party to the contract, who provides an input which is an essential ingredient in the production process. To fix ideas, we think of this party as supplier, but he could also be a skilled worker, investor, or partner.

[5] In order to introduce a meaningful trade-off in the choice of financing, we have assumed that debt is cheaper than equity, which may be due to tax shields or other subsidies in relation to debt financing, or concerns equity holders have about diluting their ownership in the firm.

surplus lost to shareholders from the strategic bankruptcy turns out to be the difference between the cash flow in solvent states (where both equity holders and the supplier contribute inputs jointly) and the same in the state of bankruptcy and liquidation where only the supplier provides input. Essentially the (incremental) surplus lost is equal to the difference of firm value in an ongoing state and the same when the firm is a bankrupt entity. The optimal debt-equity ratio equates this cost of strategic bankruptcy to the benefit of cheap debt at the margin.

Based on this trade-off we derive the optimal debt-equity ratio and show that this depends negatively on the (a) marginal cost of the supplier's input, and (b) riskiness of cash flow but depends positively on the (c) gross profitability of the project, and (d) firm's own marginal cost of production of its own input. These variables affect the optimal debt-equity ratio because they impact either the probability of strategic bankruptcy or the lost incremental surplus in case of strategic bankruptcy.

We show how all possible coalitions between three stakeholders emerge in equilibrium for different configurations of the parameters of our model. When the state of nature is favourable, the firm never encounters a hostile coalition of supplier and debt holders as profits are high enough for debt to be repaid. On the other hand, no coalition can save the firm from going bankrupt when the state of nature is sufficiently unfavourable. However for states of nature between these extremes, the supplier compares his own pay-off between solvency (when he supplies his input jointly with equity holders and shares the pie with both equity and debt holders) and bankruptcy where he provides the input and shares the proceeds with the debt holders alone. Since the supplier alone cannot trigger bankruptcy without forming a coalition with the debt holders, issuing debt ex-ante imposes the cost of potential strategic bankruptcy on the equity holders.

Our paper makes three contributions to the literature investigating the impact of stakeholders' complex relationship on a firm's choice of financing and organizational design. First, we find a novel cost of debt, hitherto unexplored in the literature, arising out of stakeholders' tendency to form coalitions against equity holders during economic distress. This cost leads to a choice of expensive equity financing over cheaper debt. Moreover this cost is relevant only for industries dependent on supply of a critical input. Hence, a primary contribution of this paper is to provide explanations for why supplier dependent industries tend to use a smaller volume of debt in spite of its other benefits. Known as "debt conservatism" or "zero leveraged firms", a large empirical literature establishes that firms eschew cheap debt and issue equity wherever supply considerations of crucial input play a big role.[6] While the existing explanations of the costs of debt including "risk shifting", bankruptcy costs, or debt overhang, etc. hold true in general, the novelty of our result is to show the existence of a cost of debt that particularly arises when suppliers can withdraw a critical input triggering both strategic bankruptcy and destruction of firm value.

[6] Titman and Wessels (1988), Qian (2003), Strebulaev (2007), Strebulaev and Yang (2013), Kale and Shahrur (2007), and Banerjee, Dasgupta, and Kim (2008) among others, have examined the relationship between a firm and its primary stakeholders, such as suppliers and customers, and studied the effect of such relationships on the choice of capital structure. These empirical papers find that firms producing durable goods use specialised and non-substitutable inputs. Such firms that depend heavily on suppliers or supply chains for procurement of inputs tend to underuse debt as a source of external finance.

Second, we show that this possibility of strategic stakeholder behaviour affects not just the composition of financing but organizational design by forcing the firm to inefficiently expand its boundaries by eliminating the supplier and producing its critical input in-house. As a consequence, we derive new empirical predictions described as follows: Beyond a critical size, a firm will exclusively use cheaper debt but then produce its critical input in-house, which could be obtained cheaper outside. However below the critical size, a firm will outsource the production of its critical input but will be forced to use equity over cheaper debt. That is, debt conservatism, as found in the empirical literature would be more common in smaller or medium sized firms while larger firms will rely on debt but produce critical inputs in-house.

Finally, the existing papers on the stakeholders' theory of capital structure explain the strategic advantage of debt as a commitment device where equity holders can use threat of bankruptcy against suppliers. For example, Dasgupta and Sengupta (1993) show that debt emerges as the optimal security since it resolves underinvestment problem associated with bilateral bargaining between the firm and its suppliers when contracts are incomplete. Hennessy and Livdan (2009) show, that the optimal leverage is the outcome of the trade-off between equity holders' increased bargaining power due to higher debt, and the overhang effect which shrinks the set of feasible contracts to the workers/suppliers due to adverse incentive effects. Habib and Johnsen (1999) show that secured debt is the optimal security as it induces efficient re-deployment of an asset in bad states (bankruptcy). In all these papers, individual debt holders are small and act in isolation and not in groups. However once debt holders are large stakeholders (say, bank debt), we show that such advantages disappear and this affects both capital structure and organization of production in a non trivial manner.

Such hold-up problems become even more difficult to manage in multilateral relationships as in our set up. First of all, with multiple contracts, it is necessary to take into account the interactions of all contractual arrangements between the parties. Second and more interestingly, multiple contracting parties may create the potential for multilateral opportunistic behaviour. For example, under certain circumstances, opportunistic behaviour may lead stakeholders to form a coalition whose aim is to exploit the excluded parties.[7] This creates two new difficulties that need to be addressed: (i) opportunistic coalitions may not be easy to prevent, and (ii) an initial contract cannot include, or specify, the terms of a potential future contract between the members of a defecting coalition in the same way that a first marriage contract cannot specify the terms of a potential second marriage contract that may be signed if the first marriage breaks down and the parties re-marry. In this sense, the inability to control future potential coalitions makes initial contracts among the parties "more incomplete". Consequently, efficiency is even harder to achieve in our framework.

[7] In a multilateral contracting environment, a coalition's payoff generically depends on its composition, as well as the composition of its complement. This is referred to as the problem of "externalities" . At this point, there is no general model in the literature on coalition formation that adequately deals with such externalities. For a discussion see Bloch (1996), Shenoy (1979), Yi (1997), and Ray (2007). For a discussion of externalities in the context of multiple contracts, see Segal (1999).

The plan of the paper is as follows. Section 2 outlines the time line of the model. In sections 3 and 4 we analyse the firm's capital structure and post-bankruptcy negotiations under outsourcing and in-house production respectively. Section 5 presents the firm's choice between in-house production and outsourcing. Section 6 concludes.

2 Structure and time-line

Consider a firm that uses three inputs x, y, and K in its production process. The gross profits of the firm are

$$R(x, y, k, \theta) = \begin{cases} R_E(x) + R_S(y) + \theta & \text{if } k \geq K \text{ and } y > 0 \\ \theta & \text{if } k \geq K \text{ and } y = 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where K denotes the minimum capital services required to start the enterprise, x is the input of the equity holder, and a random component θ captures the uncertainty facing the firm. We abuse notation slightly and use θ as both the random variable and its realisation. Input y is a critical input required for production, and it is either sourced from an outside supplier or produced in-house by the firm. Note that input y is critical since the absence of y implies that there is no additional return from investing x. The convex cost functions for x and y in case of outsourcing are denoted by $C_E(x)$ and $C_S(y)$. We assume that $C_E'''(x) \geq 0$, and $C_S'''(y) \geq 0$.[8] The key difference between outsourcing and in-house production is that the costs for x and y in case of in-house production are $\phi C_E(x)$ and $\phi C_S(y)$ where $\phi > 1$. That is, we assume that the costs of production are greater when the input is produced in-house.[9] Furthermore the input of the equity holders and the supplier is non-verifiable and consequently, not contractible.

We assume that the random variable θ, is distributed over $[0, \infty)$ according to a density function $g(\theta)$ that is differentiable, and a corresponding cumulative distribution $G(\theta)$. The functions $R_E(x)$ and $R_S(y)$ are assumed to be differentiable, increasing and concave in x and y respectively, with $R_E(0) = R_S(0) = 0$. We assume that $R_E'''(x) \leq 0$, and $R_S'''(y) \leq 0$. We also assume that $R_S(y)$ satisfies the Inada condition: $\lim_{y \to 0} R_S'(y) = \infty$. Finally, we assume that capital k fully depreciates at the end of the period.

We consider a multi-stage contracting game of symmetric information[10] with uncertainty. The game involves a principal (equity holders) and two agents: debt holders and the supplier.

[8] The signs of these third derivatives ensure that the desired second-order conditions in the equity holders' problem are satisfied.

[9] Our assumption of costlier in-house production of the specialised input is based on the traditional assumption of larger transaction costs, losses in flexibility of in-house production, etc. See for example, Buzzell (1983) and Perry (1989). This is also supported by hypotheses related to information friction within and outside firms as summarised in Lafontaine and Slade (2007). The assumption that in-house production raises the cost of x and y equally by ϕ simplifies the analysis as it ensures that the optimal mix of x and y remains the same with outsourcing and in-house production. However our results remain unaffected if instead we assume that the cost of y increases by ϕ with in-house production whereas the cost of x remains unaffected.

[10] The effects of asymmetric information on the firm's capital structure have been discussed in the literature extensively. See Jensen and Meckling (1976), and Leland and Pyle (1977).

2.1 Time-line

The time line, described in Figure 1, is as follows. At the start the equity holders decide whether to purchase the specialised input from the supplier or produce it in-house. The upper segment of the time line sketches series of events that unfold following the decision to buy the input from an outside supplier, which we describe as "outsourcing". The lower segment describes the events following the decision of the equity holders to produce the input in-house, which is akin to vertical integration. We do not analyse the question of whether the supplier is "bought" by the equity holders and made to produce the input in a vertically integrated set up, or if the equity holders sets up an independent production unit for the input without relying on the supplier's machinery and skills. In our model, both interpretations are feasible and would lead to the supplier being eliminated as a strategic player from the game.

Stage 0: Choice of in-house production or outsourcing In Stage 0, the equity holders choose between outsourcing and in-house production.

Stage 1: Contract with the debt holders In Stage 1, the equity holders make their financing decision. In particular we assume that the enterprise requires a fixed capital investment of K. The financing decision involves the choice of capital structure and is summarised by the contract with debt holders. This contract specifies the level of debt, k_D, and the corresponding payment of D to the debt holders. Both these are endogenously derived. The amount of investment made through equity is then given by $k_E = K - k_D$. The capital market is assumed to be efficient, in the sense that there are no borrowing constraints.

Stage 2: Contract with the supplier In case equity holders choose outsourcing in Stage 0, in Stage 2 they sign a contract with the supplier. They sign an incomplete contract that specifies that the supplier will receive a share, $0 \leq \gamma(\Pi) \leq 1$, of the surplus left over after the debt holders are paid, where the share itself may depend on the realisation of Π, the profits of the firm In addition to profit sharing, they also agree on a side payment, w, to be paid at the signing of the contract. The contract is therefore defined by the pair $\{\gamma(\Pi), w\}$, and is endogenously determined. The contract, however, cannot specify precisely the amount of the inputs x and y as these are assumed to be non-contractible.

Stage 3: Realisation of uncertainty At the "beginning" of Stage 3, uncertainty about θ is resolved (and is common knowledge). Then, after the realisation of θ, in the case of outsourcing, the equity holders and the supplier simultaneously choose inputs x and y, respectively. The equity holders have to meet the legal obligations to the claimants: a payment of D to the debt holders and a share $\gamma(\Pi)$ of the profits to the supplier. The outcome is constrained by the realised state of the world, the previously determined contracts and by the existing legal structure. Specifically, the threat points in this bargaining game and consequently its outcome, are affected by bankruptcy laws and in particular by the seniority of claims.[11]

Depending on the equilibrium coalitional structure (which in turn depends on the state of the world and the legal structure), there are three possible outcomes. First, if the firm can meet all of its obligations we have solvency. In this case, the overall game ends. On the other hand, if the firm's terminal assets, $R(x, y, k, \theta) \geq 0$, are insufficient to meet its obligations in full, the firm is in default and goes into bankruptcy.[12] But, there are two possible "types" of bankruptcy; strategic and non-strategic. Strategic bankruptcy occurs when an otherwise viable firm is forced into bankruptcy by an opportunistic coalition between the supplier and the debt holders. On the other hand non-strategic bankruptcy occurs if the firm cannot meet its obligations due to an unfavourable state of the world (low θ), even when there is no opportunistic behaviour. In either case, if the firm goes into bankruptcy, its assets are distributed in accordance with the seniority of claims. We assume that the debt holders are secured creditors (thus first claimants), where their claim can be applied against $R_S(y) + R_E(x) + \theta$. The supplier is an unsecured creditor, and the equity holders are residual claimants.

In case of in-house production, the equity holders choose both x and y. In this case there is no question of the supplier strategically engineering bankruptcy. There is still the possibility of non-strategic bankruptcy when the revenues of the firm fall below the claims of the debt holders. Just as with outsourcing, we will see that this occurs when the realisation of θ is below an endogenously determined threshold.

Stage 4: Post bankruptcy bargaining With outsourcing, if bankruptcy occurs in Stage 3, the debt holders and the supplier enter into post-bankruptcy bargaining in which the level of the input y to be supplied is renegotiated and the parties' shares of the gains from trade are determined. This stage is absent in the case of in-house production. We now examine the subgame perfect Nash Equilibrium of this game.

We derive the sub-game perfect equilibrium of this game by solving the game backwards. In section 3, we analyse Stages 4 to 1 with outsourcing and in section 4 we do the same with in-house production. Finally, in section 5, we analyse whether the equity holders outsource y or choose in-house production.

[11]Bankruptcy laws in several jurisdictions provide the "guidelines" that govern this bargaining process. The constraints imposed by the bankruptcy rules and the parties' relative strength in the bargaining process, determine the actual payoffs. For example, according to the bankruptcy laws in the US and Canada (See Altman (1983), Willes and Willes (2003), White (1980), White (1989)), secured creditors receive the saleable value of the assets which are subject to security. If the value of the security is insufficient to satisfy the claims, the secured creditor is entitled to claim the remainder as an unsecured creditor. Unsecured assets are distributed, according to the US and Canadian laws in the following order: (i) administrative costs, (ii) taxes, (iii) wages and rents, (iv) unsecured creditors, and finally (v) equity holders. If several claimants have the same priority, they are paid on a pro-rata basis.

[12]See Titman (1984), White (1989), for examples of discussions of the corporate bankruptcy decision. See Aghion, Hart, and Moore (1992), for a discussion of efficient bankruptcy procedures. See also Hart and Moore (1998) for a model of default with renegotiations.

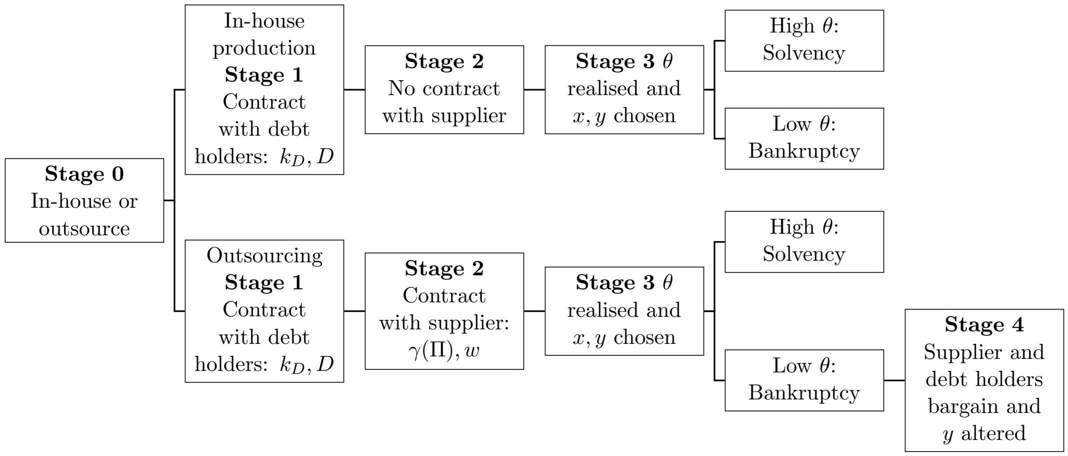

Figure 1: Timeline

3 Outsourcing: Stages 4 to 1

3.1 Stage 4: Post bankruptcy bargaining

If bankruptcy occurs in Stage 3 (the condition under which this happens will be discussed below), the debt holders and the supplier engage in post-bankruptcy bargaining in the last stage. Note that this stage only arises with outsourcing. In this bargaining game, they renegotiate the input y to be supplied. We assume that the supplier makes a take it or leave it offer to the debt holders.

Consider the solution to this bargaining game. The threat payoff of the debt holders is θ since they can get this even when the supplier supplies no input. When y is supplied, the gain from trade is given by $R_S(y) - C_S(y)$. Since the joint surplus, $R_S(y) + \theta - C_S(y)$, is maximised at

$$R'_S(y^*) = C'_S(y^*), \tag{2}$$

it is clear that the bargaining solution must be such that $y = y^*$. The payoffs of the supplier, debt holders, and equity holders are respectively $p(\theta)$, $D(\theta)$ and $e(\theta)$ where

$$\begin{aligned} p(\theta) &= R_S(y^*) - C_S(y^*), \\ D(\theta) &= \theta, \\ e(\theta) &= 0. \end{aligned} \tag{3}$$

Instead of giving full bargaining power to supplier, we could have allowed Nash bargaining between the supplier and debt holders.[13] Our results are unchanged when the supplier's bargaining power is assumed to be $\beta \in (0, 1)$. The outside option of debt holders when the supplier's input is $y = 0$, is

[13] In fact, an earlier version of this paper considered Nash bargaining between the two. The simplification used in the current treatment follows a referee's helpful suggestion. Similarly, we could assume that θ, currently the outside option of the debt holders, is instead shared by the supplier and debt holders. This modification would change the thresholds for strategic bankruptcy but will not qualitatively affect the results.

θ and the additional surplus generated by y is $R_S(y) - C_S(y)$. Therefore the payoffs of the supplier and the debt holders are $\beta(R_S(y) - C_S(y))$ and $\theta + (1-\beta)(R_S(y) - C_S(y))$, respectively. The current formulation is the special case when $\beta = 1$, that is, when bargaining power is allocated entirely to the supplier by allowing him to make a take it or leave it offer to the debt holders. The equity holders' payoffs is 0 since bankruptcy has occurred. If bankruptcy *does not* occur, the game ends in Stage 3.

3.2 Stage 3: Realisation of uncertainty

The state of the world θ is revealed in the beginning of Stage 3. Thereafter the equity holders and the supplier simultaneously choose x and y respectively. Finally the payoffs and the firm's solvency are determined. If the firm can meet its obligations, the game ends in Stage 3. If, on the other hand, the firm cannot meets its obligations, it goes into bankruptcy. Bankruptcy may or may not occur in this stage. If the realisation of the state is very poor, the firm enters bankruptcy and the game continues as outlined in the previous section. On the other hand, if both supplier and equity holders expect solvency, then each chooses his own input to maximise his own payoff, given their respective shares γ and $1-\gamma$. In this case equity holders maximise

$$\max_x \, (1-\gamma)(R_S(y) + R_E(x) + \theta - D) - C_E(x). \tag{4}$$

Similarly the supplier maximises:

$$\max_y \, \gamma(R_S(y) + R_E(x) + \theta - D) - C_S(y). \tag{5}$$

Let $x(\gamma)$ and $y(\gamma)$ be the solutions to the first order conditions such that

$$(1-\gamma)R'_E(x(\gamma)) = C'_E(x(\gamma)) \qquad \text{and} \qquad \gamma R'_S(y(\gamma)) = C'_S(y(\gamma)). \tag{6}$$

Unique solutions $x(\gamma)$ and $y(\gamma)$ are guaranteed to exist by concavity of the revenue functions and convexity of the costs.

Finally note that bankruptcy cannot be triggered when $\theta > D$ since the debt holders receive θ in the post bankruptcy negotiations, which cannot be greater than D, the amount they receive in case of solvency.[14] We are now ready to characterise the possible equilibria in the sub games described in Stage 3 and 4.

$$\begin{aligned} &\text{Define} \quad v_S(x, y; \theta) := \gamma(R_S(y) + R_E(x) + \theta - D) - C_S(y), \\ &\text{and} \qquad v_E(x, y; \theta) := (1-\gamma)(R_S(y) + R_E(x) + \theta - D) - C_E(x) \end{aligned} \tag{7}$$

[14] We could instead assume that the supplier is allowed to push the firm into bankruptcy even when $\theta > D$ by supplying $y = 0$. This is problematic since the debt holders will receive $\theta > D$ in bankruptcy, and this is perverse as bankruptcy is typically thought of as the state where the debt holders don't receive the full amount owed to them. Such a formulation will give the equity holders even greater incentives to avoid debt financing.

as the payoffs of the supplier and the equity holders in case of solvency with inputs x and y, and θ.

Assumption 1. *$v_S(x(\gamma), y(\gamma); \theta)$ is non decreasing in γ.*

This assumption ensures that the payoff of the supplier in the case of solvency is non-decreasing in the share of the profits γ allocated to him.

Lemma 1. *With outsourcing there exists $\theta_L < \theta_H$ such that there is*
a) non-strategic bankruptcy when $\theta \leq \theta_L$,
b) strategic bankruptcy when $\theta_L < \theta < \theta_H$, and
c) solvency when $\theta_H \leq \theta$.

Proof. All proofs in the appendix. □

Figure 2 illustrates the result in Lemma 1. When $\theta \geq \theta_H$, the firm has sufficient funds even in the absence of input by the supplier. As a result there is no bankruptcy. Similarly when $\theta \leq \theta_L$, there is bankruptcy even if the equity holders and the supplier provide the inputs based on the ex-ante contractual arrangement γ. The novel region is $\theta_L < \theta < \theta_H$ when the supplier's input is pivotal for ensuring solvency of the firm. In the following sections we describe each of these three possibilities.

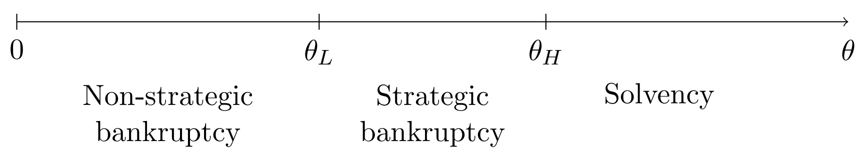

Figure 2: Outcomes for different values of θ with outsourcing

3.2.1 Solvency

The solvency region is when either $\theta_H \leq \theta$. If θ is large enough, the firm can always meet all its obligations to claimants, regardless of how much of y is actually supplied. If the supplier provides $y = 0$ he gets the contracted share $v_S(x(\gamma), 0; \theta)$. But, by supplying $y = y(\gamma)$, he receives an additional $\gamma R_S(y(\gamma)) - C_S(y(\gamma))$. Hence $y(\gamma)$ is chosen by the supplier as it is the optimal input for the supplier when he receives a share γ. The payoff of the supplier, debt holders, and equity holders respectively is

$$\begin{aligned} p(\theta) &= v_S(x(\gamma), y(\gamma); \theta), \\ D(\theta) &= D, \\ e(\theta) &= v_E(x(\gamma), y(\gamma); \theta). \end{aligned} \tag{8}$$

3.2.2 Strategic bankruptcy

Strategic bankruptcy occurs when $\theta_L < \theta < \theta_H$. Within this region, the supplier can *force* the firm into bankruptcy by not supplying his input, since this triggers a missing payment of $D - \theta > 0$ to the debt holders. To determine whether it is indeed in the interest of the supplier to exercise his option to force default, we compare his payoffs if he forces bankruptcy to when he does not. We know that if he forces bankruptcy, he will move to post-bankruptcy bargaining with debt holders in Stage 4. In case of default, at the end of post-bankruptcy bargaining he receives $R_S(y(1)) - C_S(y(1))$ since the optimal effort choice in case of strategic bankruptcy derived earlier is $y^* = y(1)$. Due to Assumption 1, when $\theta < \theta_H$ we have $R_S(y(1)) - C_S(y(1)) > v_S(x(\gamma), y(\gamma); \theta)$ and consequently when $\theta < \theta_H$, he is always better off *actually* forcing bankruptcy.

Thus, in this region the supplier first forces bankruptcy by choosing $y = 0$, and later in Stage 4 chooses $y = y(1)$. The equity holders, anticipating that the supplier will force bankruptcy, supply $x = 0$ since the supplier's input is critical in production. The players' payoffs are the same as the ones described in (3). This case implies that, under some circumstances due to opportunistic behaviour, the firm will not be able to prevent bankruptcy, even if it is viable (in the sense that it can survive if $y(\gamma)$ is supplied). This result is reminiscent of Hart and Moore (1998), where the entrepreneur cannot prevent foreclosure.

3.2.3 Non-strategic bankruptcy

When $\theta \leq \theta_L$ the firm will always default on its obligations to the debt holders, so that the equity holders are out of the picture and supply $x = 0$. The supplier chooses $y = 0$ and the game moves to Stage 4, post-bankruptcy bargaining. Assuming that the sharing rule does not depend on whether the firm went bankrupt, or was forced into bankruptcy, the gains from trade and the outcome of bargaining will be the same as the strategic bankruptcy case. That is $y = y(1)$ and the payoff of parties is the one described in (3).

3.3 Stage 2: Contract with the supplier

In Stage 2, after the contract with debt holders is signed in Stage 1, equity holders sign a contract with the supplier if outsourcing is used. This stage is absent with in-house production. The contract with the supplier specifies that he will provide his services and in return, he will receive a share $\gamma(\Pi)$ of the surplus left over after the debt holders are paid and a side payment of w will be paid at the signing of the contract. The input service, however, is non-verifiable so that the parties cannot sign a contract that is based on the level of y. We allow the share $\gamma(\Pi)$ to be a function of the revenue of the firm.

Consider the firm's problem. Given the contract with debt holders, the outcomes of the multilateral bargaining game and the supplier's choice of $y(\gamma)$, equity holders' receipt in the last stage will be:

$$e(\theta) = \begin{cases} v_E(x(\gamma(\Pi)), y(\gamma(\Pi)); \theta) & \text{if solvency, and} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

Equity holders choose a sharing arrangement, that is the function $\gamma(\Pi)$, and the fixed payment w

that maximises

$$\mathbb{E}(e(\theta)) - w - \rho_E k_E \tag{10}$$

where $\rho_E k_E$ is the capital requirement that is funded through equity decided in Stage 1 (see section 3.4) subject to the participation constraint of the supplier given by

$$\begin{aligned} & & \mathbb{E}(p(\theta)) + w &\geq 0 \\ \iff & & G(\theta_H)(R_S(y(1)) - C_S(y(1))) + \int_{\theta_H}^{\infty} v_S(x(\gamma(\Pi)), y(\gamma(\Pi)); \theta) g(\theta) d\theta + w &\geq 0. \end{aligned} \tag{11}$$

The left hand side of the inequality in (11) is the pay-off of the supplier which is an expectation over bankruptcy and solvency. As shown in (3), the first term is the payoff in the state of bankruptcy multiplied by its probability. The second term is the conditional expectation of the supplier's payoff in case of solvency, as seen in (7).

Note that this is a functional analysis problem since the equity holders choose a function $\gamma(\Pi)$ and a constant w that maximises their payoff. Since w can be used to extract all surplus from the supplier, (11) must bind with equality and the maximisation problem can be analysed in two parts: First, the problem of choosing $\gamma(\Pi)$ that maximises the total surplus, and second, the problem of choosing w such that the participation constraint of the supplier binds. We are now ready to characterise the choice of $\gamma(\Pi)$ by the equity holders.

Proposition 1. *With outsourcing, the equity holders' agreement with the supplier is characterised as follows.*
a) A unique constant γ^* exists that is chosen by the equity holders.
b) The interior solution for γ^* satisfies

$$\frac{1-\gamma^*}{\gamma^*} = -\frac{R'_E(x(\gamma^*))}{R'_S(y(\gamma^*))} \cdot \frac{x'(\gamma^*)}{y'(\gamma^*)} \tag{12}$$

and this is increasing in the return to the supplier's input relative to the equity holders' input.

This proposition establishes that the agreement between the equity holders and the supplier takes the form of $\gamma(\Pi)$ being a constant. This result arises from the additive separability of the firm's revenue in θ. Equation (12) can be rewritten as

$$\varepsilon_{R_E,\gamma} R_E(x(\gamma^*)) = \varepsilon_{R_S,(1-\gamma)} R_S(y(\gamma^*)) \tag{13}$$

where $\varepsilon_{R_E,\gamma}$ is the elasticity of $R_E(x(\gamma^*))$ with respect to γ. The proposition shows that a unique γ exists that maximises the payoff of the equity holders. As expected, this is increasing in the returns from the input of the supplier – it is in the interest of the equity holders to increase the supplier's share as the supplier's input is non-contractible and a greater γ induces a greater input. Since the choice of inputs in case of solvency does not depend on the level of θ, $\gamma(\Pi)$ is constant. The equity holders will take γ^* as given when they make their financing decision.

3.4 Stage 1: Contract with the debt holders

In the first stage, equity holders sign a contract with debt holders. The contract specifies the level debt, k_D and D which is the corresponding payment. Note once again that, for an exogenously given K, the choice of k_D also determines the amount of capital funded through equity since $k_D + k_E = K$. The cost of equity financing is therefore $\rho_E(K - k_D)$ where $\rho_E > 0$ represents the parameter capturing the cost of equity. Similarly the interest rate available to the debt holders if they don't invest in the enterprise is $\rho_D > 0$. We assume that $\rho_E \geq \rho_D$.[15] More generally $\rho_D k_D$ is the minimum interest payment on debt k_D.

The debt holders' participation constraint is,

$$\begin{aligned} \rho_D k_D &\leq \mathbb{E}(D(\theta)) \\ &\leq \int_0^D \theta g(\theta) d(\theta) + (1 - G(D))D \end{aligned} \tag{14}$$

This participation constraint defines the market debt supply function.

Given the outcomes in stages 2, 3, and 4, and noting that $\theta_H := D$, equity holders' receipts are given by

$$\begin{aligned} & \mathbb{E}(e(\theta)) - w - \rho_E(K - k_D) \\ = \; & \int_D^\infty (R_S(y(\gamma^*)) + R_E(x(\gamma^*)) + \theta - D - C_S(y(\gamma^*)) - C_E(x(\gamma^*)))g(\theta)d\theta \\ & + G(D)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D) \end{aligned} \tag{15}$$

Equity holders choose a contract with debt holders to maximise the expected net present value of their receipts (15), subject to debt holders' participation constraint (14), and the supplier's incentive constraint from (6). Note that since we used the supplier's participation constraint from (11) in the derivation of (15), its satisfaction has already been imposed. Furthermore, the optimal contract with the supplier, that is, $\gamma = \gamma^*, w = -\mathbb{E}(p(\theta))$, has also already been taken into account in the derivation of (15). The equity holders' problem is therefore to maximise (15) with respect to k_D and D subject to the participation constraint of the debt holders in (14).

Again, it is clear that the optimal solution does not leave extra surplus for debt holders. In other words, their participation constraint will hold with strict equality:

$$\mathbb{E}(D(\theta)) = \rho_D k_D \tag{16}$$

If we substitute this strict participation constraint into (15) by substituting for k_D we get expected net present value of equity which the equity holders maximise

$$\max_D \mathbb{E}(e(\theta)) + \mathbb{E}(p(\theta)) + \frac{\rho_E}{\rho_D}\mathbb{E}(D(\theta)) - \rho_E K, \tag{17}$$

which is simply the expected net present value of the firm: the expected value of profits, less the true costs of capital and supplier services. We can now show that:

[15] All our results for the case $\rho_E = \rho_D$ also apply to the case when equity is cheaper than debt, that is, $\rho_E < \rho_D$.

Proposition 2. *With outsourcing,*

a) when $\rho_E > \rho_D$ and the objective function is concave, there is a unique optimal $k_D^* \in [0, K]$.

b) when $\rho_E = \rho_D$, the probability of an opportunistic bankruptcy is zero and the optimal capital structure is $k_D^* = 0$ and all capital is financed through equity.

The proposition above gives us an important property of the optimal capital structure, that there is no debt financing, and consequently no strategic bankruptcy in equilibrium, when debt is as expensive as equity. In this case there is no strategic bankruptcy as the supplier has no one to collude with in case of low θ. The proposition also characterises what the optimal capital structure for the firm is. Define

$$\Delta TS := R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - (R_S(y(1)) - C_S(y(1))) \qquad (18)$$

as the increase in the net surplus generated from the equilibrium inputs from the supplier and equity holders as we move from bankruptcy to solvency. When D^*, the solution to (17) is interior, it is characterised by the following first order condition.

$$(\rho_E - \rho_D)(1 - G(D^*)) - \Delta TS g(D^*)\rho_D = 0. \qquad (19)$$

The second-order condition that guarantees concavity is derived in the proof of the proposition in the appendix. Since the participation constraint of the debt holders binds, there is a positive monotonic relationship between k_D^* and D^*. Therefore the properties of D^* that we derive also apply to k_D^*, which is the level of debt financing.

Equation (19) shows the trade-off behind the optimal choice of financial structure. The left hand side is the gain resulting from cheaper debt financing times the probability that the firm is solvent. The cost of debt financing is the loss of the extra surplus due to bankruptcy times the increase in the probability of strategic bankruptcy that such extra debt induces. As such, this is the key tradeoff between choosing to finance the firm's capital requirement through debt vs. equity. First note that an interior solution obtains only if $\rho_E > \rho_D$. When debt is cheaper than equity, on one hand by marginally increasing D, and therefore k_D, which is the capital financed through debt, the firm makes a lower payment relative to equity financing. This is captured in the first term $(\rho_E - \rho_D)(1 - G(D^*))$. On the other hand, increasing D and therefore k_D introduces the possibility of strategic bankruptcy, which is costly. In particular, strategic bankruptcy leads to zero input by the equity holders and consequently a loss of surplus of ΔTS. D^* is the value of D that balances this trade-off. Hence optimal debt is determined by the trade-off between cheaper debt on the one hand and the surplus lost due to strategic bankruptcy on the other. This is illustrated in the example below.

Example If θ is uniformly distributed on the interval $[0, \overline{\theta}]$ we can rewrite the first order condition from (19) as

$$(\rho_E - \rho_D)\left(1 - \frac{D^*}{\overline{\theta}}\right) = \rho_D \frac{\Delta TS}{\overline{\theta}} \tag{20}$$

and

$$D^* = \overline{\theta} - \frac{\rho_D}{\rho_E - \rho_D} \Delta TS \tag{21}$$

We observe that the optimal debt depends negatively on the cost of debt ρ_D, positively on the cost of equity ρ_E, and negatively on ΔTS, the incremental surplus lost in strategic bankruptcy.

ΔTS itself depends on basic parameters like costs of inputs provided by the equity holders and the supplier. Moreover, with a general distribution of θ, the properties of the distribution such as its mean and variance will also affect the optimal level of debt. The proposition below derives the impact of these deep parameters on the equilibrium debt in the form of comparative statics that form the basis of the testable implications of the paper.

Proposition 3. *Assume the interior solution to D^* obtains.*

a) Let distribution of θ improve from $G(\cdot)$ to $F(\cdot)$ with the corresponding densities $g(\cdot)$ and $f(\cdot)$, with $F(\cdot)$ dominating $G(\cdot)$ in the sense of the monotone hazard-rate condition

$$\frac{f(\theta)}{1 - F(\theta)} \leq \frac{g(\theta)}{1 - G(\theta)} \qquad \forall\theta. \tag{22}$$

Then

$$D^*(F(\cdot)) \geq D^*(G(\cdot)) \tag{23}$$

b) When marginal cost of inputs x and y is constant at c_E and c_S respectively, then

$$\frac{\partial D^*}{\partial c_E} \geq 0 \qquad \text{and} \qquad \frac{\partial D^*}{\partial c_S} \leq 0 \tag{24}$$

This proposition shows some comparative statics results that shed light on how the capital structure of the firm varies with the environment. Part a) of the proposition shows that as the distribution of θ improves in the sense of the monotone hazard-rate condition (MHRC), the debt-equity ratio increases.[16]

Note that MHRC implies first order stochastic dominance, which in turn implies second order stochastic dominance. Hence if $F(\cdot)$ dominates $G(\cdot)$ in the MHRC sense, then it must also dominate $G(\cdot)$ in the first and second order stochastic dominance sense.[17] An decrease in the hazard rate $\frac{f(\theta)}{1-F(\theta)}$ implies that the realisation of the random component of the cash flow is more likely to be inclined towards the better states, and this reduces the probability and expected costs associated with strategic bankruptcy. Since the hazard rate bears a negative relationship with the riskiness of cash flow, our model predicts an inverse relationship between the riskiness of cash flow and optimal

[16]Since total capital requirement K is constant, an increase in D implies an increase in k_D and an increase in the debt-equity ratio $\frac{k_D}{K - k_D}$.

[17]See Proposition 4.3 in Wolfstetter (2002).

debt-equity ratio.

Part b) of the proposition shows how the capital structure of the firm changes in the cost of inputs for the equity holders and the supplier. We see that D^* and therefore k_D^*, which is the level of capital financed through debt, is increasing in the marginal cost of equity holders' input and decreasing in the marginal cost of the supplier's input. As the equity holders' input becomes more expensive, it becomes more attractive to rely on the supplier's input. This reduces the cost of strategic bankruptcy which is a state where only the supplier makes an input as the equity holders are shut out. Therefore D^* increases, increasing the likelihood of strategic bankruptcy. Conversely as the input of the supplier becomes costlier, it becomes more attractive to rely on the input of the equity holders, and the state of strategic bankruptcy where only the supplier provides an input, becomes less attractive. Therefore D^* decreases, reducing the likelihood of strategic bankruptcy.

Finally, it is worth summarising the general predictions and empirical implications of the model. Our model identifies a new cost of debt financing. This new cost is due to two factors:

1. The probability of opportunistic behaviour leading to strategic bankruptcy $G(D^*)$.
2. The firm cannot control the contract of a potential post-bankruptcy supplier/debt holders coalition.

Due to this new cost, the firm faces a new consideration that tends to limit its use of debt. Since, in general, there are other motives for using debt, our conclusion simply implies that under these conditions, the debt-equity ratio will tend to be lower. At the same time, it is important to remember that it is the incompleteness of the contract with the supplier that facilitates opportunistic behaviour in the first place.

4 In-house production stages 4 to 1

Note that there is no Stage 4 and Stage 2 with in-house production as the supplier is absent (see Figure 1). Hence we can focus on Stage 3 and Stage 1.

4.1 Stage 3: Realisation of uncertainty

Once θ is realised, as there is no supplier, the equity holders' problem is

$$\max_{x,y} R_E(x) + R_S(y) + \theta - D - \phi C_E(x) - \phi C_S(y). \tag{25}$$

The positive optimal in-house (i) inputs x_i, y_i as the values of x and y that satisfy the first order conditions

$$R_E'(x_i) = \phi C_E'(x_i) \qquad \text{and} \qquad R_S'(y_i) = \phi C_S'(y_i). \tag{26}$$

Hence the solution to the equity holders' problem is to choose

$$x^*, y^* = \begin{cases} x_i, y_i & \text{if } R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i) \geq 0 \\ 0, 0 & \text{otherwise.} \end{cases} \tag{27}$$

With this we can characterise the outcomes in Stage 3 with in-house production.

Lemma 2. *With in-house production there exists a θ_B such that there is*
a) non-strategic bankruptcy when $\theta < \theta_B$, and
b) solvency when $\theta_B \leq \theta$.

Figure 3 illustrates the result in Lemma 2. The characterisation with in-house production is similar to that of outsourcing with the exception that there is no possibility of the supplier engineering bankruptcy with a view to collude with the debt holders after bankruptcy. With in-house production there are two possibilities that we discuss below.

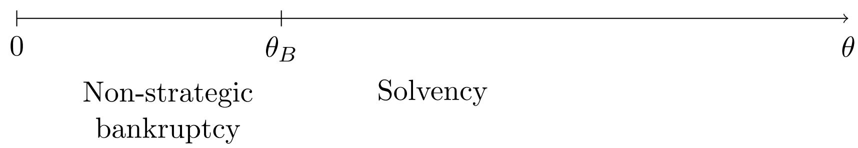

Figure 3: Outcomes for different values of θ with in-house production

4.1.1 Solvency

If the realisation of θ is high enough (greater than θ_B), such that the revenues of the firm are sufficient to meet the debt holders' claim of D, the firm remains solvent. In this case the payoffs of the debt and equity holders are

$$\begin{aligned} D(\theta) &= D \\ e(\theta) &= R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i) \end{aligned} \tag{28}$$

respectively.

4.1.2 Non-strategic bankruptcy

If the realisation of θ is low enough (less than θ_B), such that the revenues of the firm are insufficient to meet the debt holders' claim of D, the firm goes bankrupt. In this case the payoffs of the debt and equity holders are

$$\begin{aligned} D(\theta) &= \theta \\ e(\theta) &= 0 \end{aligned} \tag{29}$$

respectively.

4.2 Stage 1: Contract with the debt holders

Given the outcomes in stages 2, 3, and 4, and noting that $\theta_B := D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)$, equity holders' receipts are given by

$$\begin{aligned} & \mathbb{E}(e(\theta)) - \rho_E(K - k_D) \\ = \; & \int_{\theta_B}^{\infty}(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + \theta - D)g(\theta)d\theta - \rho_E(K - k_D). \end{aligned} \tag{30}$$

Again, it is clear that the optimal solution does not leave extra surplus for debt holders. In other words, their participation constraint will hold with strict equality:

$$\mathbb{E}(D(\theta)) = \rho_D k_D. \tag{31}$$

If we substitute this strict participation constraint into (30) by substituting for k_D we get expected net present value of equity which the equity holders maximise

$$\max_D \mathbb{E}(e(\theta)) + \frac{\rho_E}{\rho_D}\mathbb{E}(D(\theta)) - \rho_E K, \tag{32}$$

We are now ready to characterise the capital structure with in-house production.

Proposition 4. *With in-house production,*
a) when $\rho_E > \rho_D$ and the objective function is concave, there is a unique optimal $k_D^* \in [0, K]$.
b) only equity financing is chosen when $\rho_E = \rho_D$.

The second-order condition that guarantees concavity is derived in the proof of the proposition in the appendix. We observe that even with in-house production, the choice of financing doesn't just depend on whether $\rho_E > \rho_D$. This is because even with in-house production there are realisations of θ, in particular when $\theta < \theta_B$, when sub-optimal $x = y = 0$ is chosen. This would change if we instead assume that the equity holders can renegotiate with debt holders in a way that the equity holders can retain the net returns from their input x and y namely $R_E(x)+R_S(y)-\phi C_E(x)-\phi C_S(y)$. With this modification, the efficient level of $x = x_i$ and $y = y_i$ will always be provided. Moreover, capital will either be financed only through debt if $\rho_E > \rho_D$, or only through equity if $\rho_E = \rho_D$. All other results will remain qualitatively unchanged. We rule out such renegotiation between the equity and debt holders to maintain consistency; since this is what we have also assumed about outsourcing.

5 Stage 0: In-house production or outsourcing

In this section, the equity holders choose between in-house production and outsourcing of the critical input y. We start with analysing how the capital structure varies with $\frac{\rho_E}{\rho_D}$ with outsourcing and in-house production. We assume that

Assumption 2. *There exists* $\gamma \in (0,1)$ *such that*

$$R_S(y(\gamma)) + R_E(x(\gamma)) - C_E(x(\gamma)) - C_S(y(\gamma)) \geq R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)$$

This assumption ensures that in-house production is less efficient compared to outsourcing for at least some γ. By the optimality of γ^* from proposition 2, this assumption will naturally hold for γ^*. The right hand side of the inequality represents the net surplus from in-house production, and this is decreasing in ϕ, the additional cost with in-house production. Therefore the assumption will hold when ϕ is large enough. We make this assumption to make outsourcing more efficient to highlight the trade-off between choosing the more efficient outsourcing on the one hand, and the more strategic bankruptcy proof in-house production on the other. The other obvious benefit of in-house production is that it mitigates the problem of giving incentives to the supplier in the form of share γ. Allowing for this possibility would make in-house production even more attractive. Assumption 2 allows us to shut down this well understood channel and focus instead on inefficiencies caused by the possibility of bankruptcy caused by strategic interaction between the supplier and debt holders.

Proposition 5. *With both outsourcing and in-house production, the debt-equity ratio is weakly increasing in* $\frac{\rho_E}{\rho_D}$*, the relative cost of financing through equity.*

This shows that as the relative cost of equity increases, the amount of debt financing relative to equity financing increases.

We now turn to the choice of outsourcing versus in-house production of the critical input y. Using the payoffs derived from the previous stage, we can compare the profits of the firm, that is the payoff of the equity holders, from in-house production and outsourcing. Define $\overline{D} := D^*(K)$ as the value of D^* with outsourcing when only debt financing is used, so that $k_D^* = K$.

Proposition 6. *When*
a) $\rho_E = \rho_D$*, outsourcing is preferred over in-house production.*
b) $\rho_E > \rho_D$ *and*

$$\begin{gathered} R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) > \max\{\rho_D K, \\ (1 - G(\overline{D}))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) + G(\overline{D})(R_S(y(1)) - C_S(y(1)))\}, \end{gathered} \tag{33}$$

there exists a threshold $\overline{K}$ *such that the firm chooses in-house production when* $K \geq \overline{K}$ *and outsourcing when* $K < \overline{K}$.

Proposition 6 characterises how equity holders choose between outsourcing and in-house production of y. The intuition for the result is the following. Equity financing eliminates the possibility of collusion between the supplier and the debt holders and therefore eliminates strategic bankruptcy. It is therefore the preferred mode of financing when $\rho_E = \rho_D$, that is when equity is as cheap as debt. We see this in row 1 of table 1, which summarises the financing and coalitional outcomes of

Row	$\rho_D < \rho_E$	$K < \overline{K}$	$\theta_H \leq \theta$	Production of y	Financing	Organisational outcome
1	✗	–	–	Outsource	Equity	Supplier-Equity holders
2	✓	✗	–	In-house	Debt	Equity-Debt holders
3	✓	✓	✗	Outsource	Mix	Supplier-Debt holders
4	✓	✓	✓	Outsource	Mix	All three parties

Table 1: Financing and organisational design outcomes

the model. We see that the equity holders choose outsourcing and rely completely on equity when $\rho_D \geq \rho_E$.

When $\rho_D < \rho_E$, the equity holders face a trade-off. Although debt is attractive because it is cheaper, it creates problems of ex-post collusion between the supplier and the debt holders. This tension is resolved in one of two ways: If K is small enough, the equity holders choose outsourcing and typically finance it with a mix of equity and low levels of debt as seen in rows 3-4 of table 1. This is because the cost of using expensive equity is increasing in the level of capital requirement. In this case if the realisation of θ is high enough (row 4), the interest of all three stakeholders is aligned with maximising the firm value, and strategic bankruptcy is avoided in an outcome that we can think of as the grand coalition that includes all stakeholders. If the realisation of θ is low enough (row 3), the supplier triggers strategic bankruptcy by colluding with the debt holders.

If on the other hand, the capital requirement K is large, then the attraction of cheap debt induces the equity holders to choose the less efficient in-house production as seen in row 2 of table 1. Although in-house production is less efficient, it resolves the problem of collusion between the supplier and the debt holders by eliminating the supplier from the picture.

5.1 Discussion

The mechanics of our model can be summarised as follows. Ex-ante the equity holders choose $\gamma = \gamma^*$ as the supplier's share of revenue. Once θ is realised there are two possibilities. If $\theta \geq D$ there are sufficient funds to pay the debt holders and the contract that is made ex-ante can be enforced. If however $\theta < D$, the supplier's effort is needed to ensure solvency and we assume that this shifts the bargaining power to the supplier allowing him to pick γ. Furthermore the setup also assumes that renegotiation between the supplier and equity holders is not possible at this stage, and in particular the supplier cannot offer a side payment to the equity holders for their effort. If this is relaxed, strategic bankruptcy will not lead to inefficiency since the supplier and equity holders will simply stick to the efficient choice of γ^* regardless of the realisation of θ.

Here are some implications of the model:

1) The possibility of opportunistic behaviour between the supplier and debt holders in case of distress implies that the equilibrium capital structure is not the first best. In particular there exists a range where $\rho_E > \rho_D$, and though debt is cheaper, equity financing is chosen to eliminate the possibility of strategic bankruptcy.

2) The first best is feasible only when $\rho_E = \rho_D$, that is when equity is as cheap as debt. When $\rho_E > \rho_D$ there is one of two inefficiencies:
   (a) Either the equity holders substitute away from cheap debt towards costly equity to avoid ex-post collusion between the debt holders and the supplier, or
   (b) The equity holders substitute away from more efficient outsourcing to less efficient in-house production, once again to avoid ex-post collusion between the debt holders and the supplier.
3) The distribution of θ affects whether input y is produced in-house or outsourced. For instance if $G(\theta_H) = 0$, there is no threat of strategic bankruptcy and the equity holders will choose outsourcing. As seen in Proposition 3, as the distribution of θ improves, the likelihood of strategic default decreases, allowing the firm to increase the level of debt.
4) Our analysis perhaps suggests a slightly different interpretation of bankruptcy: bankruptcy is the only mechanism through which departure from the agreed upon allocations can be achieved. In particular, bankruptcy makes it possible for the supplier to increase his share of the payoff from the initially contracted upon $\gamma^* < 1$ to 1. Total payoff decreases as a result of equity holders' zero investment in such case, but the supplier's payoff nonetheless increases. Note that debt holders' payoff also decreases: that $v_E(x(\gamma), y(\gamma); \theta)$ and $v_S(x(\gamma), y(\gamma); \theta)$ are both positive for $\theta_L \leq \theta \leq \theta_H$ means that payoff, in case investment is made, is large enough to permit the servicing of debt, yet debt holders' receive $\theta \leq \theta_H = D$. As such debt holders do not profit from bankruptcy. Of course, the debt holders will not benefit from a regime where strategic bankruptcy is exogenously ruled out since their interest payment would adjust downwards to reflect to the improvement in the probability of repayment.
5) Finally, the likelihood of government bailouts (as in the Skeena case mentioned in the introduction) may have an effect on strategic stakeholder behaviour, the choice of capital structure, and the boundary of the firm. For example, if we add labour as one of the affected parties, losses from a breakup due to a defecting coalition may, depending on the political environment, make it more likely for the government to provide a bailout (this possibility will, of course, be taken into account by the parties involved).

6 Conclusion

We have addressed the issue of stakeholder coalitions during bankruptcy and the impact this has on the ex-ante choice of capital structure and the choice of outsourcing or in-house production of a critical input. We have shown that both decisions are influenced by the risk and cost associated with strategic bankruptcy, triggered by a coalition formed by the supplier and debt holders. Due to friction in the contracting environment, namely the inability of equity holders to control the behaviour of other stakeholders after bankruptcy, equity holders choose a capital structure and firm boundaries to minimise the effect of ex-post strategic stakeholder behaviour. In particular, if the endogenous strategic bankruptcy costs are too high, one of two things happens:

1. The firm shuts out the supplier and produces the critical input in-house even though the cost

of in-house production is greater than outsourcing.

2. The firm shuts out the debt holders and funds the capital requirement through more expensive equity.

Since strategic bankruptcy is the central theme of our paper, it would be interesting to examine how bankruptcy laws in different countries affect strategic stakeholder behaviour. For example, Chapter 11 restructuring allows for an automatic stay on payments of interests, emergency loan provision (debtor-in-possession financing), and a cram-down of reorganisation plans by judges on stakeholders. Our model predicts that such provisions may affect both the capital structure and the degree of vertical integration. It may be interesting to investigate this empirically in future work.

7 To do list

1. $\gamma(\pi)$ rather than $\gamma(\Pi)$.

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A Appendix

Proof of Lemma 1. Recall that

$$\begin{aligned} & v_S(x, y; \theta) := \gamma(R_S(y) + R_E(x) + \theta - D) - C_S(y), \\ \text{and} \quad & v_E(x, y; \theta) := (1 - \gamma)(R_S(y) + R_E(x) + \theta - D) - C_E(x) \end{aligned} \tag{A.1}$$

where $x(\gamma)$ and $y(\gamma)$ are the solutions to the first-order conditions such that

$$(1 - \gamma)R'_E(x(\gamma)) = C'_E(x(\gamma)) \qquad \text{and} \qquad \gamma R'_S(y(\gamma)) = C'_S(y(\gamma)). \tag{A.2}$$

Unique solutions $x(\gamma)$ and $y(\gamma)$ are guaranteed to exist by concavity of the revenue functions and convexity of the costs.

Define $\theta_H := D$ and $\theta_L := \max\{\theta\}$ such that

$$\begin{aligned} &\text{either} \quad v_S(x(\gamma), y(\gamma); \theta) \le 0, \\ &\text{or} \quad v_S(0, y(\gamma); \theta) \le 0 \qquad \text{and} \quad v_E(x(\gamma), y(\gamma); \theta) \le 0 \end{aligned} \tag{A.3}$$

implying

$$\theta_L = \max\{ \frac{C_S(y(\gamma))}{\gamma} + D - R_S(y(\gamma)) - R_E(x(\gamma)), \\ \min\{\frac{C_S(y(\gamma))}{\gamma} + D - R_S(y(\gamma)), \frac{C_E(x(\gamma))}{1-\gamma} + D - R_S(y(\gamma)) - R_E(x(\gamma))\}\} \tag{A.4}$$

Note first that $\theta_L < \theta_H$. This is because

$$v_S(x(\gamma), y(\gamma); D) > 0 \qquad \text{and} \qquad v_E(x(\gamma), y(\gamma); D) > 0. \tag{A.5}$$

We can now analyse the three regions.

- Non-strategic bankruptcy when $\theta \leq \theta_L$.
  1. $v_S(x(\gamma), y(\gamma); \theta) \leq 0$. When $v_S(x(\gamma), y(\gamma); \theta) \leq 0$, the supplier's payoff from supplying positive input is non-positive. Hence $y = 0$ is optimal. When the supplier supplies $y = 0$ the payoff of the equity holders is 0 (since $\theta_L < D$) and hence $x = 0$ is optimal. Since $\theta \leq \theta_L < \theta_H = D$, the profit of the firm is negative and it must go bankrupt.
  2. $v_E(x(\gamma), y(\gamma); \theta) \leq 0$ and $v_S(0, y(\gamma); \theta) \leq 0$. When $v_E(x(\gamma), y(\gamma); \theta) \leq 0$, the equity holders' payoff from supplying positive input is non-positive. Hence $x = 0$ is optimal. When the equity holders supply $x = 0$ the payoff of the supplier is $v_S(0, y(\gamma); \theta) \leq 0$ and hence $y = 0$ is optimal. Since $\theta \leq \theta_L < \theta_H = D$, the profit of the firm is negative and it must go bankrupt.
- Strategic bankruptcy when $\theta_L < \theta < \theta_H$. We define strategic bankruptcy as the state of bankruptcy that is triggered by a less than optimal choice of y by the supplier in Stage 3. This is only possible when $\theta < \theta_H$, since otherwise the debt holders are guaranteed their contractually obligated amount D.

  The supplier's payoff in Stage 4 at the end of post bankruptcy negotiations with the debt holders is $R_S(y(1)) - C_S(y(1))$ from the expression in (3). $v_S(x(\gamma), y(\gamma); \theta)$ is increasing in γ. Since $y(1) = y(\gamma = 1)$ and $\theta < \theta_H$ we have that $R_S(y(1)) - C_S(y(1))$ the payoff from triggering bankruptcy is always greater than $v_S(x(\gamma), y(\gamma); \theta)$, the payoff from supplying input $y(\gamma)$ and ensuring solvency. Note that $x = 0$ is optimal when $y = 0$ since the supplier's input is critical in the production function described in (1).
- Solvency otherwise. When $\theta_H \leq \theta$, we have solvency as debt holders receive full payment D, the supplier receives $p(\theta) = v_S(x(\gamma), y(\gamma); \theta) > 0$, and the equity holders receive $e(\theta) = v_E(x(\gamma), y(\gamma); \theta) > 0$.

□

Proof of Proposition 1. Since (11) binds with equality, and substituting for w in (10) the problem reduces to choosing the function $\gamma(\Pi)$ such that

$$\max_{\gamma(\Pi)} \int_{\theta_H}^{\infty} (R_S(y(\gamma(\Pi))) + R_E(x(\gamma(\Pi))) + \theta - D - C_S(y(\gamma(\Pi))) - C_E(x(\gamma(\Pi))))g(\theta)d\theta \\ + G(\theta_H)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D) \tag{A.6}$$

where

$$\Pi := \begin{array}{l} \int_{\theta_H}^{\infty}(R_S(y(\gamma(\Pi))) + R_E(x(\gamma(\Pi))) + \theta - D - C_S(y(\gamma(\Pi))) - C_E(x(\gamma(\Pi))))g(\theta)d\theta \\ +G(\theta_H)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D) \end{array} \tag{A.7}$$

Since Π itself is a function of γ, we decompose Π into two components, the one that depends on γ and the part that is constant in γ. We find

$$\Pi := (1 - G(\theta_H))(R_S(y(\gamma(\Pi))) + R_E(x(\gamma(\Pi))) - C_S(y(\gamma(\Pi))) - C_E(x(\gamma(\Pi)))) + c \tag{A.8}$$

where

$$c := \int_{\theta_H}^{\infty}(\theta - D)g(\theta)d\theta + G(\theta_H)(R_S(y(1)) - C_S(y(1))) - \rho_E(K - k_D). \tag{A.9}$$

From this decomposition we observe that the optimal $\gamma(\Pi)$ is not affected by the realisation of θ.

We know that the first-order condition for the choice of x by equity holders and y for the supplier will hold. In particular from (5) we must have

$$\gamma(\Pi)R'_S(y(\gamma(\Pi))) = C'_S(y(\gamma(\Pi))) \qquad \text{and} \qquad (1 - \gamma(\Pi))R'_E(x(\gamma(\Pi))) = C'_E(x(\gamma(\Pi))). \tag{A.10}$$

Note that $R_S(y), C_S(y), R_E(x)$ and $C_E(x)$ are unaffected by the value of θ and therefore $\gamma(\Pi)$ is a constant.

To establish the existence and uniqueness of γ^*, note that the first-order condition for the maximisation problem in (A.6) is

$$(R'_S(y) - C'_S(y))|_{y(\gamma^*)}y'(\gamma^*) + (R'_E(x) - C'_E(x))|_{x(\gamma^*)}x'(\gamma^*) = 0 \tag{A.11}$$

and the second-order condition is

$$\begin{array}{l} (R''_S(y) - C''_S(y))|_{y(\gamma)}y'(\gamma)^2 + (R'_S(y) - C'_S(y))|_{y(\gamma)}y''(\gamma) \\ +(R''_E(x) - C''_E(x))|_{x(\gamma)}x'(\gamma)^2 + (R'_E(x) - C'_E(x))|_{x(\gamma)}x''(\gamma) \quad < 0 \end{array} \tag{A.12}$$

The second-order condition is satisfied since using the first-order conditions that define $x(\gamma), y(\gamma)$ in (A.2) we can derive $y''(\gamma)$ and $x''(\gamma)$. These are

$$y''(\gamma) = \frac{2R''_S(y)y'(\gamma) + y'(\gamma)^2(\gamma R'''_S(y) - C'''_S(y))}{C''_S(y) - \gamma R''_S(y)} < 0 \tag{A.13}$$

and

$$x''(\gamma) = \frac{-2R''_E(x)x'(\gamma) + x'(\gamma)^2((1-\gamma)R'''_E(x) - C'''_E(x))}{C''_E(x) - (1-\gamma)R''_E(x)} < 0. \tag{A.14}$$

We can see that $x'(\gamma) < 0$, $y'(\gamma) > 0$ by taking the comparative statics with respect to γ of the first-order conditions in (A.2). Moreover $R'''_E(x) \leq 0, R'''_S(y) \leq 0, C'''_S(y) \geq 0$ and $C'''_E(x) \geq 0$ by the assumptions we've made about the production and the cost functions. Therefore the solution found for γ^* in (A.11) is the unique maximum.

Using the first-order conditions in (A.2) we can substitute for $C'_E(x(\gamma))$ and $C'_S(y(\gamma))$ in (A.11) and we get

$$\frac{1-\gamma^*}{\gamma^*} = -\frac{R'_E(x(\gamma^*))}{R'_S(y(\gamma^*))} \cdot \frac{x'(\gamma^*)}{y'(\gamma^*)} \tag{A.15}$$

Note that $-\frac{x'(\gamma^*)}{y'(\gamma^*)} \geq 0$ since $x'(\gamma) \leq 0$. Holding $-\frac{x'(\gamma^*)}{y'(\gamma^*)}$ constant, note that the value of γ^* is increasing in $\frac{R'_E(x(\gamma^*))}{R'_S(y(\gamma^*))}$, which is the return from the supplier's input relative to the equity holders' input.

Assumption that $R_S(y)$ satisfies the Inada condition ensures that the first-order condition in (A.11) is greater than 0 at $\gamma = 0$. If the condition is always greater than 0, then the solution to γ^* is 1. □

Proof of Proposition 2. Consider the case when $\frac{\rho_E}{\rho_D} = \rho > 1$. From (17), the equity holders's problem is to

$$\begin{aligned}\max_D \quad & \textstyle\int_D^\infty \left(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) + \theta + (\rho-1)D - C_E(x(\gamma^*)) - C_S(y(\gamma^*))\right) g(\theta)d\theta \\ & + \textstyle\int_0^D \left(R_S(y(1)) - C_S(y(1)) + \rho\theta\right) g(\theta)d\theta - \rho_E K.\end{aligned} \tag{A.16}$$

This yields the following first-order condition

$$(\rho-1)(1-G(D^*)) - \underbrace{\left(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - R_S(y(1)) + C_S(y(1))\right)}_{\Delta TS} g(D^*) = 0. \tag{A.17}$$

The second-order condition for the problem to be concave is

$$-(\rho-1)g(D^*) - g'(D^*)\Delta TS \leq 0. \tag{A.18}$$

We only consider the case when the objective function is concave. The objective function is concave when

$$g'(\theta) \geq 0 \qquad \text{or} \qquad \Delta TS \leq -\frac{(\rho-1)g(\theta)}{g'(\theta)} \tag{A.19}$$

With concavity, there are three possible sub cases to consider.

1. If $(\rho-1)(1-G(D)) - \Delta TSg(D) < 0$ for all D, then $k_D^* = 0$ and the entire capital K is financed through equity.
2. If $(\rho-1)(1-G(D)) - \Delta TSg(D) > 0$ for all D, then $k_D^* = K$ and the entire capital K is financed through debt.
3. If there exists a D that satisfies $(\rho-1)(1-G(D)) - \Delta TSg(D) = 0$, then there is a unique interior solution to D (by concavity) and $k_D^* \in (0, K)$.

Next consider the case when $\rho_E = \rho_D$ the equity holders' maximisation problem in (17) simplifies to choosing D to maximise

$$\begin{array}{l}(1-G(D))\left(R_S(y(\gamma^*))+R_E(x(\gamma^*))-C_S(y(\gamma^*))-C_E(x(\gamma^*))\right)\\ +G(D)\left(R_S(y(1))-C_S(y(1))\right)+\int_0^\infty \theta g(\theta)d\theta-\rho_E K\end{array} \tag{A.20}$$

This is a linear programming programming problem in D. Note that $y(1)=y(\gamma=1)$. Since $R_S(y(\gamma^*))+R_E(x(\gamma^*))-C_S(y(\gamma^*))-C_E(x(\gamma^*)) > R_S(y(1))-C_S(y(1))$ by the optimality of $\gamma=\gamma^*$, the optimal $D^*=0$ implying $k_D^*=0$ by (16) and all of capital K is fully financed by equity. □

Proof of proposition 3. Defining $\frac{\rho_E}{\rho_D}=:\rho$, the first-order condition in (19) can be rearranged to

$$\frac{g(D^*)}{1-G(D^*)}=\frac{\rho-1}{\Delta TS} \tag{A.21}$$

where the right hand side is independent of the distribution of θ.

We will first prove part a) of the proposition. When the distribution $F(\cdot)$ dominates distribution $G(\cdot)$ in the sense of the monotone hazard-rate condition, we have

$$\frac{f(D^*)}{1-F(D^*)}\leq\frac{g(D^*)}{1-G(D^*)} \qquad \forall D^* \tag{A.22}$$

The second-order condition that ensures concavity is

$$-(\rho-1)g(D^*)-g'(D^*)\Delta TS\leq 0. \tag{A.23}$$

This implies that $\frac{g(D^*)}{1-G(D^*)}$ and $\frac{f(D^*)}{1-F(D^*)}$ are increasing in D^*. This implies that $D^*(F(\theta))\geq D^*(G(\theta))$ since

$$\frac{f(D^*(F(\cdot)))}{1-F(D^*(F(\cdot)))}=\frac{g(D^*(G(\cdot)))}{1-G(D^*(G(\cdot)))}=\frac{\rho-1}{\Delta TS}. \tag{A.24}$$

We now turn to part b) note that

$$\begin{aligned}\frac{\partial\Delta TS}{\partial c_S} &= \underbrace{((R_S'(y(\gamma^*))-c_S))y'(\gamma^*)+(R_E'(x(\gamma^*))-c_E)x'(\gamma^*))}_{0 \text{ by FOC for } \gamma \text{ in (A.11)}}\frac{\partial\gamma}{\partial c_S}-y(\gamma)\\ &\quad - \underbrace{(R_S'(y(1))-c_S)}_{0 \text{ by FOC for } y(1) \text{ in (2)}}\cdot\frac{\partial y(1)}{\partial c_S}+y(1)\\ &= y(1)-y(\gamma)\geq 0.\end{aligned} \tag{A.25}$$

since $y(1)=y(\gamma=1)\geq y(\gamma)$. Similarly

$$\frac{\partial\Delta TS}{\partial c_E}=\underbrace{((R_S'(y(\gamma^*))-c_S))y'(\gamma^*)+(R_E'(x(\gamma^*))-c_E)x'(\gamma^*))}_{0 \text{ by FOC for } \gamma \text{ in (A.11)}}\frac{\partial\gamma}{\partial c_E}-x(\gamma)\leq 0 \tag{A.26}$$

Hence differentiating the FOC for D^* in (19) and defining $\rho := \frac{\rho_E}{\rho_D}$ we find that

$$\frac{\partial D^*}{\partial c_S} = \frac{g(D^*)\frac{\partial \Delta TS}{\partial c_S}}{(\rho-1)g(D^*) + g'(D^*)\Delta TS} \geq 0 \tag{A.27}$$

and

$$\frac{\partial D^*}{\partial c_E} = \frac{g(D^*)\frac{\partial \Delta TS}{\partial c_E}}{(\rho-1)g(D^*) + g'(D^*)\Delta TS} \leq 0. \tag{A.28}$$

Note that the denominator in (A.27) and (A.28) is positive by the second-order condition for D^* derived in (A.18). □

Proof of Lemma 2. Recall that x_i, y_i are the values of x, y that solve the first-order conditions:

$$R_E'(x_i) = \phi C_E'(x_i) \qquad \text{and} \qquad R_S'(y_i) = \phi C_S'(y_i). \tag{A.29}$$

Define

$$\theta_B := \max\{0, D + \phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i)\}. \tag{A.30}$$

By inspection we see that if $\theta < \theta_B$, the payoff of the equity holders from exerting any positive input is negative. Hence $x = y = 0$ is optimal. In this case $D > \theta$ and consequently the firm must go bankrupt.

Similarly when $\theta_B \leq \theta$ we see that the payoff of the equity holders from supplying x_i, y_i that solve equations in (A.29) is non-negative. In this case the debt holders will be paid in full and the equity holders will claim what remains and the firm will be solvent. □

Proof of Proposition 4. Define $\rho := \frac{\rho_E}{\rho_D}$. First consider the case when $\rho > 1$. From (32), the equity holders' problem is to

$$\begin{array}{rll} & \max\limits_D & \int_{\theta_B(D)}^{\infty} \left(R_E(x_i) + R_S(y_i) + \theta - D - \phi C_E(x_i) - \phi C_S(y_i)\right) g(\theta)d\theta \\ & & +\rho\int_0^{\theta_B(D)} \theta g(\theta)d\theta + \rho(1 - G(\theta_B(D)))D - \rho_E K \\ \iff & \max\limits_D & (1 - G(\theta_B(D)))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) + \int_0^{\infty} \theta g(\theta)d\theta \\ & & +(\rho-1)\int_0^{\theta_B(D)} \theta g(\theta)d\theta + (\rho-1)(1 - G(\theta_B(D)))D - \rho_E K. \end{array} \tag{A.31}$$

This yields the following first-order condition when $\theta_B > 0$

$$(\rho-1)(1 - G(\theta_B(D_i^*))) - \rho\left(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)\right) g(\theta_B(D_i^*)) = 0, \tag{A.32}$$

where D_i^* is the optimal payment to the debt holders in case of solvency when in-house (i) production is used. The second-order condition for the problem to be concave is

$$-(\rho-1)g(\theta_B(D_i^*)) - \rho g'(\theta_B(D_i^*))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) \leq 0. \tag{A.33}$$

We only consider the case when the objective function is concave. The objective function is concave when

$$g'(\theta) \geq 0 \qquad \text{or} \qquad R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) \leq -\frac{(\rho-1)g(\theta)}{\rho g'(\theta)} \tag{A.34}$$

Recall that $\theta_B = \max\{0, D - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i)\}$. There are two cases to consider

1. $\rho_D K - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i) \leq 0$. In this case $\theta_B(D) = 0$ for any choice of $k_D \in [0, K]$, and by inspecting the second part of the expression in (A.31) we see that the optimal choice of $k_D^* = K$ and by equation (31), $D^* = \rho_D K$.
2. $\rho_D K - R_E(x_i) - R_S(y_i) + \phi C_E(x_i) + \phi C_S(y_i) > 0$. An interior solution for k_D^* is now possible. The first-order condition in equation (A.32) can be rewritten as

$$\left(1 - \frac{1}{\rho}\right)(1 - G(\theta_B(D_i^*))) - (R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))\, g(\theta_B(D_i^*)) = 0. \tag{A.35}$$

   There are 3 sub cases to consider:
   (a) If the left hand side of (A.35) is less than 0 for all D, then $k_D^* = 0$ and the entire capital K is financed through equity.
   (b) If the left hand side of (A.35) is greater than 0 for all D, then $k_D^* = K$ and the entire capital K is financed through debt.
   (c) If there exists a D that (A.35) holds, then there is a unique interior solution to D (by concavity) and $k_D^* \in (0, K)$.

Next consider the case when $\rho_E = \rho_D$, that is $\rho = 1$. The equity holders' maximisation problem simplifies to choosing D to maximise

$$(1 - G(\theta_B(D)))\,(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))) + \textstyle\int_0^\infty \theta g(\theta) d\theta - \rho_E K \tag{A.36}$$

Recall that $\theta_B(D) = \max\{0, D + \phi C_E(x_i) + \phi C_S(y_i) - R_E(x_i) - R_S(y_i)\}$. The maximum is attained when $\theta_B(D) = 0$. This is possible when $k_D^* = 0$ by (31) and all of capital K is fully financed by equity.

□

Proof of Proposition 5. Recall that $\rho := \frac{\rho_E}{\rho_D}$. As shown in Propositions 2 and 4, when $\rho \leq 1$, $k_D^* = 0$ for both in-house production and outsourcing. As we increase ρ we need to assume that the inequalities in (A.19) and (A.34) that ensure concavity continue to hold. In this case there is a threshold ρ below which $k_D = 0$ and above which there is an interior solution to k_D defined by the first-order condition in (A.17) for outsourcing and (A.32) for in-house production.

- First, consider outsourcing. In this case we derive the comparative statics of D^* with respect to ρ using (A.17) and find

$$\frac{\partial D^*}{\partial \rho} = \frac{1 - G(D^*)}{\Delta TS g'(\theta) + (\rho - 1) g(D^*)} > 0, \tag{A.37}$$

where $\Delta TS = R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*)) - R_S(y(1)) + C_S(y(1))$. This implies that in this region $\frac{\partial k_D^*}{\partial \rho} > 0$.

- Next, consider in-house production. In this case we derive the comparative statics of D_i^* with respect to ρ using (A.32) and find

$$\frac{\partial D_i^*}{\partial \rho} = \frac{1 - G(\theta_B(D_i^*)) - g(\theta_B(D_i^*))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))}{(\rho - 1)g(\theta_B(D_i^*)) + \rho(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))g'(\theta_B(D_i^*))} > 0, \tag{A.38}$$

  as the numerator is positive due to equation (A.32) and the denominator is positive by the concavity condition in inequality (A.34). Using equations (31) and (A.32), this implies that in this region

$$\rho_D \frac{\partial k_D^*}{\partial \rho} = (1 - G(\theta_B) - g(\theta_B)(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i))) \frac{\partial D^*}{\partial \rho} > 0. \tag{A.39}$$

Finally as we increase ρ there may be a threshold of ρ such that the respective first-order conditions hold for in-house production and outsourcing, such that $k_D^* = K$. For ρ greater than this, k_D^* is invariant to increase in ρ. This implies that k_D is weakly increasing in ρ when the objective function is concave.

□

Proof of Proposition 6. Recall that $\rho := \frac{\rho_E}{\rho_D}$. To begin with note that when $\rho \leq 1$ only equity financing is used, and outsourcing is preferred over in-house production due to assumption 2.

Now consider the case when $\rho > 1$. The payoff for the equity holders with outsourcing is

$$\begin{aligned} \pi_E^o = \; & (1 - G(D^*))(R_S(y(\gamma^*)) + R_E(x(\gamma^*)) - C_E(x(\gamma^*)) - C_S(y(\gamma^*))) + G(D^*)(R_S(y(1)) - C_S(y(1))) \\ & + (\rho - 1)(1 - G(D^*))D^* + (\rho - 1)\int_0^{D^*} \theta g(\theta) d\theta + \mathbb{E}(\theta) - \rho_E K. \end{aligned} \tag{A.40}$$

The payoff for the equity holders from in-house production is

$$\begin{aligned} \pi_E^i = \; & (1 - G(\theta_B))(R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)) + \mathbb{E}(\theta) \\ & + (\rho - 1)\int_0^{\theta_B} \theta g(\theta) d\theta + (\rho - 1)(1 - G(\theta_B))D_i^* - \rho_E K. \end{aligned} \tag{A.41}$$

Since $\theta_B = \max\{0, D_i^* - R_E(x_i) + R_S(y_i) + C_E(x(0))\}$ and $D_i^* = \rho_D K < R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i)$ by (33), we must have $\theta_B = 0$. This simplifies (A.41) to

$$\pi_E^i = R_E(x_i) + R_S(y_i) - \phi C_E(x_i) - \phi C_S(y_i) + \mathbb{E}(\theta) + (\rho - 1)(\rho_E - \rho_D)K. \tag{A.42}$$

Comparing the profits from outsourcing in (A.40) to in-house production in (A.42) we find that the latter is greater when

$$\begin{aligned}
&(1-G(D^*))(R_S(y(\gamma^*))+R_E(x(\gamma^*))-C_E(x(\gamma^*))-C_S(y(\gamma^*)))+G(D^*)(R_S(y(1))-C_S(y(1))) \\
&+(\rho-1)(1-G(D^*))D^*+(\rho-1)\int_0^{D^*}\theta g(\theta)d\theta \\
\leq\ & R_E(x_i)+R_S(y_i)-\phi C_E(x_i)-\phi C_S(y_i)+(\rho_E-\rho_D)K.
\end{aligned} \tag{A.43}$$

To show that this inequality is satisfied when condition (33) holds and $\rho > 1$, we consider 3 cases.

1. $k_D^* = 0$ with outsourcing. In this case D^* for outsourcing is also 0. Consequently (A.43) simplifies to

$$\begin{aligned}
&R_S(y(\gamma^*))+R_E(x(\gamma^*))-C_E(x(\gamma^*))-C_S(y(\gamma^*)) \\
\leq\ & R_E(x_i)+R_S(y_i)-\phi C_E(x_i)-\phi C_S(y_i)+(\rho_E-\rho_D)K.
\end{aligned} \tag{A.44}$$

   Let $\overline{K}_1$ be the value of K such that this condition holds with an equality. By construction, (A.44) will be true for all $K \geq \overline{K}_1$.

2. k_D^* follows the first order condition with outsourcing. In this case differentiating the value function for the equity holders from (A.40) with respect to K, we have

$$\frac{\partial \pi_E^{sc}(D^*(K),K)}{\partial K} = \underbrace{\frac{\partial \pi_E^{sc}(D^*,K)}{\partial D^*}}_{\text{0 by FOC}} \frac{\partial D^*(K)}{\partial K} + \frac{\partial \pi_E^{sc}(D^*,K)}{\partial K}|_{D^*} = -\rho_E. \tag{A.45}$$

   When we differentiate equation the value function of the equity holders for in-house production in (A.41) with respect to K, we have $\frac{\partial \pi_E^{sc}(K)}{\partial K} = -\rho_D$. Let $\overline{K}_2$ be the value of K such that condition (A.43) holds with an equality. $\overline{K}_2$ must exist since both value functions are linearly decreasing in K and $\rho_D < \rho_E$. By construction, (A.43) will be true for all $K \geq \overline{K}_2$.

3. $k_D^* = K$ with outsourcing. We have defined $D^*(K) = \overline{D}$. Since the participation constraint of the debt holders implies that

$$\rho_D K\overline{D} = \int_0^{\overline{D}} \theta g(\theta)d(\theta) + (1-G(\overline{D}))\overline{D}, \tag{A.46}$$

   the inequality (A.43) simplifies to

$$\begin{aligned}
&(1-G(\overline{D}))(R_S(y(\gamma^*))+R_E(x(\gamma^*))-C_E(x(\gamma^*))-C_S(y(\gamma^*)))+G(\overline{D})(R_S(y(1))-C_S(y(1))) \\
&+(\rho_E-\rho_D)K \\
\leq\ & R_E(x_i)+R_S(y_i)-\phi C_E(x_i)-\phi C_S(y_i)+(\rho_E-\rho_D)K,
\end{aligned} \tag{A.47}$$

   which is always true because of the condition in (33).

Define $\overline{K} = \overline{K}_1$ if case 1, $\overline{K} = \overline{K}_2$ if case 2 and $\overline{K} = 0$ if case 3. By construction, outsourcing will be preferred for $K < \overline{K}$ and in-house production will be preferred for $\overline{K} \leq K$. □